

Modeling of cloud processes: Ice microphysics

Wojciech Grabowski

National Center for Atmospheric Research, Boulder, Colorado



Fundamentals of cloud physics

ELEMENTARY CLOUD PHYSICS:

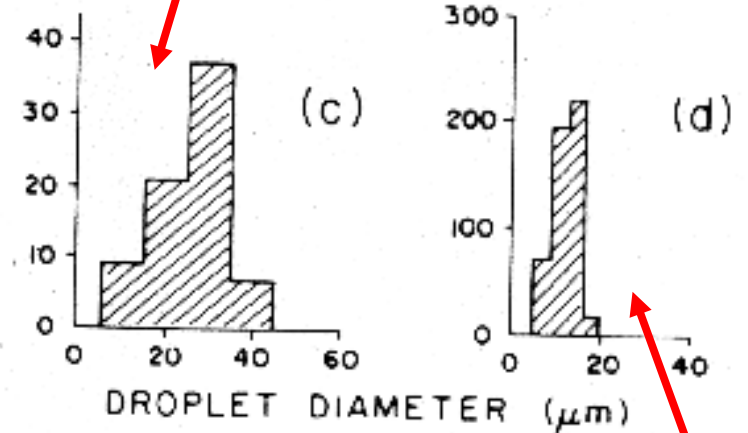
clouds form due to cooling of air (e.g., adiabatic expansion of a parcel of air rising in the atmosphere)

- *condensation*: water vapor \rightarrow cloud droplets

heterogeneous nucleation on atmospheric aerosols called Cloud Condensation Nuclei (CCN); typically highly soluble salts (sea salt, sulfates, ammonium salts, nitrates)

typically, only a small percentage of CCN used by clouds (i.e., water clouds form just above saturation)

Maritime cumulus



Continental cumulus

ELEMENTARY CLOUD PHYSICS, cont.:

- *formation of ice particles*

heterogeneous nucleation on atmospheric aerosols called Ice-forming Nuclei (IN); dominates for temperatures higher than about -40 deg C (233 K); poorly understood; various modes (contact, deposition, condensation-freezing)

IN are typically silicate particles (clays) or other compounds with crystallographic lattice similar to ice, highly insoluble (contact nucleation) or coated with soluble compound (condensation-freezing)

IN are scarce, their number depends strongly on temperature (typically, 1 per liter at -20 deg C, 10 per liter at -25 deg C).

homogeneous freezing is possible once droplet temperature is smaller than about -40 deg C.

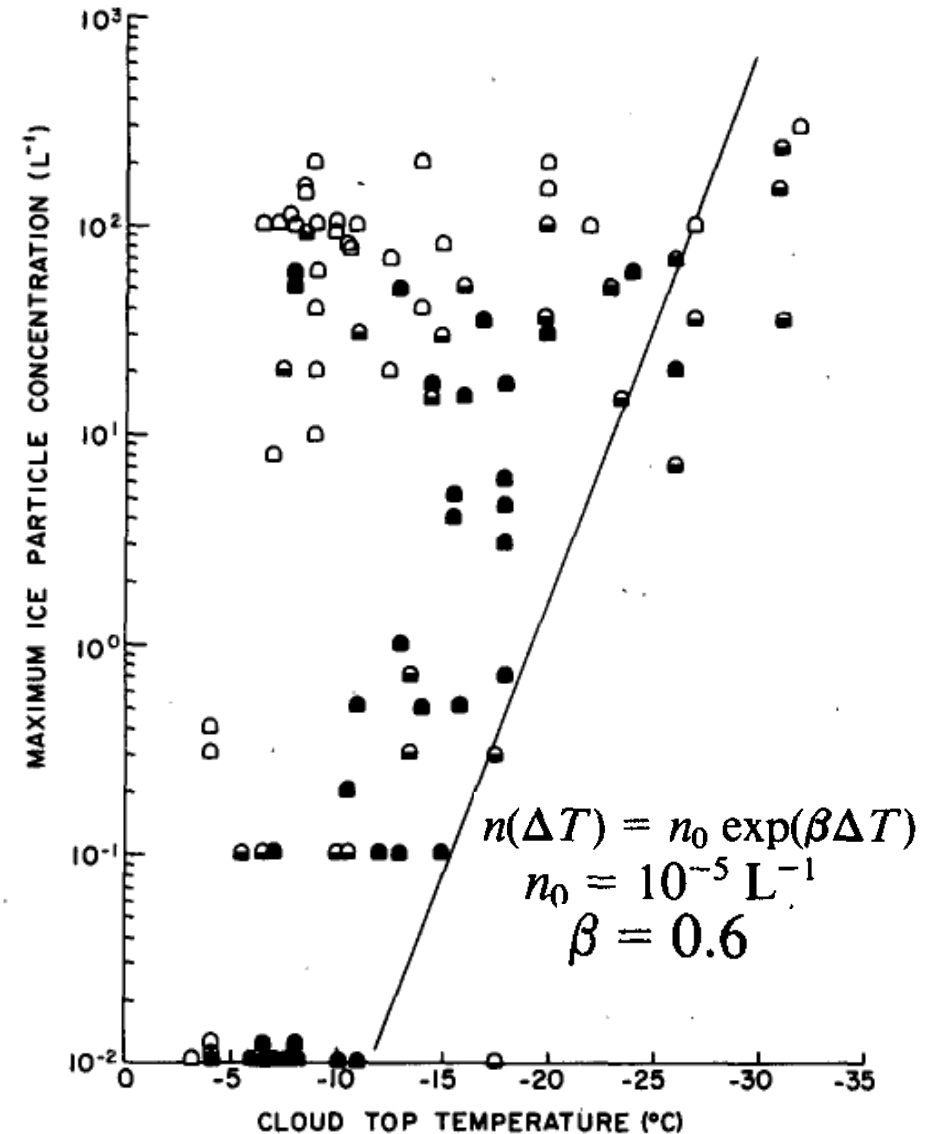


FIG. 2. Measurements of the maximum ice particle concentrations in mature and aging maritime (open humps), continental (closed humps) and transitional (half-open humps) cumuliform clouds. The line represents the concentrations of ice nuclei given by Eq. (1).

Rangno and Hobbs *JAS* 1985

From cloud droplets and ice crystals to precipitation:

WARM RAIN:

→ gravitational collision and coalescence between
cloud droplets

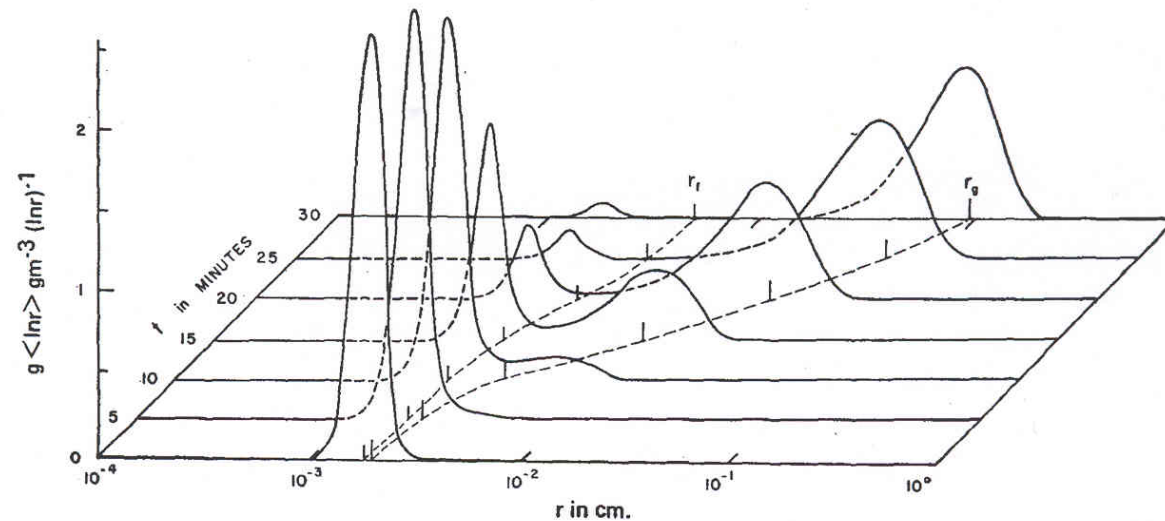


FIG. 5. Time evolution of the initial spectrum for $r_f^0 = 18 \mu\text{m}$, var $x = 0.25$.

THE DISTRIBUTION OF RAINDROPS WITH SIZE

By *J. S. Marshall and W. McK. Palmer*¹

McGill University, Montreal

(Manuscript received 26 January 1948)

$$N_D = N_0 e^{-\Lambda D}$$

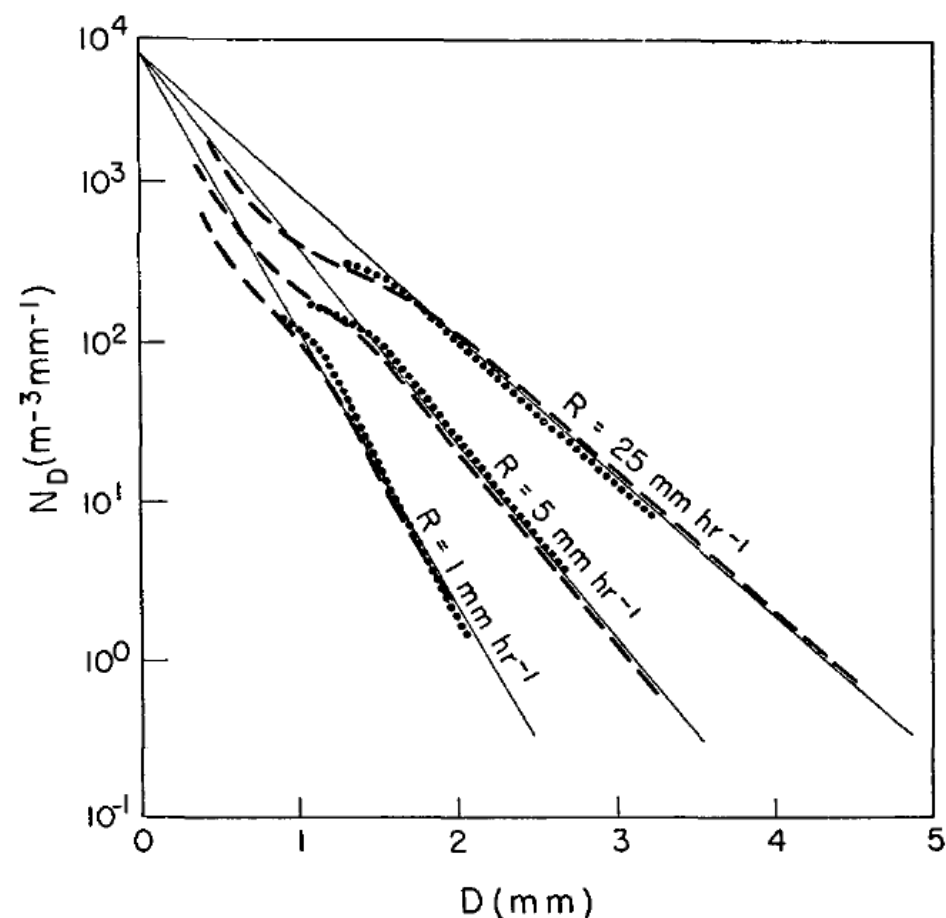


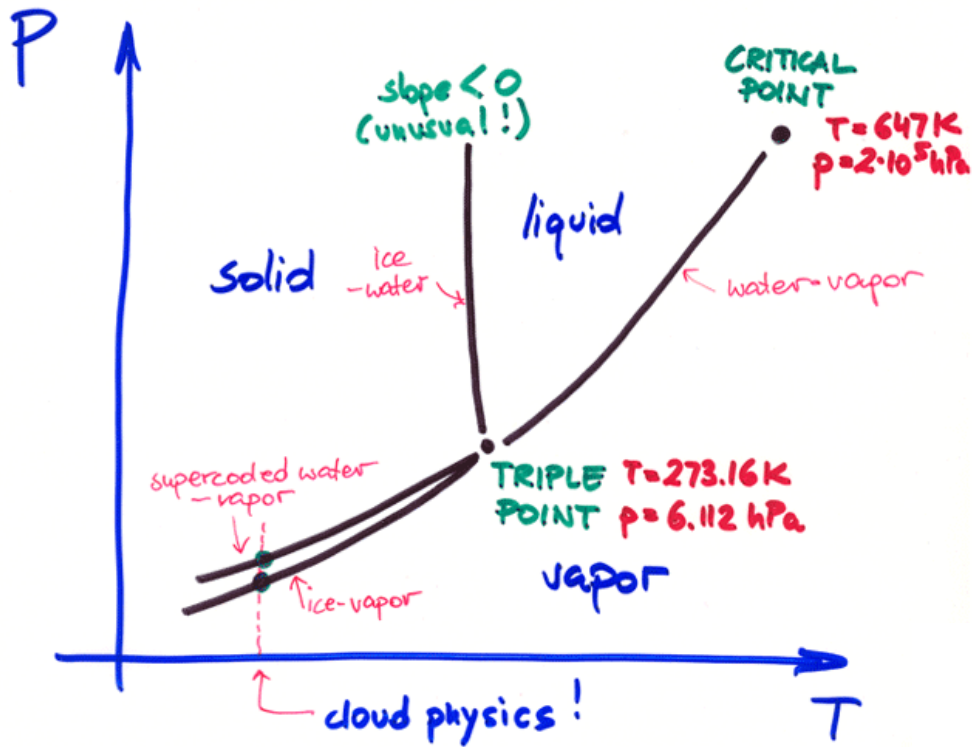
FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

From cloud droplets and ice crystals to precipitation:

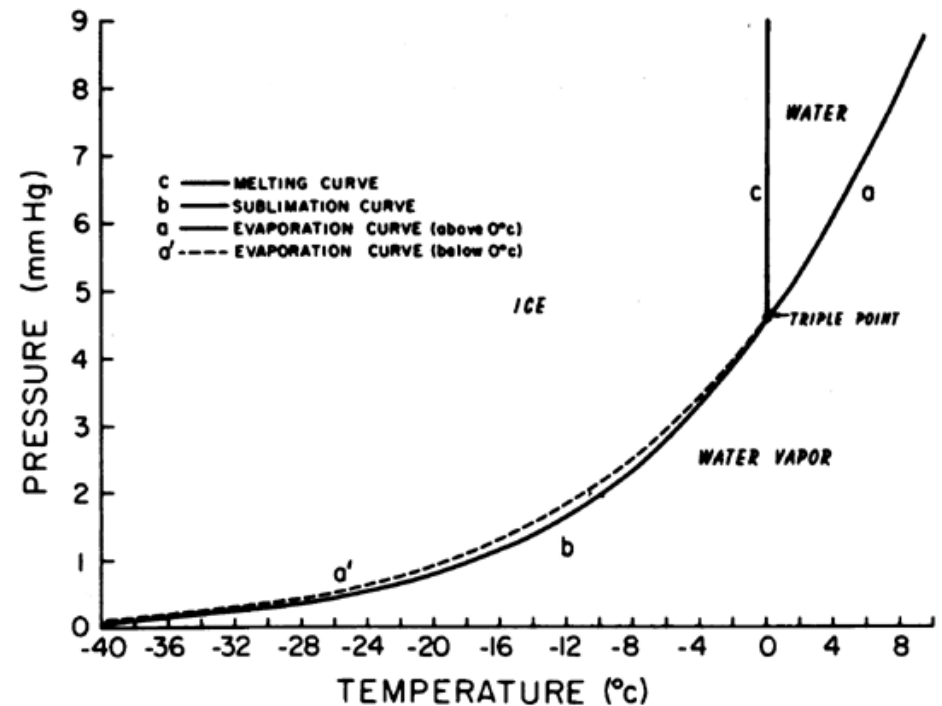
ICE PROCESSES:

→ Findeisen-Bergeron process: water vapor pressure at saturation is lower over ice than over water; it follows that once ice crystal is formed from supercooled droplet, it grows rapidly through diffusion of water vapor at the expense of cloud droplets

→ riming: falling ice crystal collects supercooled droplets that freeze upon contact (graupel, hail, etc).

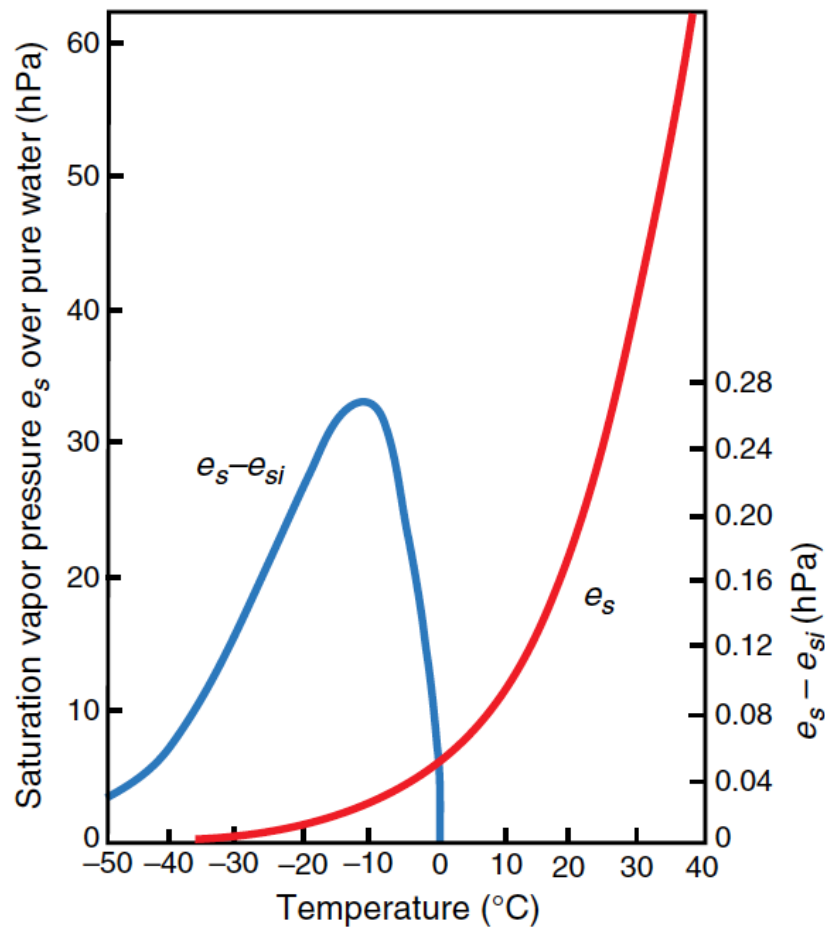


p-T phase equilibrium diagram for water substance



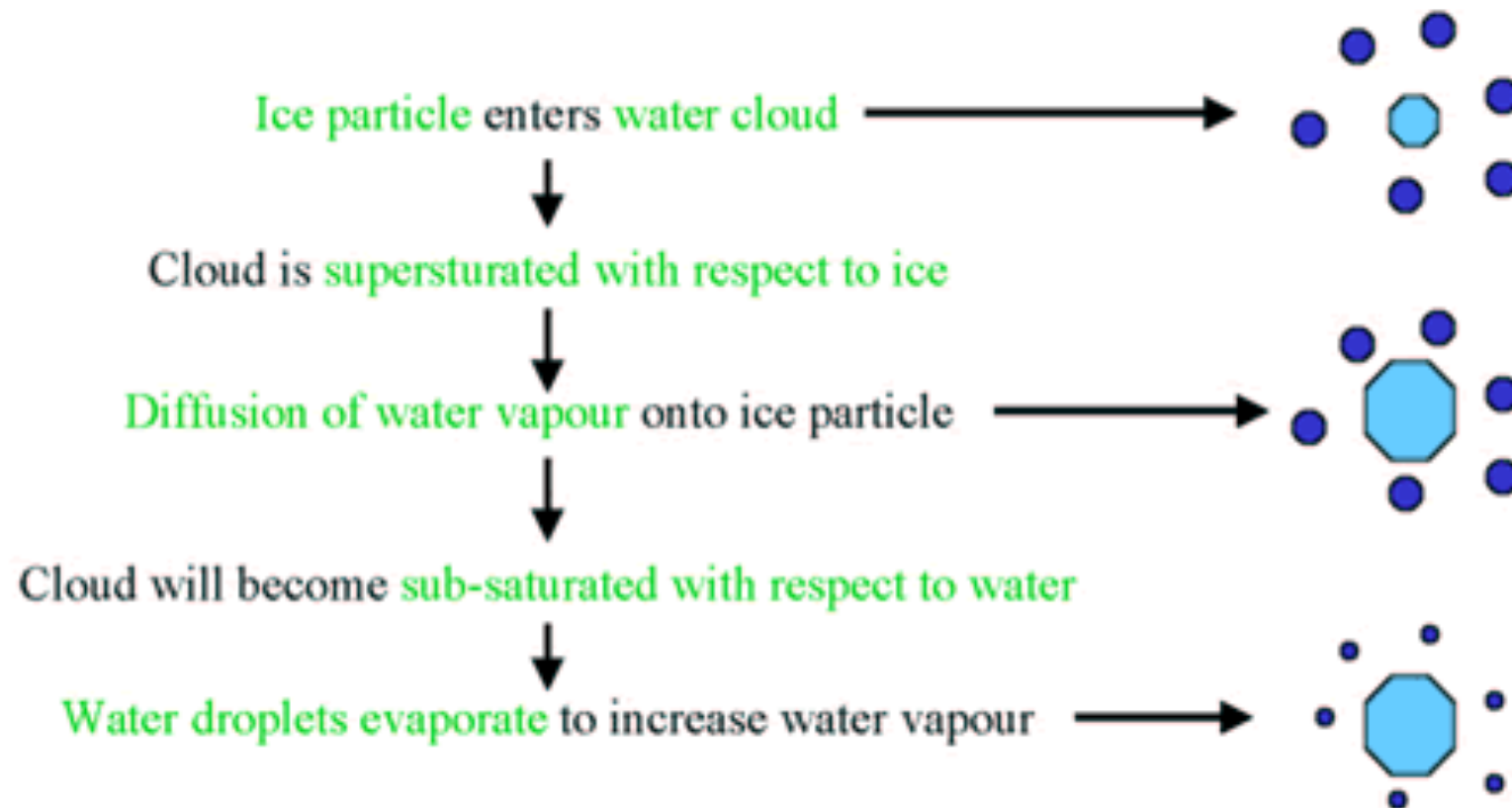
Diffusion growth (C is “capacitance” of particle in units of length, current flow to a conductor analogy for molecular diffusion):

$$\frac{dm}{dt} = 4\pi CD (\rho(\infty) - \rho_s(r))$$



$$\frac{dm}{dt} = \frac{4\pi CD s_{s,i}}{f(L_s, e_{s,i}(T))}$$

Bergeron process



Ice particles grow at the expense of water droplets

EFFECT OF PHASE DIFFERENCE

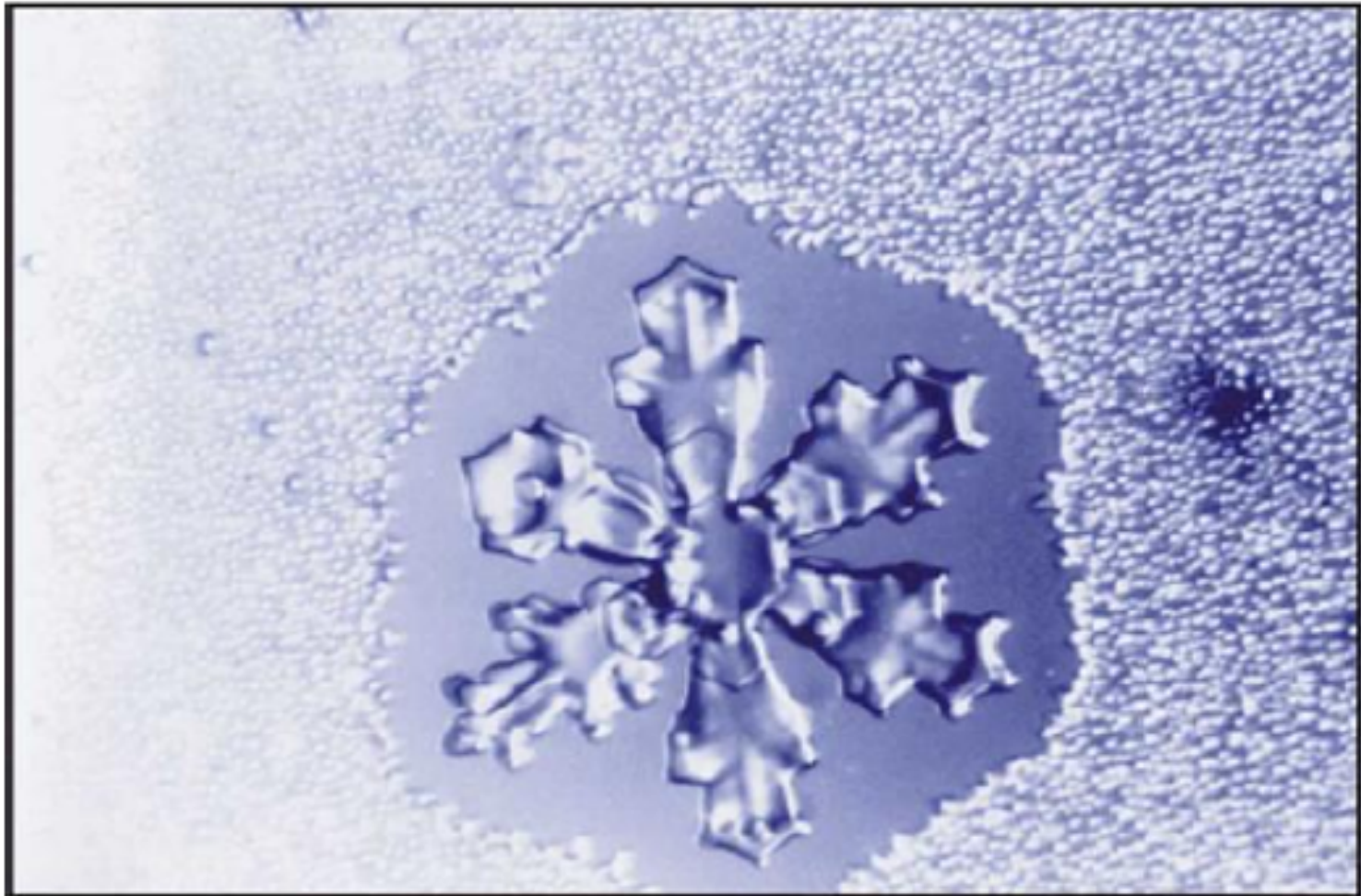
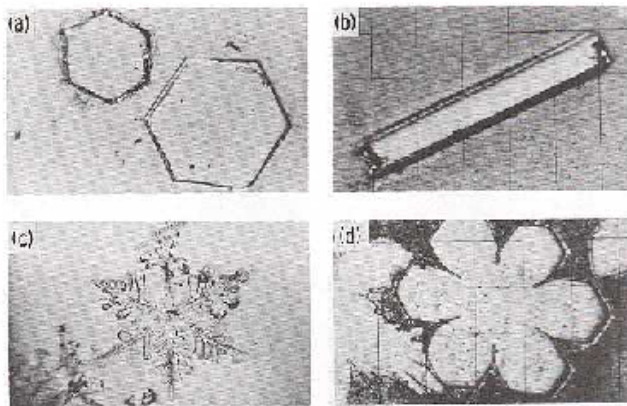


Photo by R. P. tier

Pristine ice crystals,
grown by diffusion of
water vapor (water
vapor between ice-
and water-saturation)



Snowflakes, grown by
aggregation

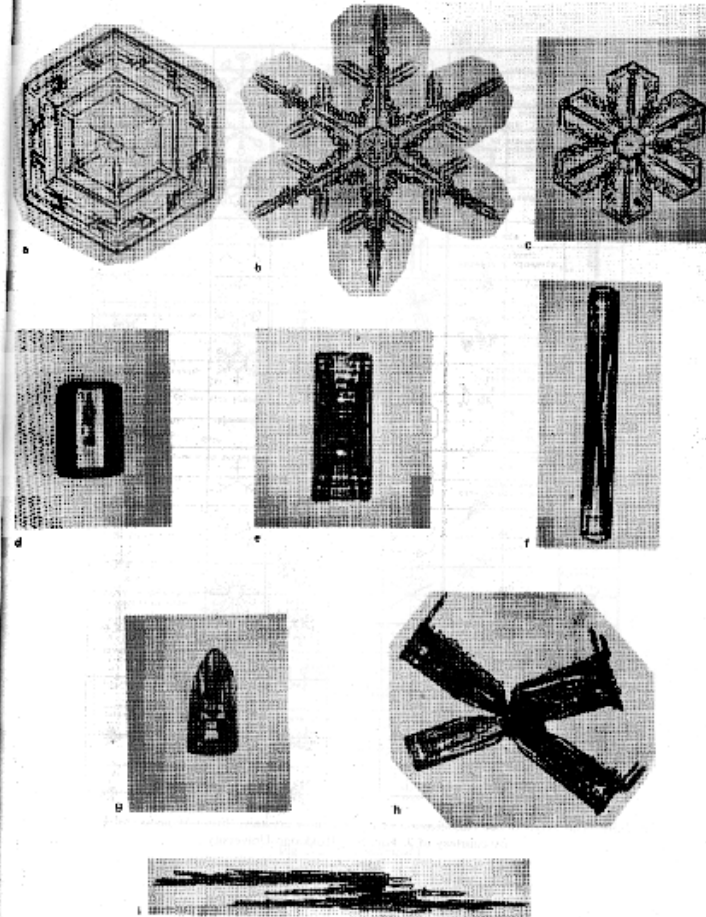
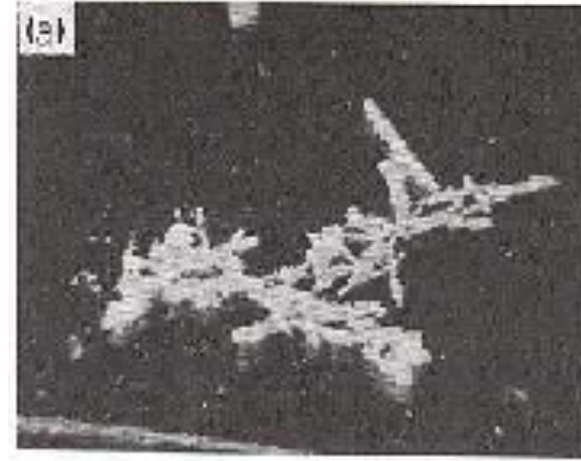
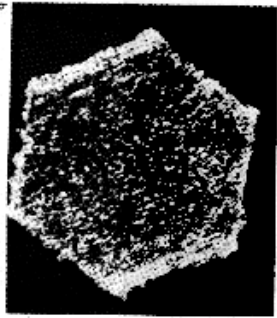
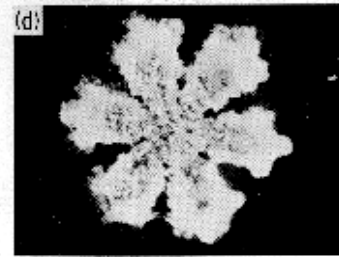
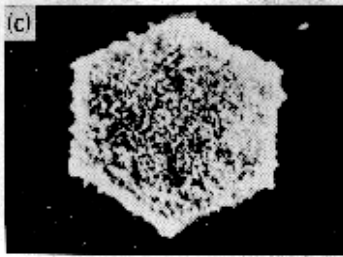


Plate 2. Major shapes of snow crystals: (a) simple plate, (b) dendrite, (c) crystal with broad branches, (d) solid column, (e) hollow column, (f) sheath, (g) bullet, (h) combination of bullets (rosette, Prisenbüschel), (i) combination of needles. (From Nakaya, 1954; by courtesy of Harvard University Press, copyright 1954 by the President and Fellows of Harvard College.)

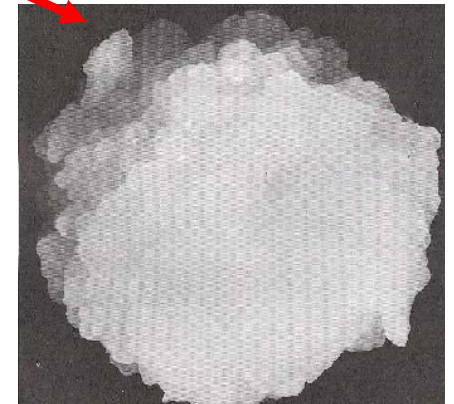
**Rimed ice crystals
(accretion of
supercooled cloud
water)**



**Graupel (heavily
rimed ice crystals)**



Hail (not to scale)



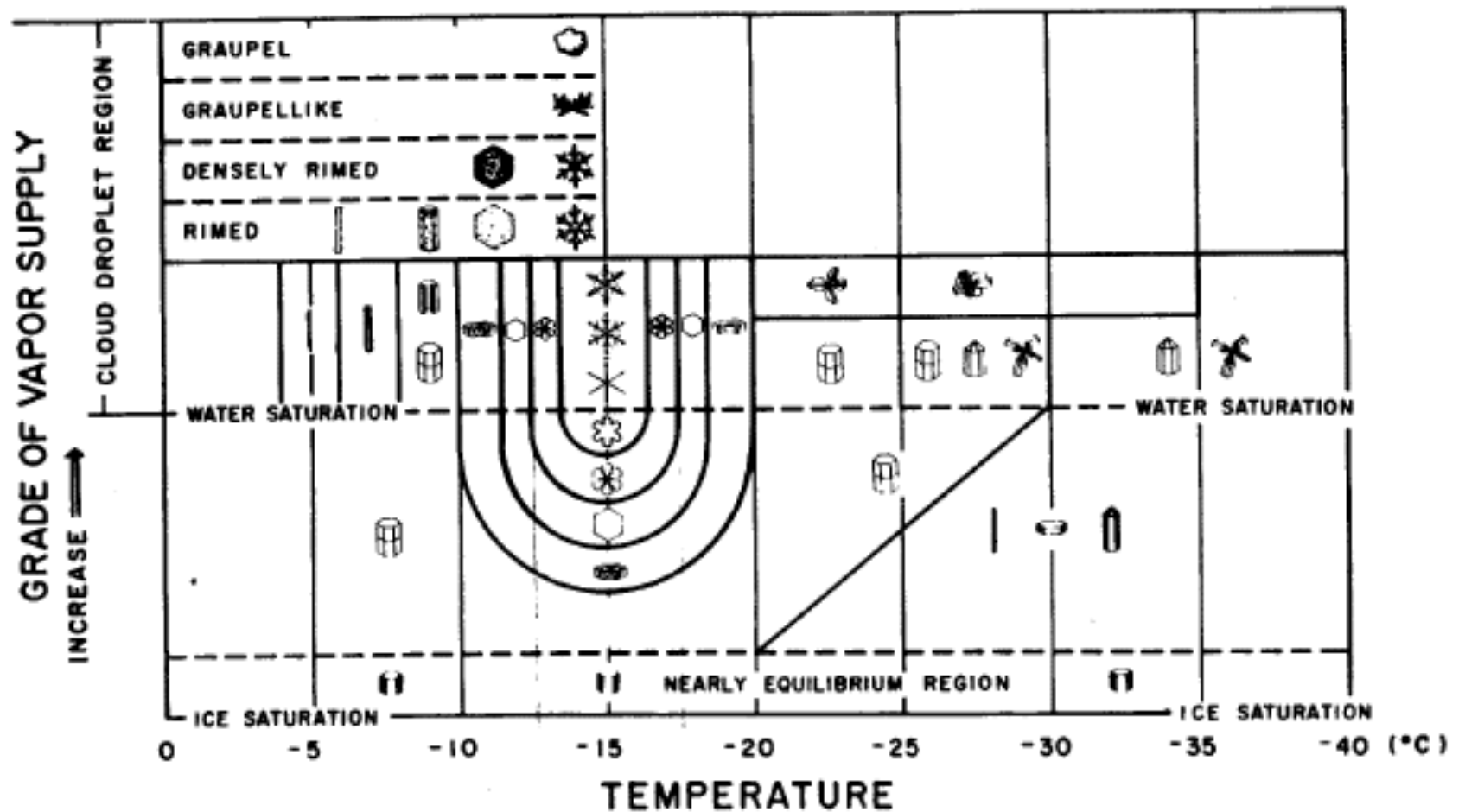
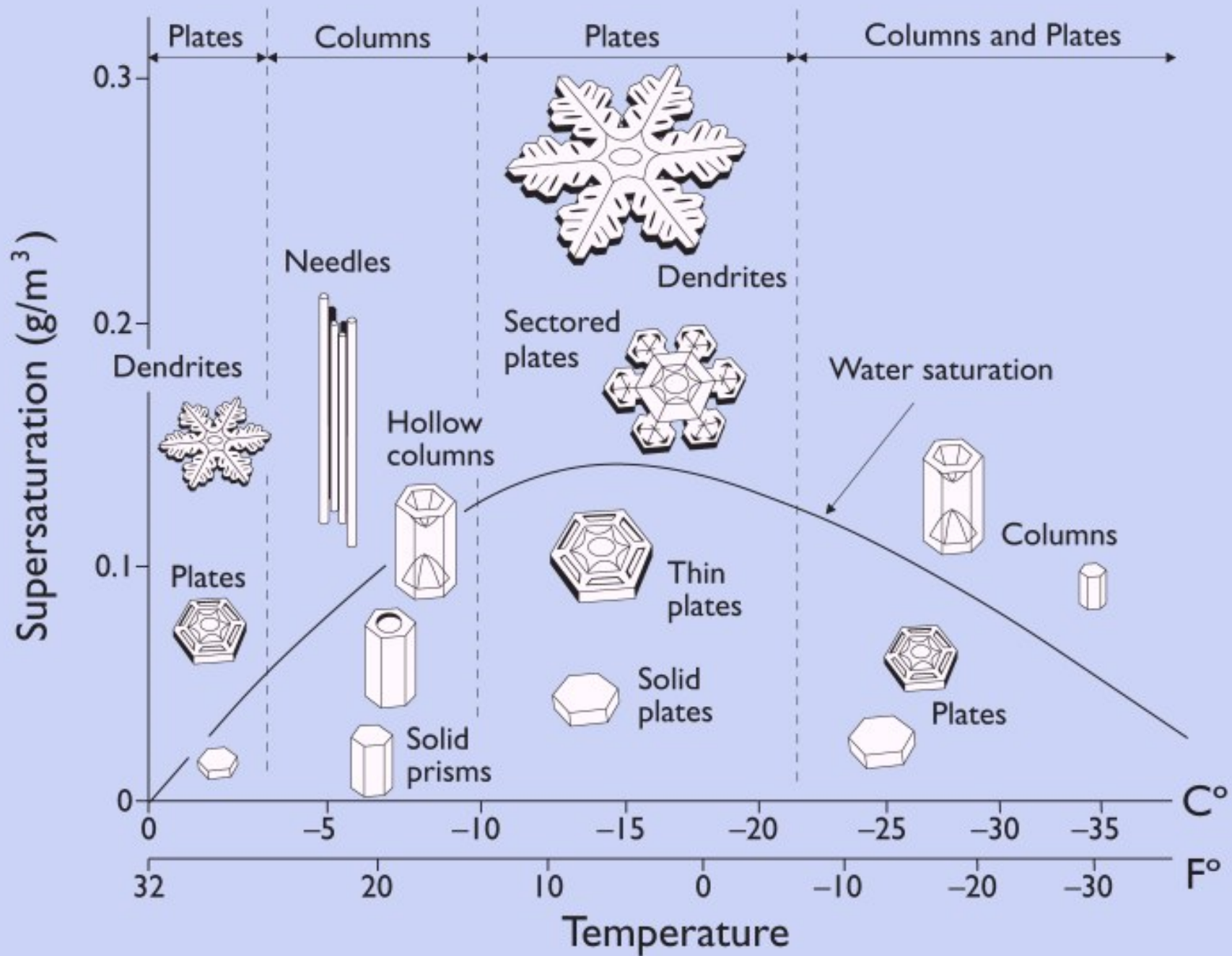
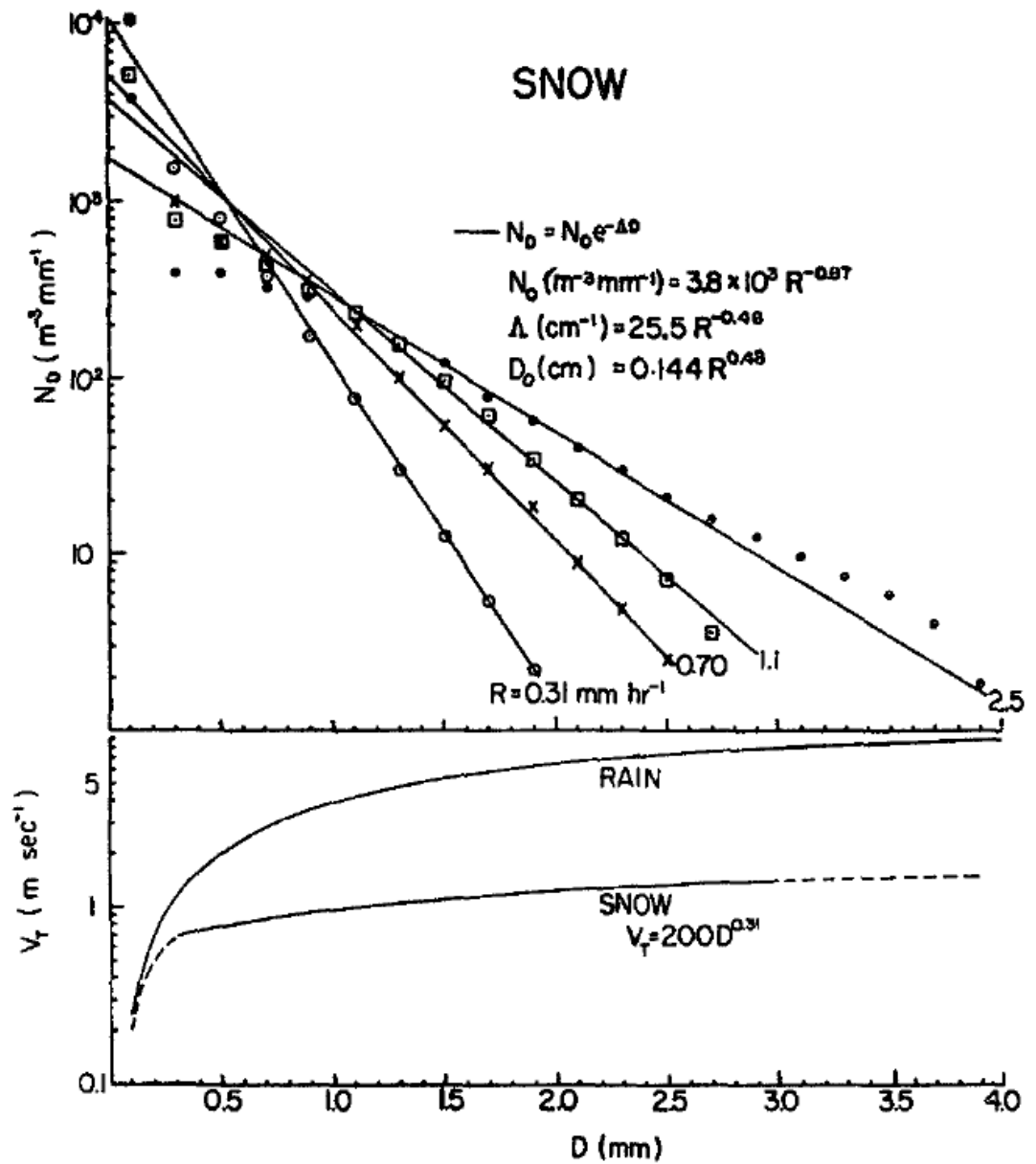


Fig. 2-26. Temperature and humidity conditions for the growth of natural snow crystals of various types. (From Magono and Lee, 1966; by courtesy of *J. Fac. Sci.*, Hokkaido University.)

Magono and Lee (1966) classification of ice crystals and their growth regimes



$$N_D = N_0 e^{-\Lambda D}$$



Gunn and Marshall JAS 1958

Ice processes:

Ice initiation is the main problem:

Primary ice nucleation - freezing of cloud droplets homogeneously for temperatures colder than about -40 deg C and heterogeneously by contact with ice-forming substance

Secondary ice nucleation - the ice multiplication.

Unlike warm-rain microphysics, where cloud droplets and rain/drizzle drops are well separated in the radius space, growth of ice phase is continuous in size/mass space. Both diffusional and accretional growth are important.

Complexity of ice crystal shapes (“habits”).

TABLE 9.1. *Temperatures at which different substances nucleate ice. (From Houghton, 1985)*

Substance	Crystal lattice dimension		Temperature to nucleate ice (°C)	Comments
	<i>a</i> axis (Å)	<i>c</i> axis (Å)		
Pure substances				
Ice	4.52	7.36	0	—
AgI	4.58	7.49	−4	Insoluble
PbI ₂	4.54	6.86	−6	Slightly soluble
CuS	3.80	16.43	−7	Insoluble
CuO	4.65	5.11	−7	Insoluble
HgI ₂	4.36	12.34	−8	Insoluble
Ag ₂ S	4.20	9.50	−8	Insoluble
CdI ₂	4.24	6.84	−12	Soluble
I ₂	4.78	9.77	−12	Soluble
Minerals				
Vaterite	4.12	8.56	−7	(Silicate)
Kaolinite	5.16	7.38	−9	
Volcanic ash	—	—	−13	
Halloysite	5.16	10.1	−13	
Vermiculite	5.34	28.9	−15	
Cinnabar	4.14	9.49	−16	
Organic materials				
Testosterone	14.73	11.01	−2	(Bacteria in leaf mold)
Chloesterol	14.0	37.8	−2	
Metaldehyde	—	—	−5	
β-Naphthol	8.09	17.8	−8.5	
Phloroglucinol	—	—	−9.4	
Bacterium	—	—	−2.6	
<i>Pseudomonas</i>				
<i>Syringae</i>				

- Contact
 - Water droplet freezes instantaneously upon contact with ice nuclei



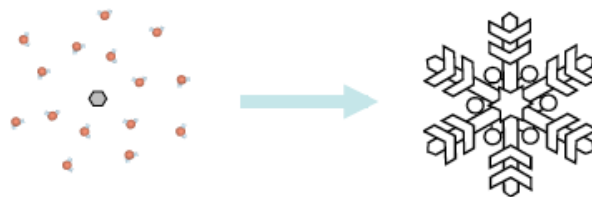
- Condensation followed by instantaneous freezing
 - Nuclei acts as CCN, then insoluble component freezes droplet



- Immersion
 - Ice nuclei causes freezing sometime after becoming embedded within droplet

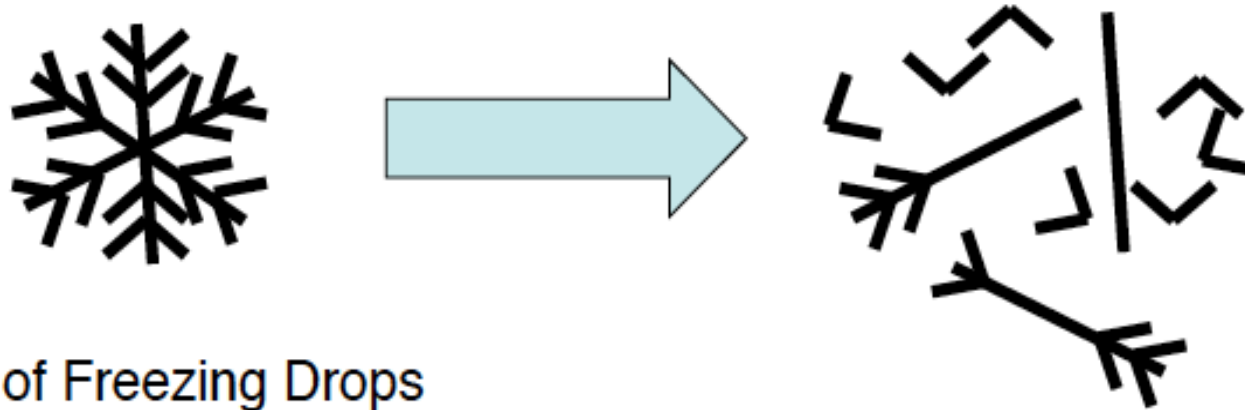


- Deposition
 - Ice forms directly from vapor phase



Ice Multiplication Process

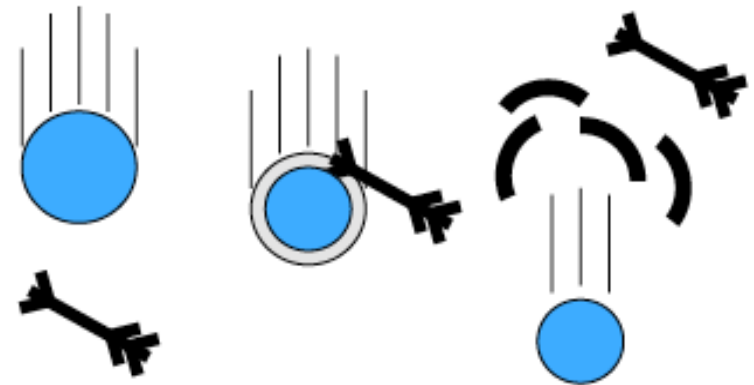
- Fracture of Ice Crystals



- Splintering of Freezing Drops

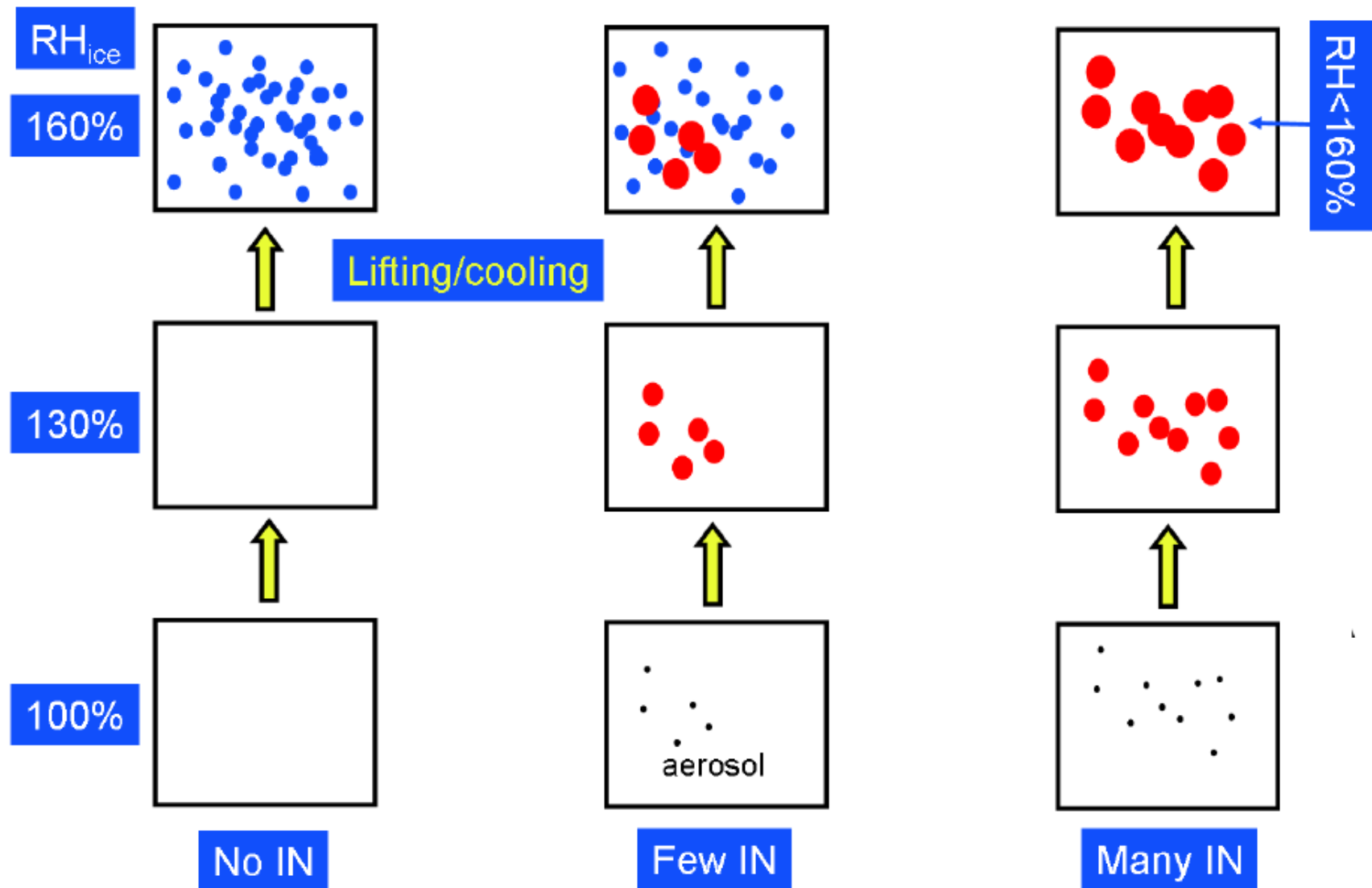
During ice particle riming under very selective conditions:

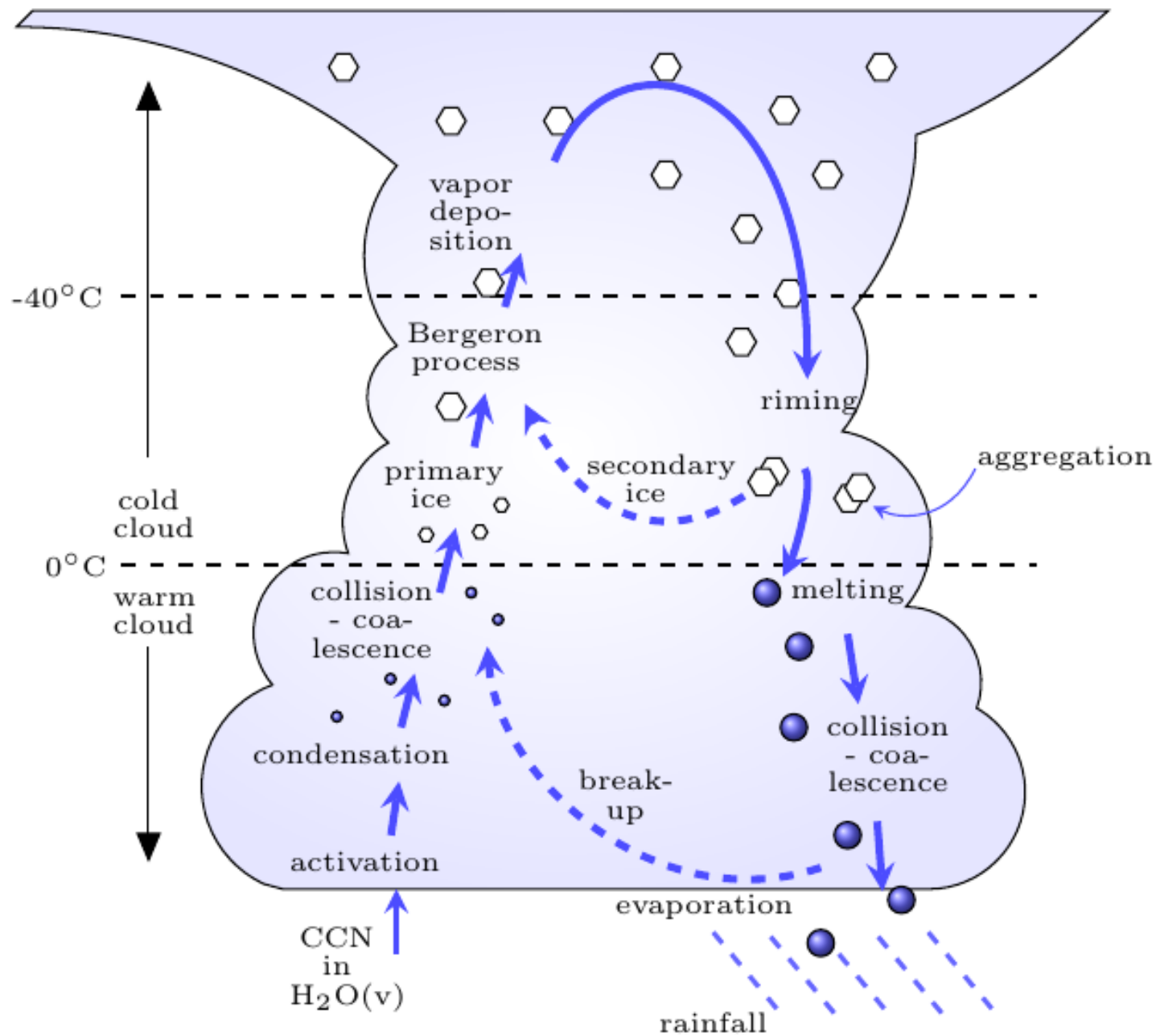
1. Temperature in the range of -3° to -8°C .
2. A substantial concentration of large cloud droplets ($D > 25\ \mu\text{m}$).
3. Large droplets coexisting with small cloud droplets.



Hallet-Mossop process

Ice processes provide much more diversity in particle growth and different pathways to precipitation. Competition between homogeneous and heterogeneous ice initiation is part of it...





Microphysical processes in a deep convective cloud (U. Lohmann, ETH).

Bulk ice physics modeling

- equilibrium approach: a simple extension of the warm-rain scheme
- non-equilibrium approach: more comprehensive schemes
- single-moment versus multi-moment schemes

Bin ice microphysics

Lagrangian (particle-based) methods

OUTLINE:

Bulk ice physics modeling

- equilibrium approach: a simple extension of the warm-rain scheme
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- single-moment versus multi-moment schemes

Bin ice microphysics

Lagrangian (particle-based) methods

Toward Cloud Resolving Modeling of Large-Scale Tropical Circulations: A Simple Cloud Microphysics Parameterization

WOJCIECH W. GRABOWSKI

National Center for Atmospheric Research, Boulder, Colorado*

(Manuscript received 9 September 1997, in final form 9 February 1998)

ABSTRACT

This paper discusses cloud microphysical processes essential for the large-scale tropical circulations and the tropical climate, as well as the strategy to include them in large-scale models that resolve cloud dynamics. The emphasis is on the ice microphysics, which traditional cloud models consider in a fairly complex manner and where a simplified approach is desirable. An extension of the classical warm rain bulk parameterization is presented. The proposed scheme retains simplicity of the warm rain parameterization (e.g., only two classes of condensed water are considered) but introduces two important modifications for temperatures well below freezing: 1) the saturation conditions are prescribed based on saturation with respect to ice, not water; and 2) growth characteristics and terminal velocities of precipitation particles are representative for ice particles, not raindrops. Numerical tests suggest that, despite its simplicity, the parameterization is able to capture essential aspects of the cloud microphysics important for the interaction between convection and the large-scale environment. As an example of the application of this parameterization, preliminary results of the two-dimensional cloud-resolving simulation of a Walker-like circulation are presented.

**Equilibrium approach: a simple extension
of the warm-rain scheme)**

J. Atmos. Sci.

Equilibrium approach (a simple extension of the warm-rain scheme)

SIMPLE BULK ICE MODEL (Grabowski JAS 1998):

$$\frac{D\theta}{Dt} = \frac{L_v\theta}{c_p T} (COND - DIFF)$$

$$\frac{Dq_v}{Dt} = -COND + DIFF$$

$$\frac{Dq_c}{Dt} = COND - AUTC - ACCR$$

$$\frac{Dq_p}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho q_p v_t) + AUTC + ACCR - DIFF$$

θ - potential temperature

q_v - water vapor mixing ratio

q_c - cloud condensate (water or ice) mixing ratio

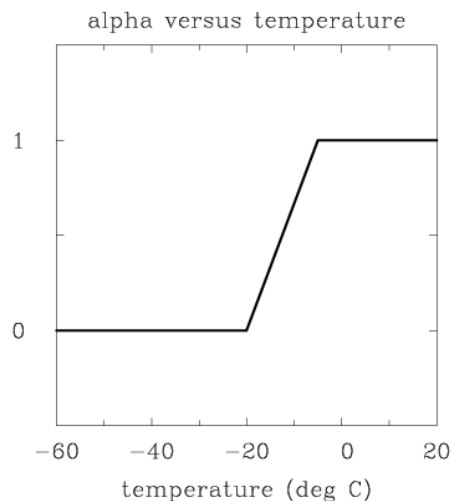
q_p - precipitation water (rain or snow) mixing ratio

$COND$ - condensation rate (saturation adjustment)

$DIFF$ - diffusional growth rate

$AUTC$ - “autoconversion” rate: $q_c \rightarrow q_p$

$ACCR$ - accretion rate: $q_c, q_p \rightarrow q_p$



saturation: $q_{vs} = \alpha q_{vw} + (1 - \alpha) q_{vi}$

cloud water: $q_w = \alpha q_c$; cloud ice: $q_i = (1 - \alpha) q_c$

rain: $q_r = \alpha q_p$; snow: $q_s = (1 - \alpha) q_p$

$$DIFF = DIFF_r + DIFF_s$$

$$AUTC = AUTC_r + AUTC_s$$

$$ACCR = ACCR_r + ACCR_s$$

$$v_t = \alpha v_t(q_r) + (1 - \alpha) v_t(q_s)$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \theta) = \frac{L_v \theta_e}{c_p T_e} (\text{CON} + \text{DEP}) + D_\theta, \quad (1a)$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_v) = -\text{CON} - \text{DEP} + D_{q_v}, \quad (1b)$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_c) = \text{CON} - \text{ACC} - \text{AUT} + D_{q_c}, \quad (1c)$$

$$\frac{\partial \rho_o q_p}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T \mathbf{k}) q_p] = \text{ACC} + \text{AUT} + \text{DEP} + D_{q_p}. \quad (1d)$$

saturation: $q_{vs} = \alpha q_{vw} + (1 - \alpha)q_{vi}$

cloud water: $q_w = \alpha q_c$; cloud ice: $q_i = (1 - \alpha)q_c$

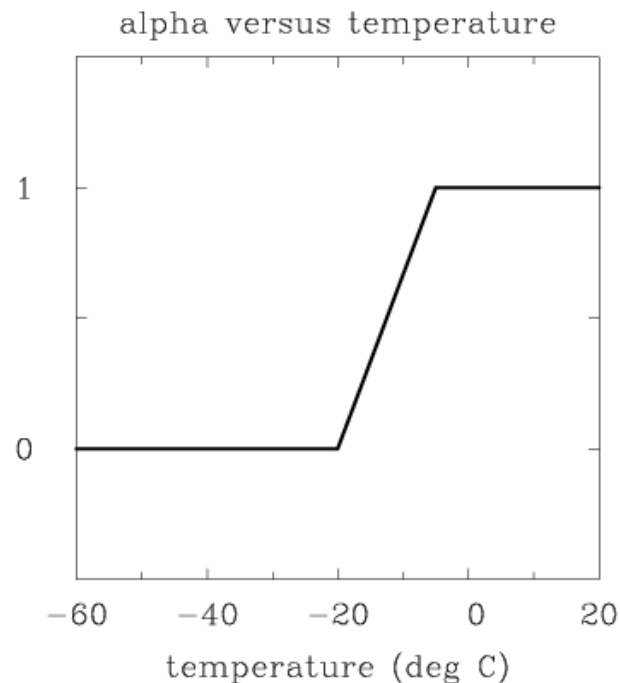
rain: $q_r = \alpha q_p$; snow: $q_s = (1 - \alpha)q_p$

$$DIFF = DIFF_r + DIFF_s$$

$$AUTC = AUTC_r + AUTC_s$$

$$ACCR = ACCR_r + ACCR_s$$

$$v_t = \alpha v_t(q_r) + (1 - \alpha)v_t(q_s)$$



$$q_{vs} = \frac{\varepsilon e_s}{p_e - e_s} \quad (2)$$

(see discussion in section 7 and appendix A of Lipps and Hemler 1982), where $\varepsilon = R_d/R_v$ (R_d and R_v are gas constants for the dry air and for the water vapor, respectively), p_e is the environmental pressure profile, and e_s is the saturated water vapor pressure given either by

$$e_s(T) = e_{oo} \exp \left[\frac{L_v}{R_v} \left(\frac{1}{T_{oo}} - \frac{1}{T} \right) \right] \quad (3a)$$

for the saturation over water or

$$e_s(T) = e_{oo} \exp \left[\frac{L_s}{R_v} \left(\frac{1}{T_{oo}} - \frac{1}{T} \right) \right] \quad (3b)$$

for the saturation over ice, where L_s denotes the latent heat of sublimation, $T = \theta(p_e/p_{oo})^{R_d/c_p}$, $p_{oo} = 10^5$ Pa, $e_{oo} = 611$ Pa, and $T_{oo} = 273.16$ K. The values of latent heats ($L_v = 2.53 \times 10^6$ J kg⁻¹, $L_s = 2.84 \times 10^6$ J kg⁻¹), assumed constant in (3), have been selected to provide as accurate values as possible of the saturated water vapor pressure over water and ice and their ratio over a wide range of temperatures.

precipitation (rain or snow),
 $N_0 = \text{const}$

$$n(D) = N_o \exp(-\lambda D)$$

precipitation particles
mass-size and terminal
velocity-size relationships:

$$m = aD^b,$$

$$v_t = cD^d.$$

raindrops: $a = \frac{\pi}{6} \rho_w; \quad b = 3; \quad c = 130; \quad d = 0.5$

snowflakes: $a = 2.5 \times 10^{-2}; \quad b = 2; \quad c = 4; \quad d = 0.25$

$$\lambda = \left[\frac{aN_o \Gamma(b+1)}{\rho_o q_p} \right]^{1/(b+1)}$$

Grabowski JAS 1998

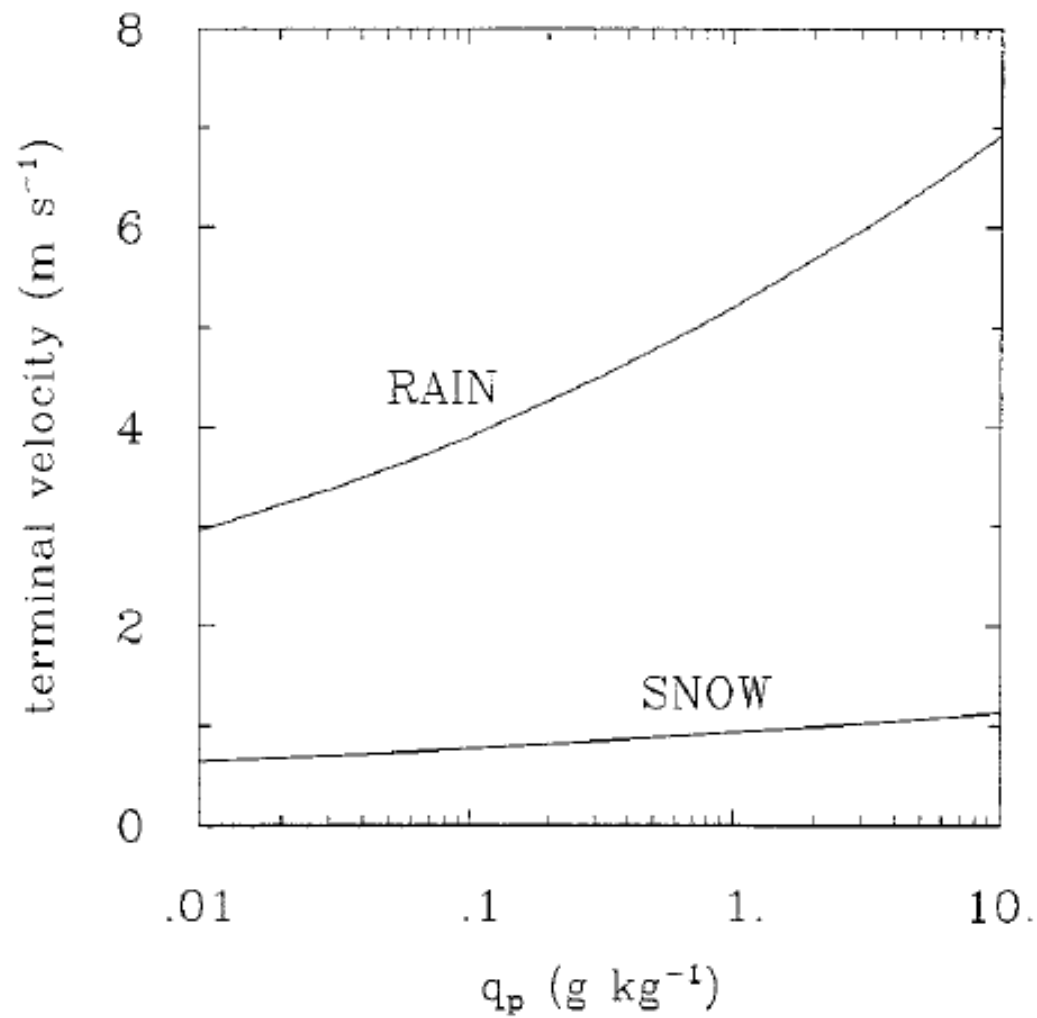


FIG. 2. Terminal velocity of rain and snow field as given by (17) as a function of the precipitation mixing ratio q_p .

SNOW:

cm^{-3} . For the snow, the autoconversion term is parameterized as

$$\text{AUT} = \frac{\rho_o q_c}{\tau_a}, \quad (10)$$

where τ_a is the conversion timescale assumed equal to a time required to grow an ice crystal by diffusion of water vapor in water saturated conditions up to a size of small precipitation particle (mass of 10^{-9} kg). This timescale is estimated using formulas for ice crystal growth developed by Koenig (1971) and approximated by a simple quadratic function decreasing from $\tau_a = 10^3$ s at 0°C to $\tau_a = 200$ s at -15°C and increasing back to $\tau_a = 10^3$ s at -30°C . For even colder temperatures, $\tau_a = 10^3$ s is used. Note that when the temperature is between T_w and T_i , q_c in (8) and (10) represents either cloud water part or cloud ice part of the cloud condensate, not the entire cloud condensate.

The precipitation growth terms (ACC, DEP) are estimated using characteristics of the particle with the average mass, that is,

$$\text{ACC} = \bar{n} \left(\frac{d\bar{m}}{dt} \right)_{\text{ACC}} \quad (11a)$$

$$\text{DEP} = \bar{n} \left(\frac{d\bar{m}}{dt} \right)_{\text{DEP}}, \quad (11b)$$

where \bar{n} is the mean concentration of precipitation particles

$$\bar{n} = \frac{N_o}{\lambda}; \quad (12)$$

\bar{m} is the mean mass of a precipitation particle,

$$\bar{m} = \frac{\rho_o q_p}{\bar{n}}, \quad (13)$$

and $(d\bar{m}/dt)_{\text{ACC}}$, $(d\bar{m}/dt)_{\text{DEP}}$ are growth rates of the mean particle due to accretion of cloud condensate and deposition of water vapor, respectively. The growth rates are estimated according to (e.g., Grabowski 1988)

for rain are

$$E = 0.8, \quad \alpha = 1, \quad \beta = 2,$$

and for the ice,

$$E = 0.2, \quad \alpha = 0.3, \quad \beta = 3.$$

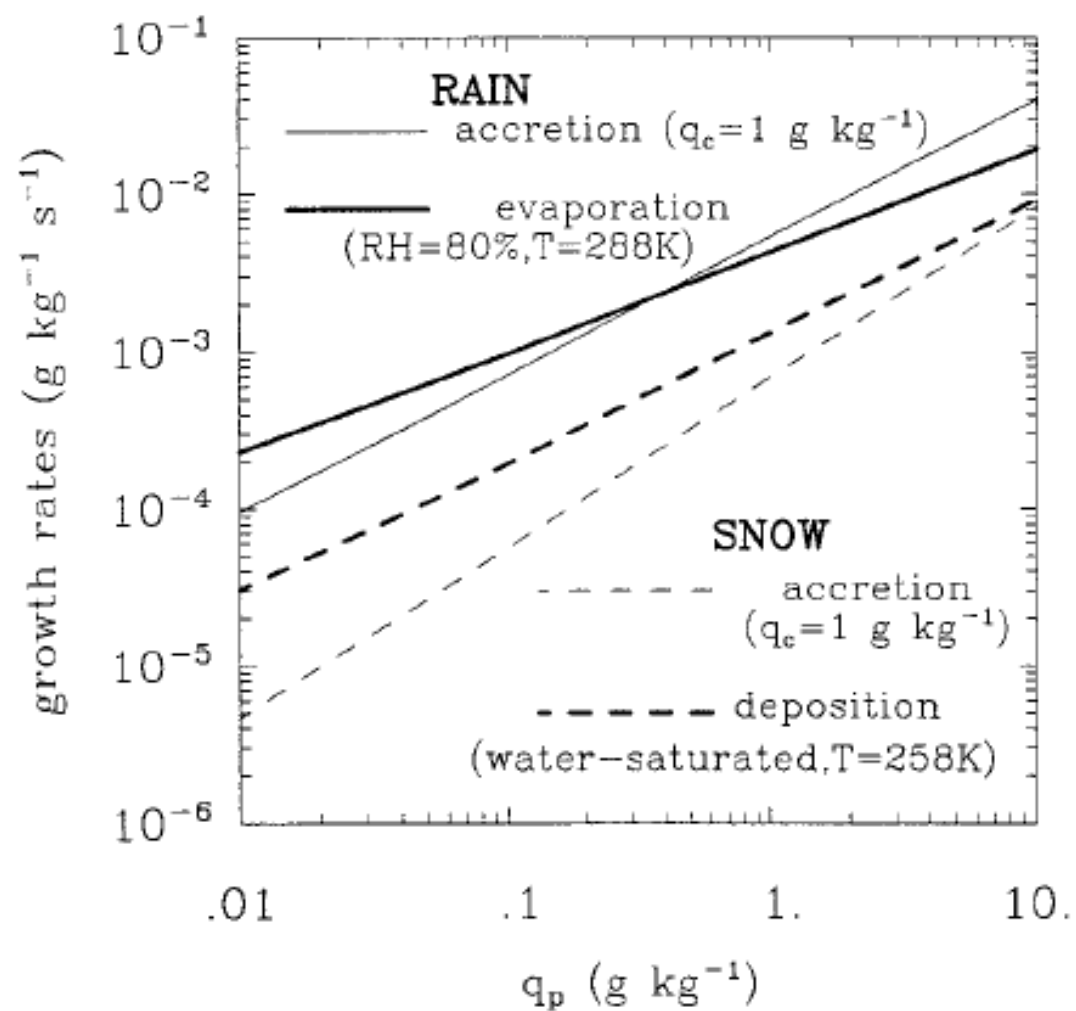
$$\left(\frac{d\bar{m}}{dt} \right)_{\text{ACC}} = \frac{\pi \bar{D}^2}{4} v_t(\bar{D}) E \alpha \rho_o q_c \quad (14a)$$

$$\left(\frac{d\bar{m}}{dt} \right)_{\text{DEP}} = \frac{4\pi \bar{D}}{\beta} (S - 1) F G(T_e), \quad (14b)$$

where \bar{D} is the diameter of a particle with average mass [calculated from \bar{m} using (5a)], E is the collection efficiency, α is the ratio of the effective area of the precipitation particle and $\pi \bar{D}^2/4$ (i.e., 1 for raindrops and smaller than 1 for snow), β is a nondimensional factor that depends on precipitation particle geometry (e.g., $\beta = 2$ for a sphere, $\beta = \pi$ for an infinitely thin circular disc, $\beta \sim 3$ for a thin needle), $S = q_v/q_{vs}$ is the saturation ratio, F is the ventilation factor [$F \approx 0.78 + 0.27 R_e^{1/2}$ for raindrops and $F \approx 0.65 + 0.39 R_e^{1/2}$ for ice particles, $R_e = \bar{D} v_t(\bar{D})/\nu$ is the Reynolds number, $\nu \approx 2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ is the kinematic viscosity of air; Pruppacher and Klett (1978)], and $G(T_e)$ is the thermodynamic function [Pruppacher and Klett (1978), Eq. (13.28) and (13.71)] approximated as

$$G(T_e) = A \left(2.2 \frac{T_e}{e_s(T_e)} + \frac{2.2 \times 10^2}{T_e} \right)^{-1}, \quad (15)$$

Grabowski JAS 1998



OUTLINE:

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- equilibrium approach - a simple extension of the warm-rain scheme

- non-equilibrium approach – more comprehensive schemes

- single-moment versus multi-moment schemes

Bin ice microphysics

Lagrangian (particle-based) methods

BULK MODEL WITH ICE MICROPHYSICS:

- potential temperature θ :

$$\frac{D\theta}{Dt} = \frac{L_v\theta_e}{c_p T_e} S_1 + \frac{L_s\theta_e}{c_p T_e} S_2 + \frac{L_f\theta_e}{c_p T_e} S_3$$

- water vapor mixing ratio q_v :

$$\frac{Dq_v}{Dt} = S_v$$

- cloud condensate variables q_c^i , $i = 1, N_c$ (typically, $N_c = 2$: cloud water, cloud ice):

$$\frac{Dq_c^i}{Dt} = S_c^i$$

- precipitating water variables q_p^i , $i = 1, N_p$: (typically, $N_p = 3$: rain, snow, graupel/hail):

$$\frac{Dq_p^i}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho q_p^i v_t^i) + S_p^i$$

S – various sources/sinks due to phase changes

S_1 – condensation/evaporation
 S_2 – sublimation/resublimation
 S_3 – freezing/melting

Single-moment schemes:

q – mass mixing ratios
(3rd moment of particle size distribution; PSD)

Double-moment schemes:

q – mass and number mixing ratios
(3rd and 1st moments of PSD)

...

Lin et al. 1983


Rutledge and Hobbs 1984


BULK RAIN/ICE MODEL


(Lin et al. 1983, Rutledge and Hobbs 1984)


$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \theta) = \frac{L_v \theta_e}{c_p T_e} S_1 + \frac{L_s \theta_e}{c_p T_e} S_2 + \frac{L_f \theta_e}{c_p T_e} S_3 + D_\theta$$


Water vapor  $\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_v) = S_4 + D_{q_v}$

Cloud water  $\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_c) = S_5 + D_{q_c}$

Cloud ice  $\frac{\partial \rho_o q_i}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_i) = S_6 + D_{q_i}$

Rain  $\frac{\partial \rho_o q_r}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^r \mathbf{k}) q_r] = S_7 + D_{q_r}$

Snow  $\frac{\partial \rho_o q_s}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^s \mathbf{k}) q_s] = S_8 + D_{q_s}$

Graupel
or hail  $\frac{\partial \rho_o q_g}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^g \mathbf{k}) q_g] = S_9 + D_{q_g}$

JAS 1983

**The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation
in Midlatitude Cyclones. VIII: A Model for the “Seeder-Feeder” Process
in Warm-Frontal Rainbands¹**

STEVEN A. RUTLEDGE AND PETER V. HOBBS

Department of Atmospheric Sciences, University of Washington, Seattle, WA 98195

JAS 1984

**The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation in
Midlatitude Cyclones. XII: A Diagnostic Modeling Study of Precipitation
Development in Narrow Cold-Frontal Rainbands**

STEVEN A. RUTLEDGE¹ AND PETER V. HOBBS

Atmospheric Sciences Department, University of Washington,² Seattle, WA 98195

(Manuscript received 20 May 1983, in final form 6 August 1984)

Traditional approach to bulk cloud microphysics

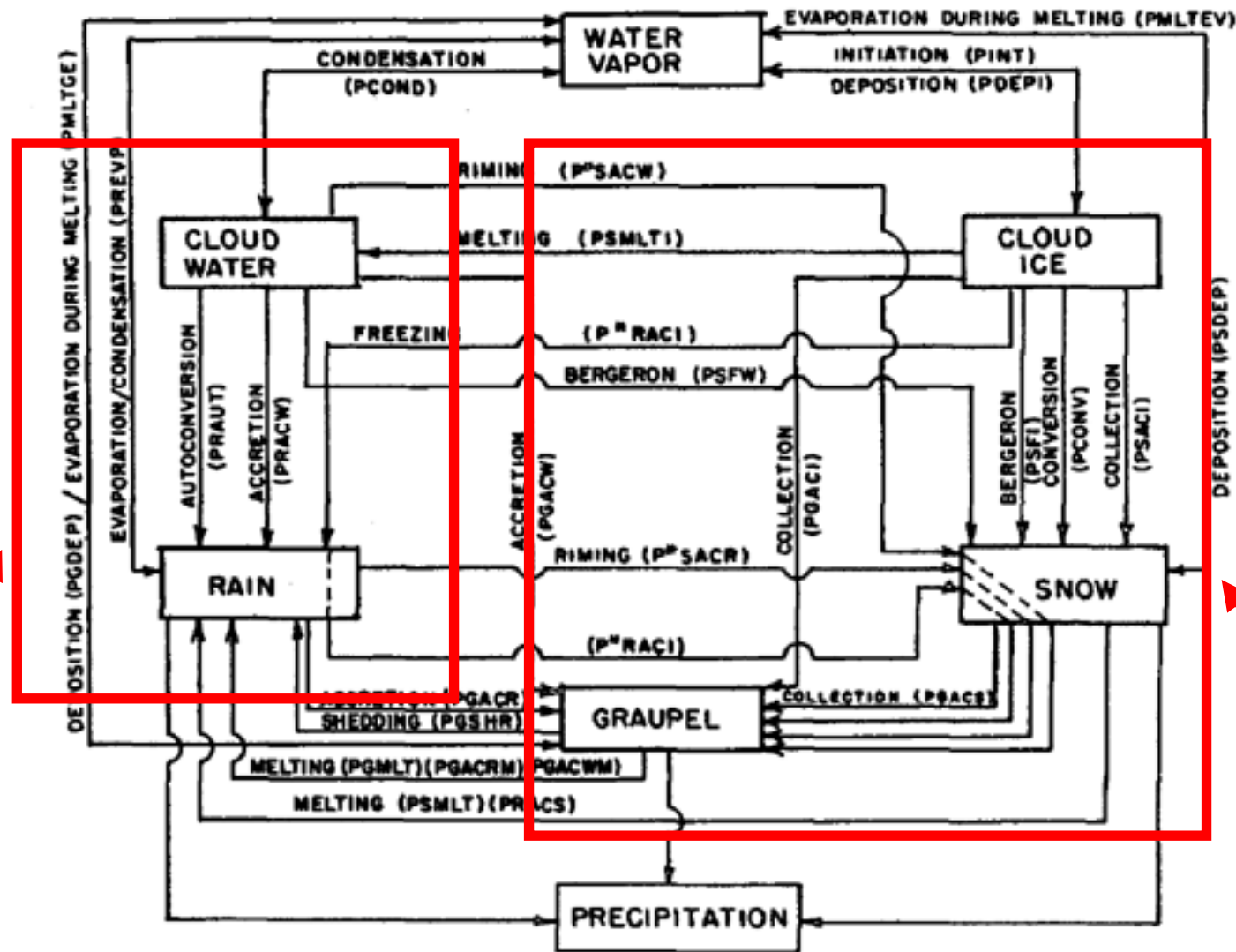


FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.

The main problem with such an approach to ice microphysics parameterization is that the ice scheme should produce various types of ice (cloud ice, snow, graupel/hail) just by the physics of particle formation and growth.

Partitioning ice particles a priori into separate categories introduces unphysical “conversion rates” and may involve “thresholding behavior” (i.e., model solutions diverge depending whether the threshold is reached or not).

Unfortunately, the schemes designed in the 1980ies (with the logic taken from the warm rain physics...) remain the mainstream of ice parameterization methods today...

OUTLINE:

Bulk ice physics modeling

- equilibrium approach - a simple extension of the warm-rain scheme
- non-equilibrium approach – more comprehensive schemes
- single-moment versus multi-moment schemes

Bin ice microphysics

Lagrangian (particle-based) methods

TRADITIONAL ICE MICROPHYSICS:

cloud ice: q_i

snow: q_s

graupel / hail: q_g

-

EXTENDING TRADITIONAL APPROACH TO 2-MOMENT ICE MICROPHYSICS:

cloud ice: q_i, N_i

snow: q_s, N_s

graupel / hail: q_g, N_g

-

Is such an approach justified?

Not really!

The ice scheme should produce various types of ice (cloud ice, snow, graupel/hail) just by the physics of particle growth. Partitioning ice particles a priori into separate categories introduces unphysical “conversion rates” and may involve “thresholding behavior” (i.e., model solutions diverge depending whether the threshold is reached or not).

A two-moment three-variable ice scheme:

$$\frac{\partial N}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - \underline{V_N \mathbf{k}}) N] = \mathcal{F}_N$$

Number concentration of ice crystals, N

$$\frac{\partial q_{dep}}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - \underline{V_q \mathbf{k}}) q_{dep}] = \mathcal{F}_{q_{dep}}$$

Mixing ratio of ice mass grown by diffusion of water vapor, q_{dep}

$$\frac{\partial q_{rim}}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - \underline{V_q \mathbf{k}}) q_{rim}] = \mathcal{F}_{q_{rim}}$$

Mixing ratio of ice mass grown by riming (accretion of liquid water), q_{rim}

Morrison and Grabowski (JAS 2008)

Case without riming



- Growth by vapor deposition



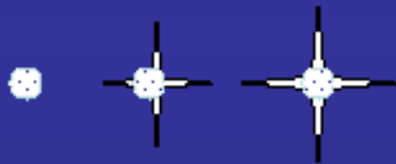
- Growth by aggregation
- Growth by vapor deposition

Smooth transition from small spherical ice to larger nonspherical crystals to aggregates.

Case with riming

Stage 1: Unrimed crystal

- Particle dimension and mass determined by vapor deposition



- Vapor depositional growth

Stage 2: Partially-rimed crystal

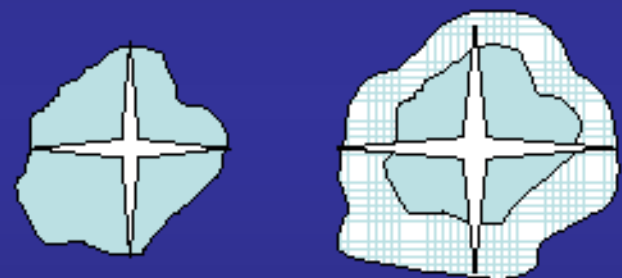
- Particle dimension determined by vapor deposition
- Mass determined by vapor deposition and riming



- Riming of crystal interstices
- Vapor depositional growth

Stage 3: Graupel

- Particle dimension determined by vapor deposition and riming
- Mass determined by vapor deposition and riming



- Complete filling-in of interstices with rime
- Further growth by riming and vapor deposition

Prediction of both riming and vapor deposition mass allows for realistic particle evolution during growth.

Ice particles assumed to follow gamma distribution (3 parameters: N_o , μ , λ)

$$N(D) = N_o D^\mu e^{-\lambda D}$$

$$N = \int_0^\infty N(D) dD$$

$$q_{dep} + q_{rim} \equiv q = \int_0^\infty m(D) N(D) dD$$

$$\frac{\partial N}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - V_N \mathbf{k}) N] = \mathcal{F}_N$$

$$\frac{\partial q_{dep}}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - V_q \mathbf{k}) q_{dep}] = \mathcal{F}_{q_{dep}}$$

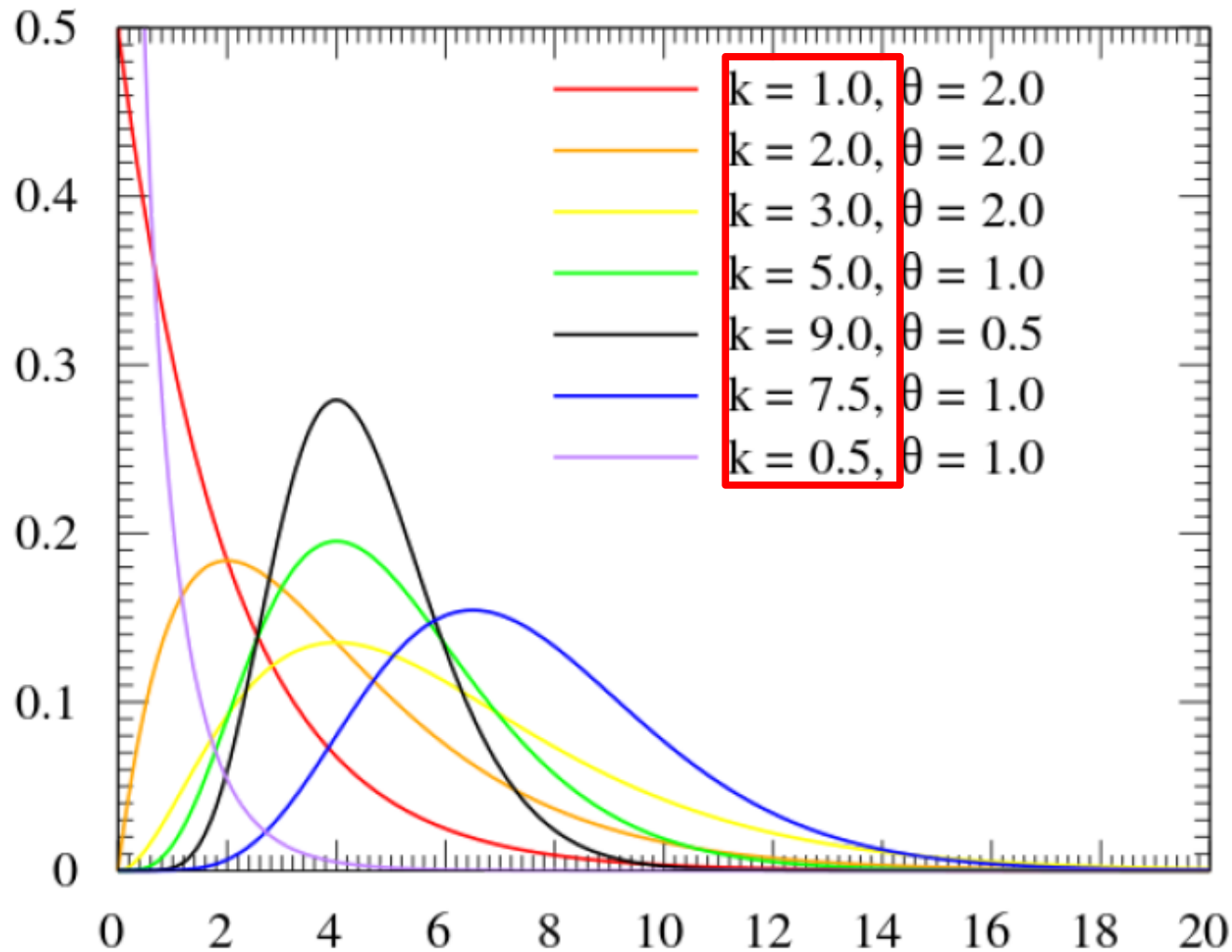
$$\frac{\partial q_{rim}}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a (\mathbf{u} - V_q \mathbf{k}) q_{rim}] = \mathcal{F}_{q_{rim}}$$

$$\mu = 0.076 \lambda^{0.8} - 2 ; \quad 0 \leq \mu \leq 6$$

Diagnostic relationship based on cloud observations (Heymsfield 2003)

Morrison and Grabowski (JAS 2008)

Gamma distribution

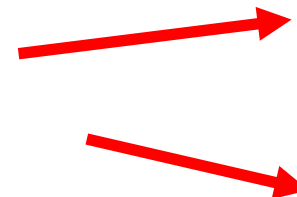


$$\frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$$

$k=1$ – exponential (M-P)

$k>1$ – distribution with a peak

Ice particle mass-dimension (m-D) and projected-area-dimension (A-D) relationships are based on observed characteristics of ice crystals, aggregates, and graupel particles (from aircraft and ground-based observations).


$$m = \alpha D^{\beta}$$
$$A = \sigma D^{\gamma}$$

Morrison and Grabowski (JAS 2008)

rimed mass fraction F_r :

$$F_r = \frac{q_{rim}}{q_{dep} + q_{rim}} \approx \frac{m_{rim}}{m_{dep} + m_{rim}}$$

assumed constant across the spectrum of ice particles

$F_r = 0$ — ice particle grown by diffusion/aggregation

$F_r \rightarrow 1$ — graupel (or hail)

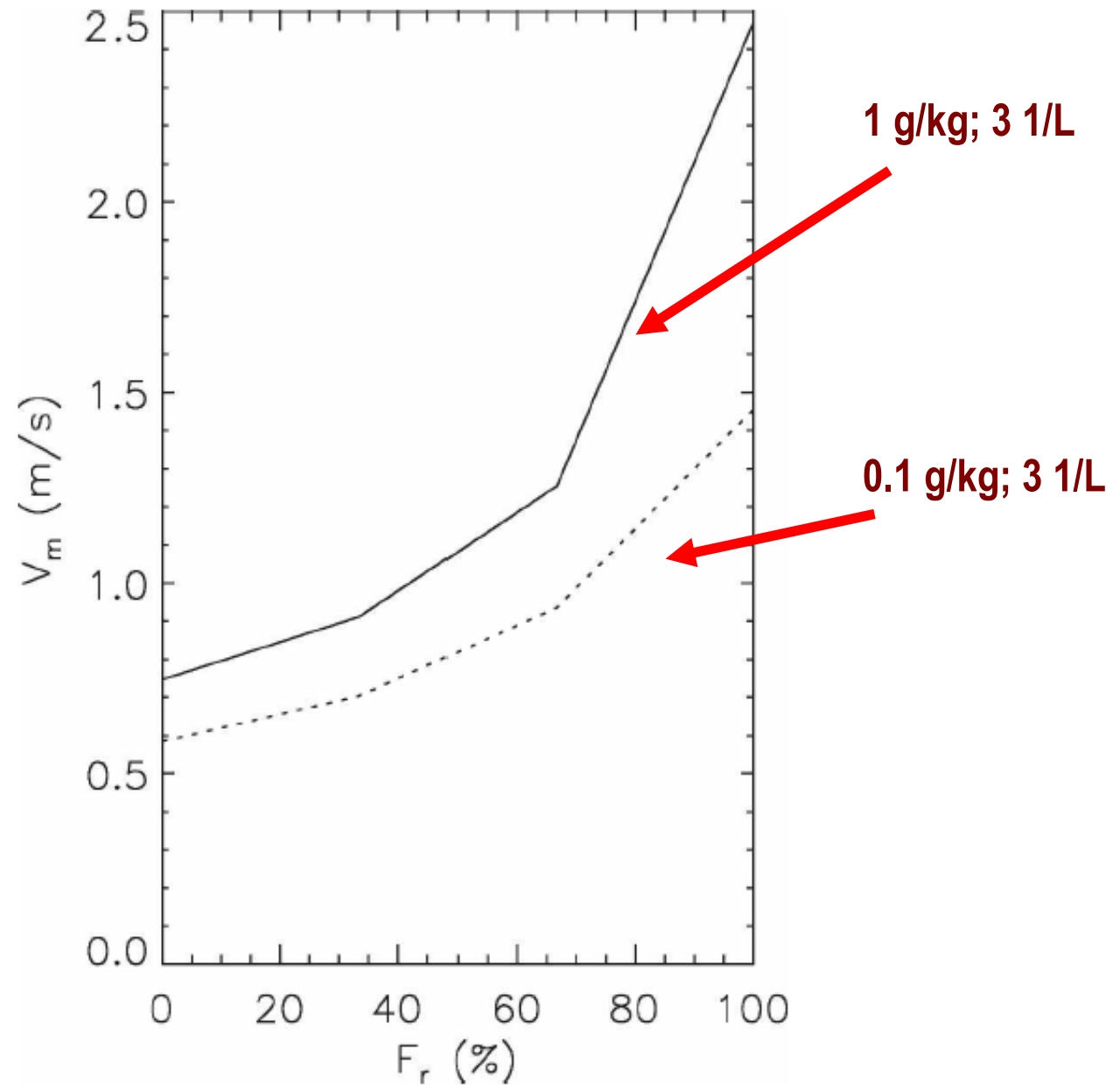
$0 < F_r < 1$ — rimed ice particle:

size D given by the mass grown by diffusion $m_{dep} = (1 - F_r)m$;

$m = m_{rim} + m_{dep}$ is the total mass of ice particle

Morrison and Grabowski (JAS 2008)

Parameterization
of ice mass
fallspeed. Note
gradual increase
with the rimed
fraction F_r



Morrison and Grabowski (JAS 2008)

Example of the application of the new ice scheme: precipitation development in a small convective cloud [2D (x-z) prescribed-flow framework with a low-level convergence, upper-level divergence, evolving-in-time updraft, and weak vertical shear]

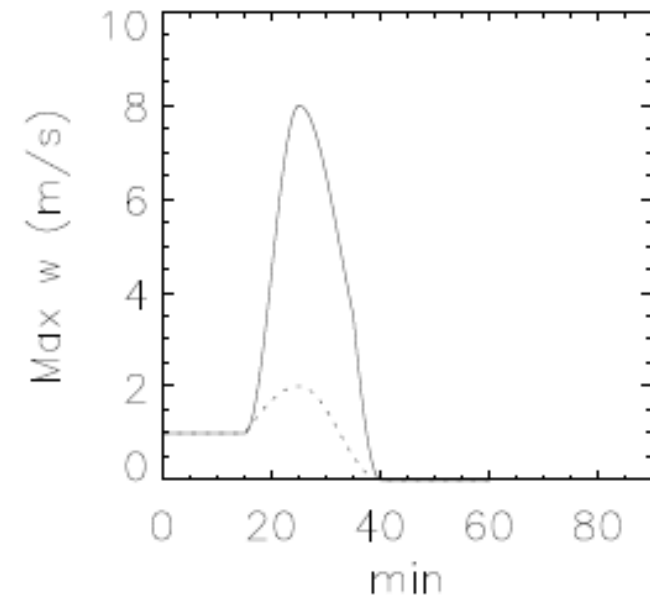
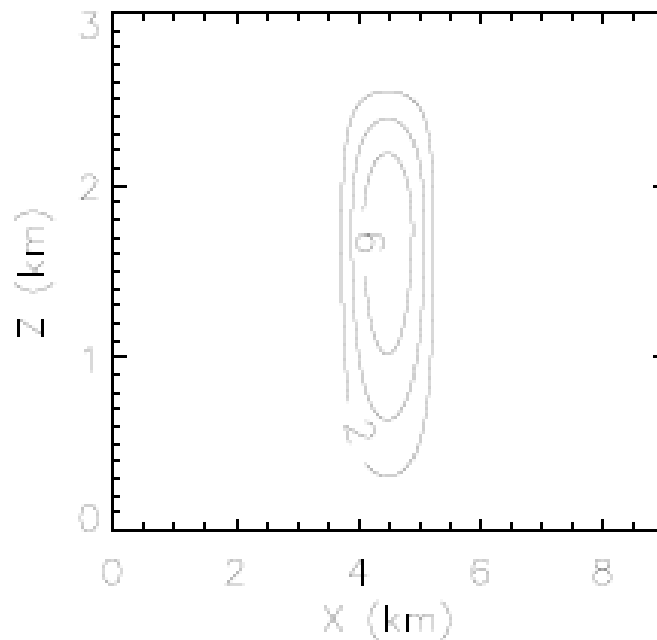
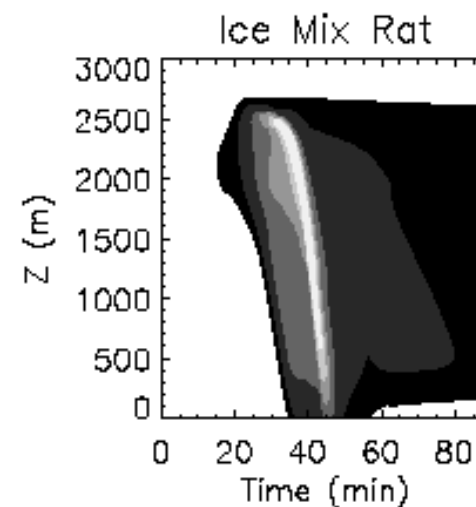
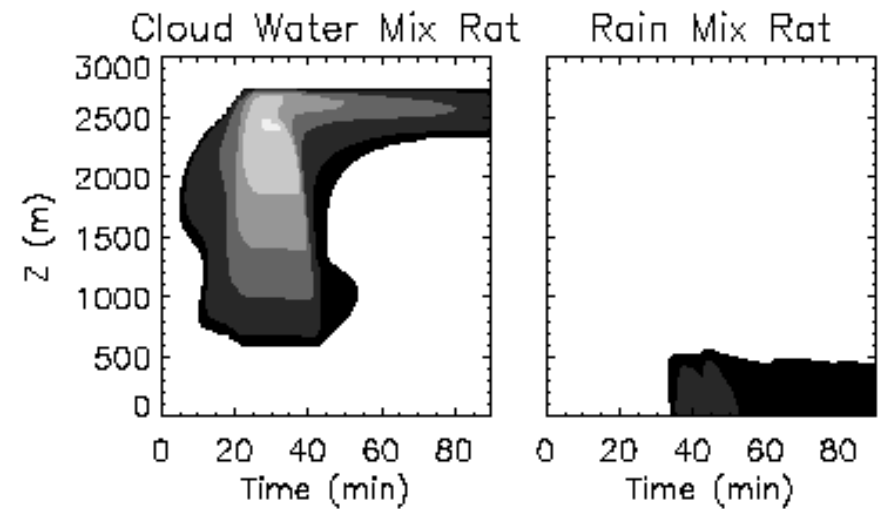
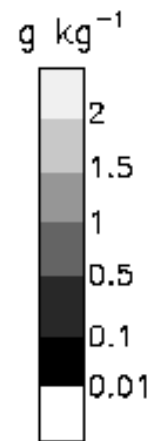
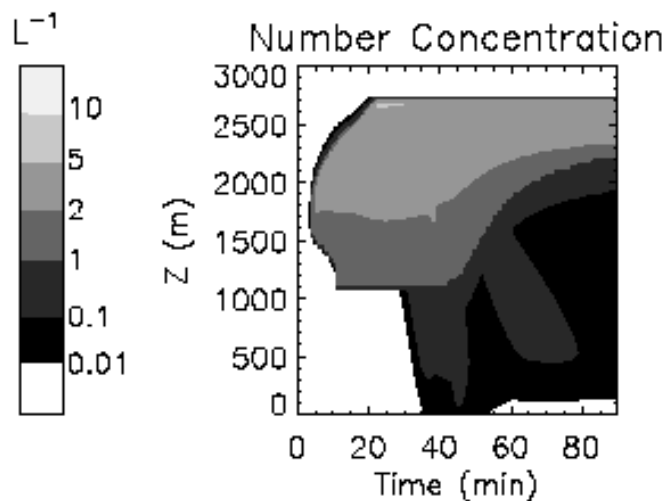
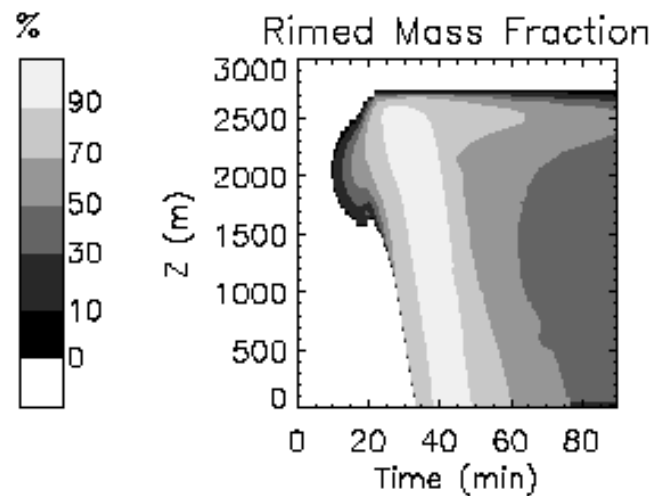


Figure 4: Maximum updraft velocity w in the X-Z plane as a function of time for peak updraft strength of 8 m/s (solid) and 2 m/s (dotted).

Morrison and Grabowski (JAS 2008)

Example of results:
evolutions of horizontal
maximum at each level



Morrison and Grabowski (JAS 2008)

Table 2: Time- and domain-average cloud liquid water path LWP (g m^{-2}), ice water path IWP (g m^{-2}), water optical depth τ_c (unitless), ice optical depth τ_i (unitless), total cloud optical depth τ_{tot} (unitless), and surface precipitation rate $PREC$ (mm/hr) for simulations with maximum updraft speed w of either 2 or 8 m s^{-1} . “NEW” and “TRADITIONAL” refer to simulations using the new and traditional ice microphysics schemes, respectively. “TH-HIGH” and “TH-LOW” refer to sensitivity tests using the traditional scheme but with the threshold ice/snow and droplet mixing ratios for graupel production during droplet collection increased or decreased, respectively (see text for details). “S1” and “H07” refer to sensitivity tests with the new scheme using the m-D relationship for side planes or from Heymsfield et al. (2007), respectively. The averaging period is from $t = 25$ to 90 min.

Run	Max w	LWP	IWP	τ_c	τ_i	τ_{tot}	$PREC$
NEW	8	228.4	237.1	26.7	8.6	35.3	0.68
TRADITIONAL	8	92.9	317.9	12.1	32.6	44.7	0.76
TH-HIGH	8	52.8	719.3	7.4	111.0	118.4	0.62
TH-LOW	8	474.7	137.5	52.6	2.8	55.4	0.46
S1	8	288.8	198.0	33.3	4.6	37.9	0.64
H07	8	477.0	136.5	52.1	2.4	54.5	0.52
NEW	2	49.8	85.3	7.1	4.5	11.6	0.23
TRADITIONAL	2	16.8	140.4	2.7	20.9	23.6	0.20
TH-HIGH	2	15.4	169.4	2.5	26.4	28.9	0.20
TH-LOW	2	122.8	34.3	16.0	0.8	16.8	0.14

Morrison and Grabowski (JAS 2008)

Bin ice microphysics

Traditional approach to **bin** cloud microphysics

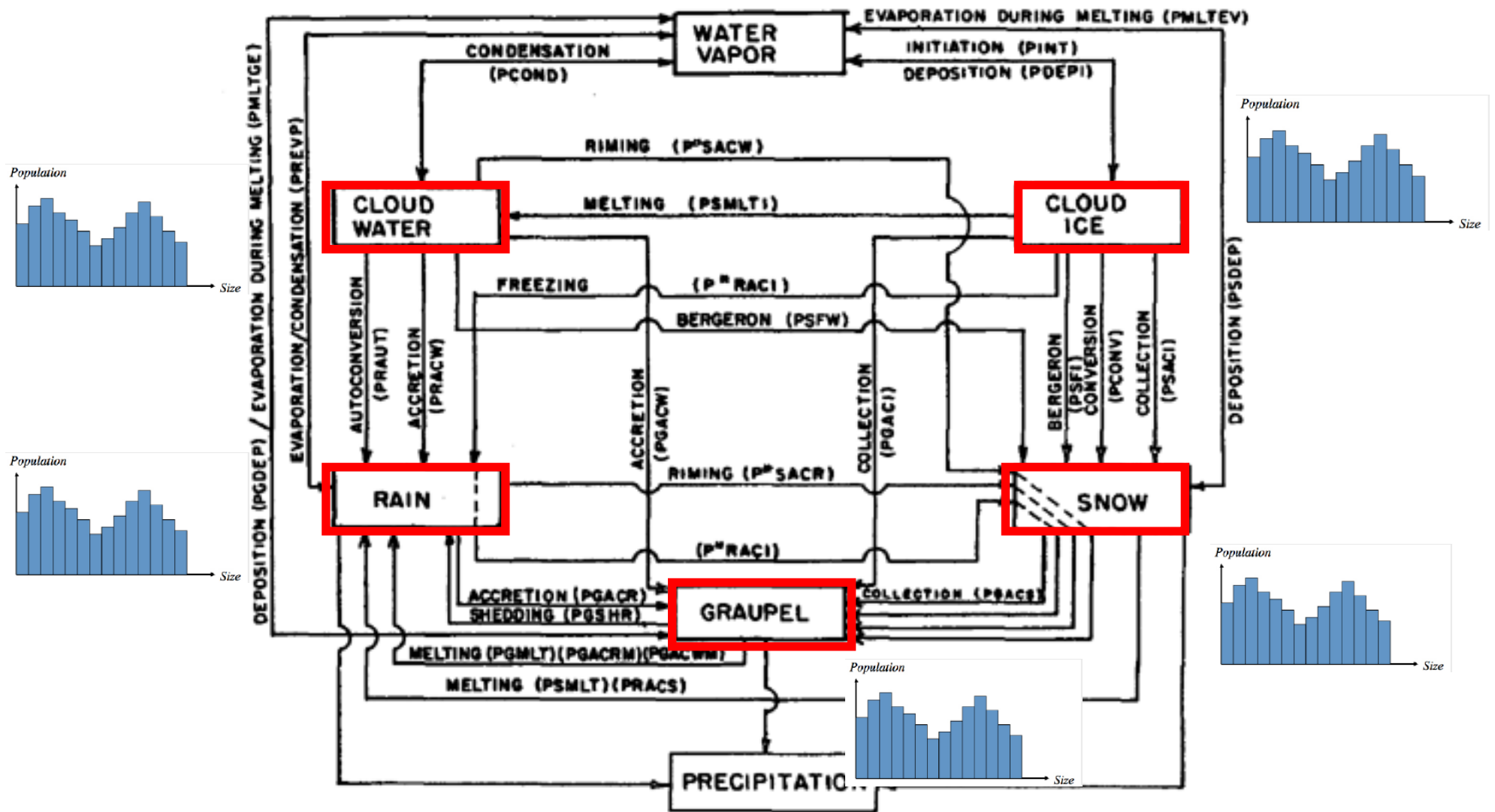


FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.

Prof. Alexander Khain, Hebrew University of Jerusalem, Israel
 Prof. Istvan Geresdi, University of Pets, Hundgary

**Idealized simulations of a squall line from the MC3E field campaign applying
three bin microphysics schemes. Part 1: Dynamic and thermodynamic
structure**

Midlatitude Continental Convective Clouds Experiment

*Lulin Xue^{*1}, Jiwen Fan², Zachary J. Lebo³, Wei Wu^{4,1}, Hugh Morrison¹, Wojciech W.*

Grabowski¹, Xia Chu³, Istvan Geresdi⁵, Kirk North⁶, Ronald Stenz⁷, Yang Gao², Xiaofeng

Lou⁸, Aaron Bansemer¹, Andrew J. Heymsfield¹, Greg M. McFarquhar^{4,1}, Roy M.

Rasmussen¹

¹ National Center for Atmospheric Research, Boulder, CO, USA

² Pacific Northwest National Laboratory, Richland, WA, USA

³ University of Wyoming, Laramie, WY, USA

⁴ University of Illinois Urbana-Champaign, Urbana, IL, USA

⁵ University of Pécs, Pécs, Hungary

⁶ McGill University, Montreal, Canada

⁷ University of North Dakota, Grand Forks, ND, USA

⁸ Chinese Academy of Meteorological Sciences, Beijing, China

in review in Monthly Weather Review

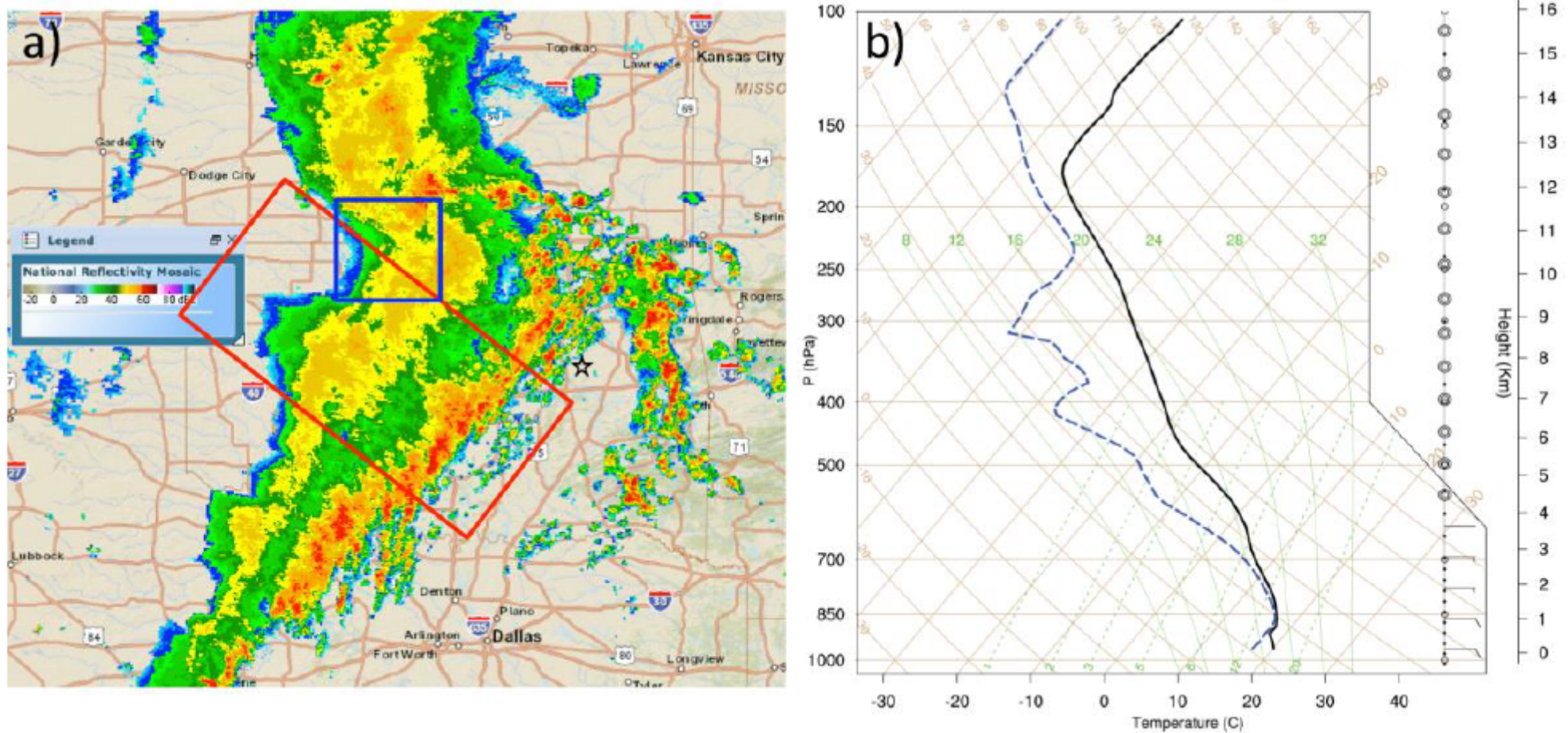


Fig. 2. (a) The S-band NEXRAD radar composite reflectivity at 1200 UTC on May 20, 2011. The red box shows the region where the vertical and the horizontal cross sections of the observed reflectivity are plotted in Figs. 3a1 and b1. The blue box indicates the 100 km by 100 km area where the vertical velocity CFAD in Fig. 4a derived from the C-band and the X-band radars is calculated. (b) Model sounding based on observations from Morris, OK, located as indicated by the star symbol in (a).

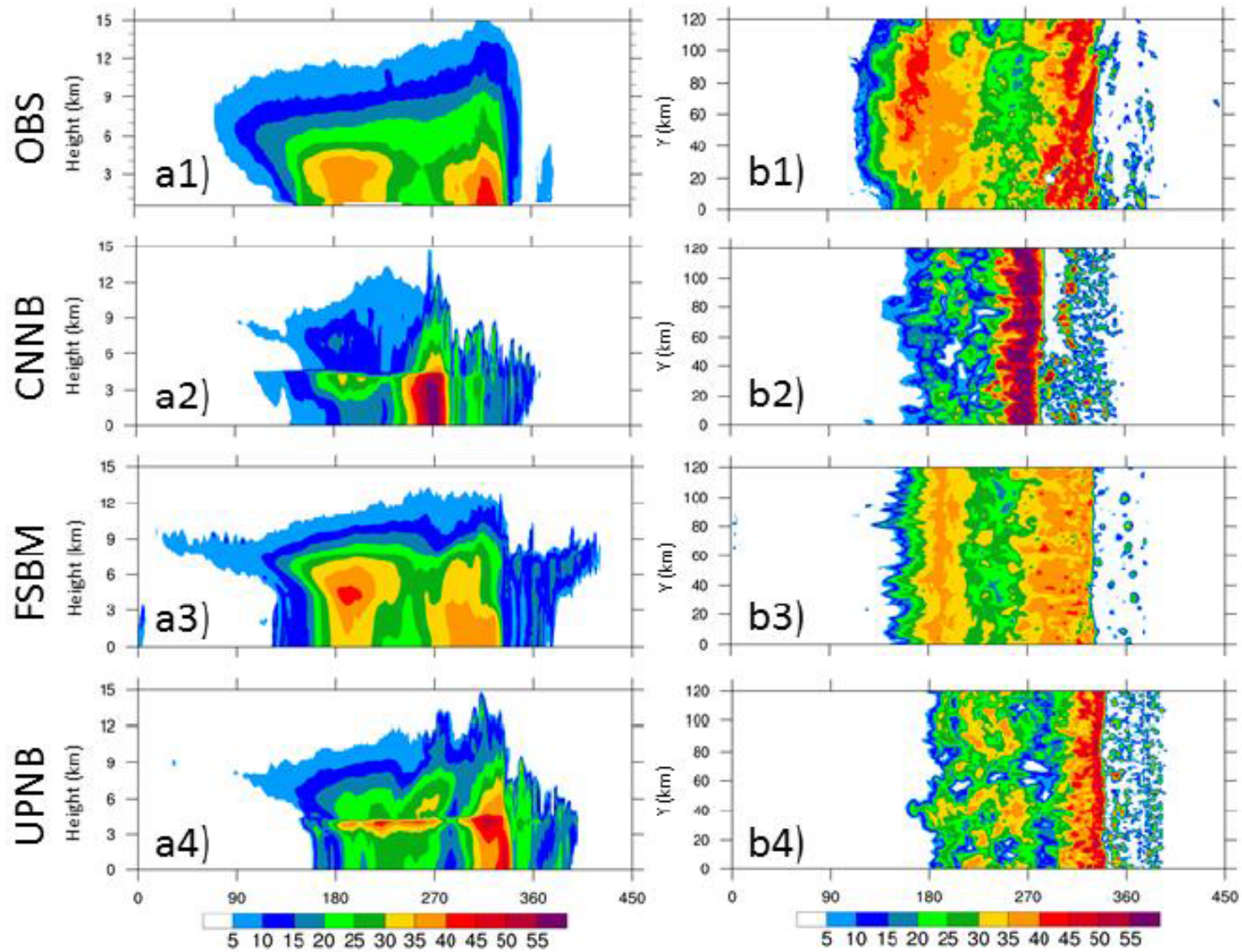


Fig. 3. Line-averaged vertical cross sections in (a) and horizontal cross sections at 2 km in (b) for 1) observed NEXRAD radar reflectivity (dBZ) and 2) to 4) model simulated S-band radar reflectivity (dBZ) from CNNB, FSBM and UPNB respectively.

New trend: Lagrangian treatment of the condensed phase:

The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model

S. Shima,^{a*} K. Kusano,^c A. Kawano,^a T. Sugiyama^a and S. Kawahara^b

Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model

M. Andrejczuk,¹ W. W. Grabowski,² J. Reisner,³ and A. Gadian¹

Large-Eddy Simulations of Trade Wind Cumuli Using Particle-Based Microphysics with Monte Carlo Coalescence

SYLWESTER ARABAS

Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland

SHIN-ICHIRO SHIMA

Graduate School of Simulation Studies, University of Hyogo, Kobe, and Japan Agency for Marine-Earth Science and Technology, Kanagawa, Japan

A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision

T Riechelmann^{1,3}, Y Noh² and S Raasch¹

Summary of the warm-rain and ice lectures:

A wide range of modeling approaches exists that one can use in modeling various aspects of cloud dynamics and microphysics. Most of them are within the framework of Eulerian modeling, but use of Lagrangian microphysics is rapidly expanding.

The selection of specific method needs to be tailored to the specific problem one would like to study. If multiscale dynamics (e.g., convectively coupled waves in the Tropics) is the focus, application of as simple microphysics as possible makes sense to use the computer time to widen the range of spatial scales resolved. If small-scale dynamics–microphysics interactions is the focus, more emphasis on microphysics is needed.

The multiscale nature of clouds (the range of spatial scales), difficulties of cloud observations (in-situ and remote sensing), and increasing appreciation of the role of clouds in weather and climate make the cloud physics an appealing area of research.