

# Growth of cloud droplets in turbulent clouds

Wojciech W. Grabowski

Mesoscale and Microscale Meteorology Laboratory

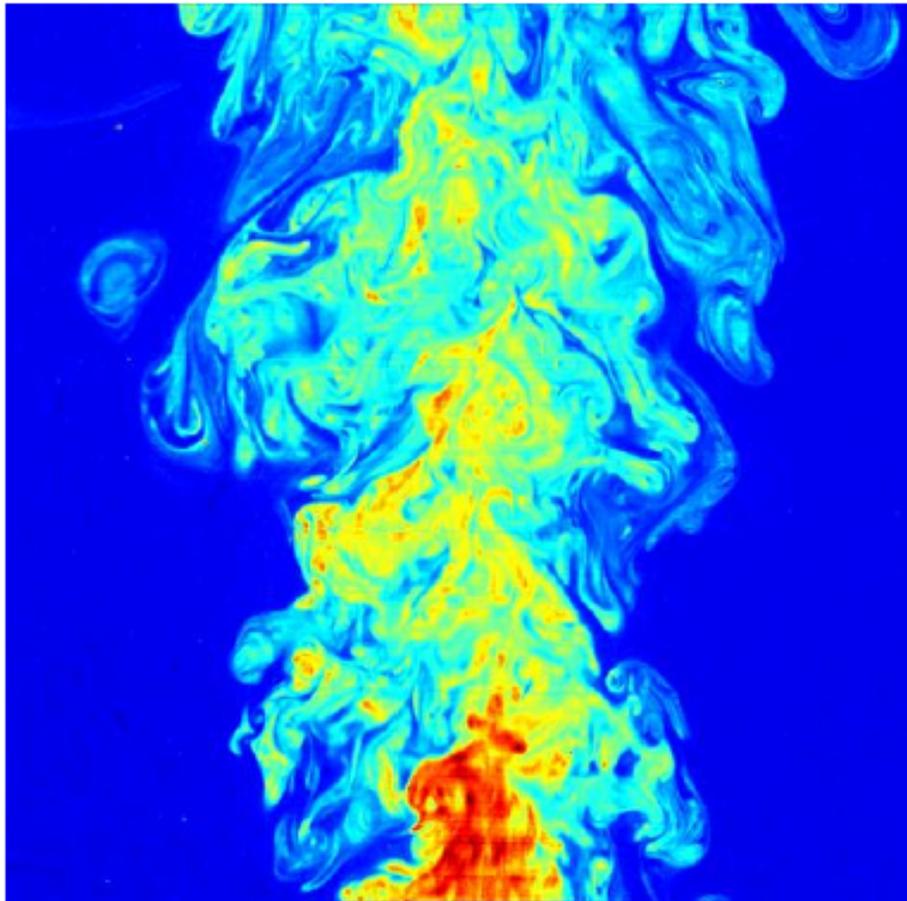
NCAR, Boulder, Colorado, USA



UNIVERSITY  
OF WARSAW



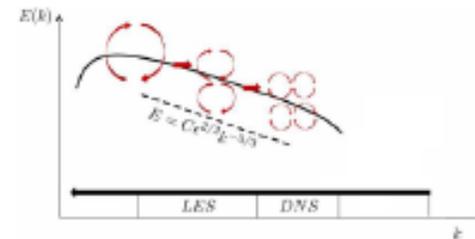
# Clouds are turbulent, but what does it mean?



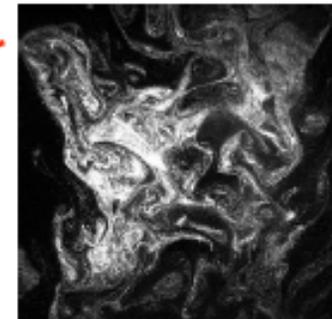
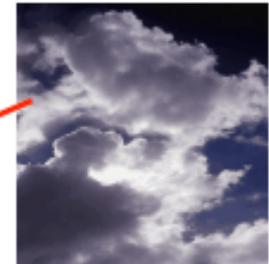
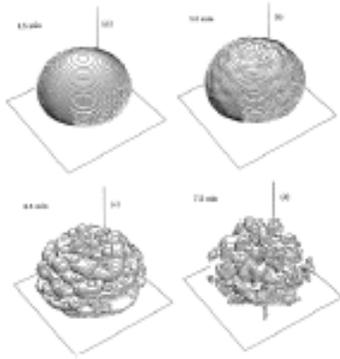
Turbulent jet in the laboratory

Lewis F. Richardson's phrase:

“Big whirls have little whirls  
Which feed on their velocity,  
And little whirls have lesser whirls,  
And so on to viscosity.”



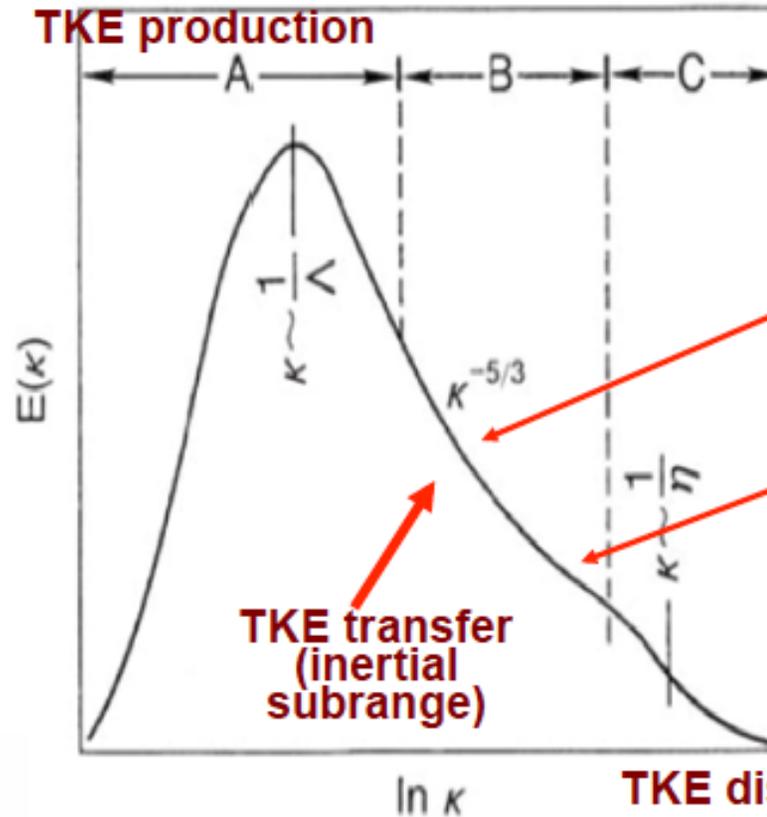
# TKE – turbulent kinetic energy



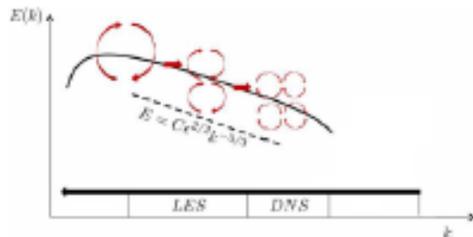
$A$  – energy-containing scales ( $\sim 100$  m)

$\eta$  – Kolmogorov microscale ( $\sim 1$  mm)

## TKE production



## TKE dissipation

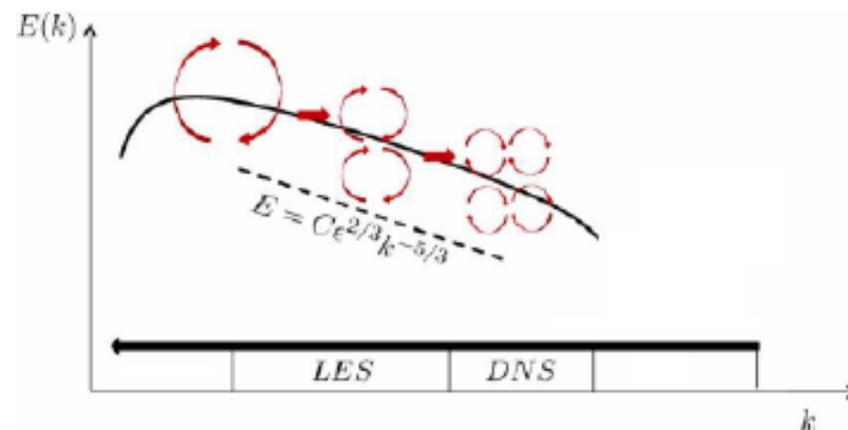
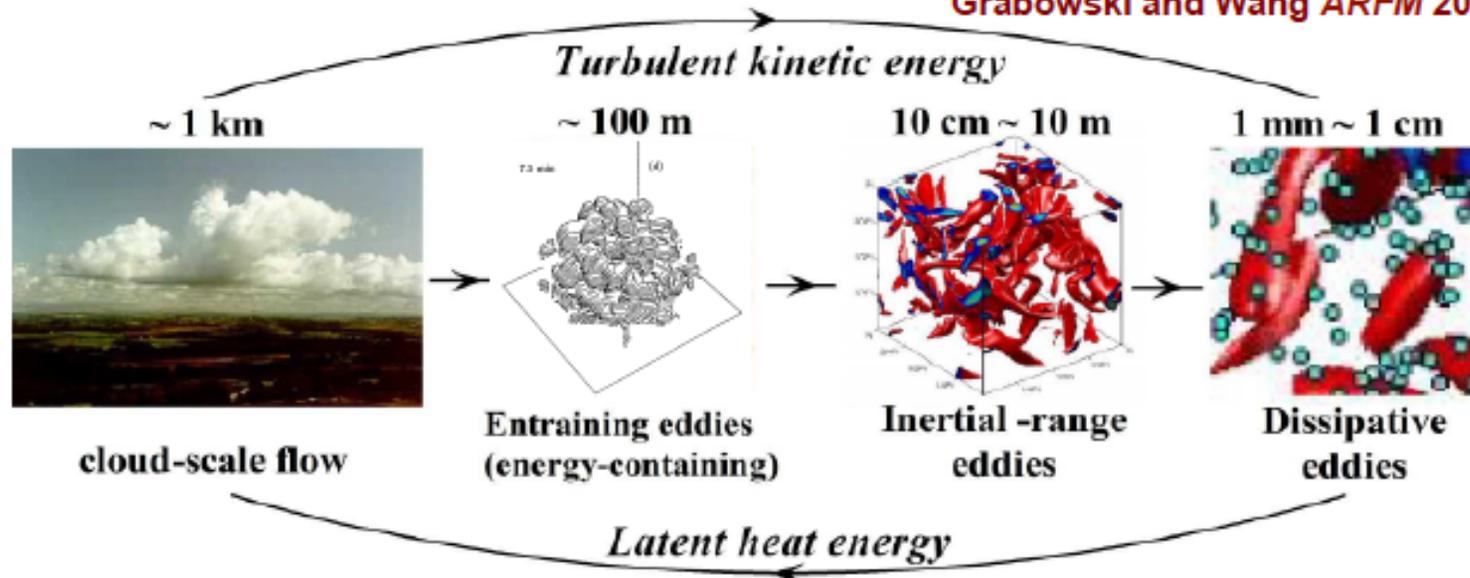


Kolmogorov length scale	$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$
Kolmogorov time scale	$\tau_\eta = \left(\frac{\nu}{\epsilon}\right)^{1/2}$
Kolmogorov velocity scale	$u_\eta = (\nu\epsilon)^{1/4}$

$\epsilon$ ( $\text{cm}^2 \text{s}^{-3}$ )	$\tau_k$ (s)	$\eta$ (cm)	$u_k$ ( $\text{cm s}^{-1}$ )
10	0.1304	0.1488	1.142
100	0.0412	0.0837	2.031
400	0.0206	0.0592	2.872

# Multiscale interactions in atmospheric clouds

Grabowski and Wang *ARFM* 2013

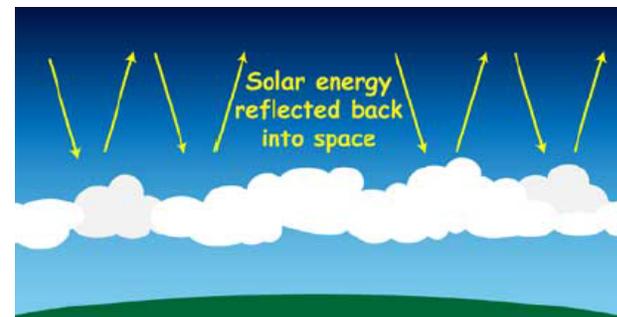
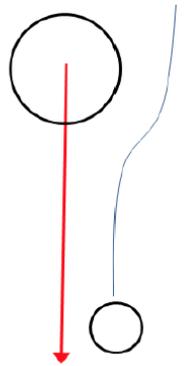


*Cloud droplets grow by the diffusion of water vapor (i.e., by condensation) and by collision/coalescence.*

*For both growth mechanisms, cloud turbulence plays a significant and still poorly understood role.*

*For gravitational collisions, width of the droplet spectrum grown by diffusion is the key...*

*The width of the droplet spectrum also affects the amount of solar radiation reflected back to space by clouds...*



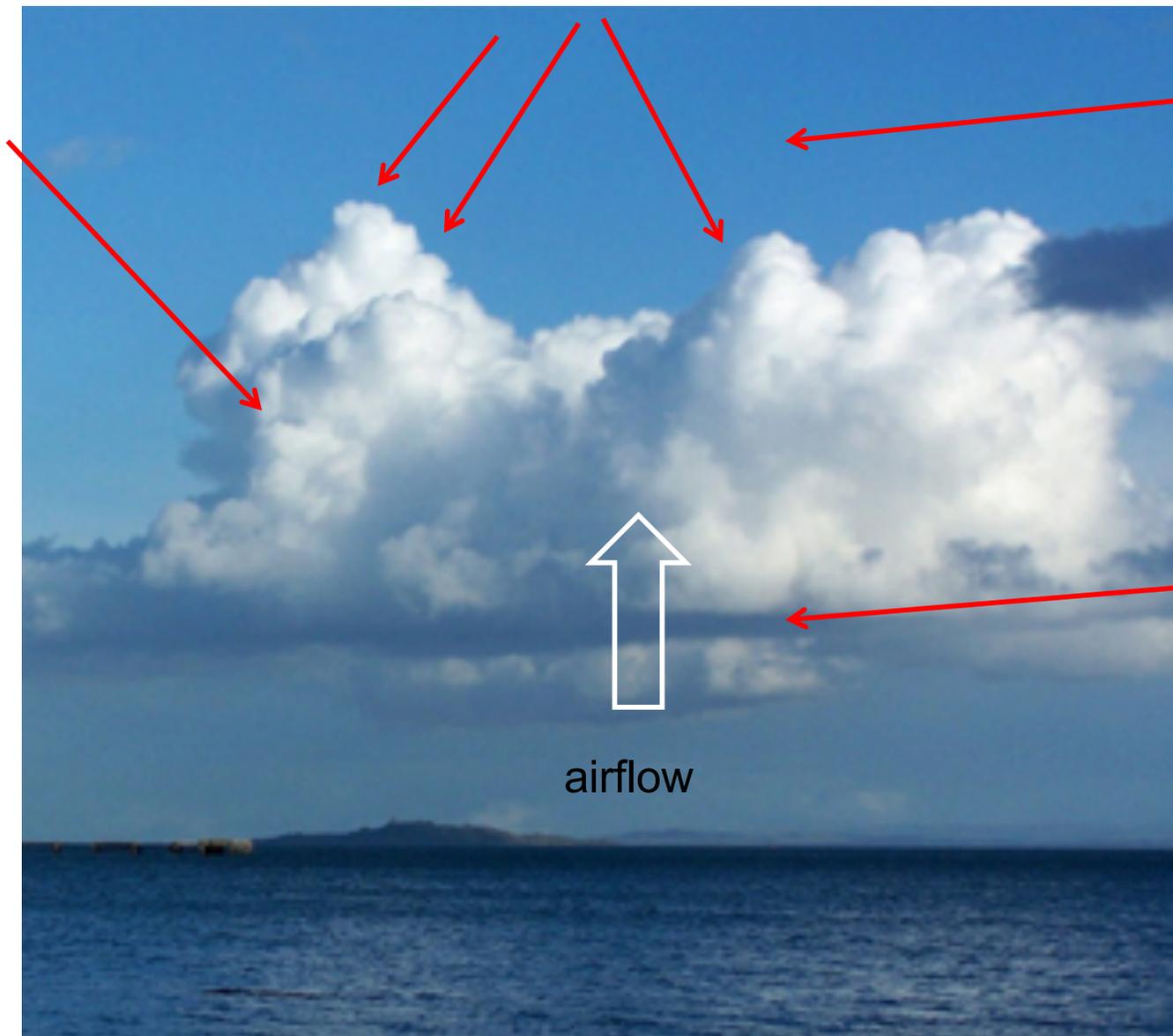
turbulent  
cloud

interfacial  
instabilities

calm (low-  
turbulence)  
environment

cloud base  
(CCN activation)

airflow





droplet spectra

vertical and along-track velocity

liquid water content

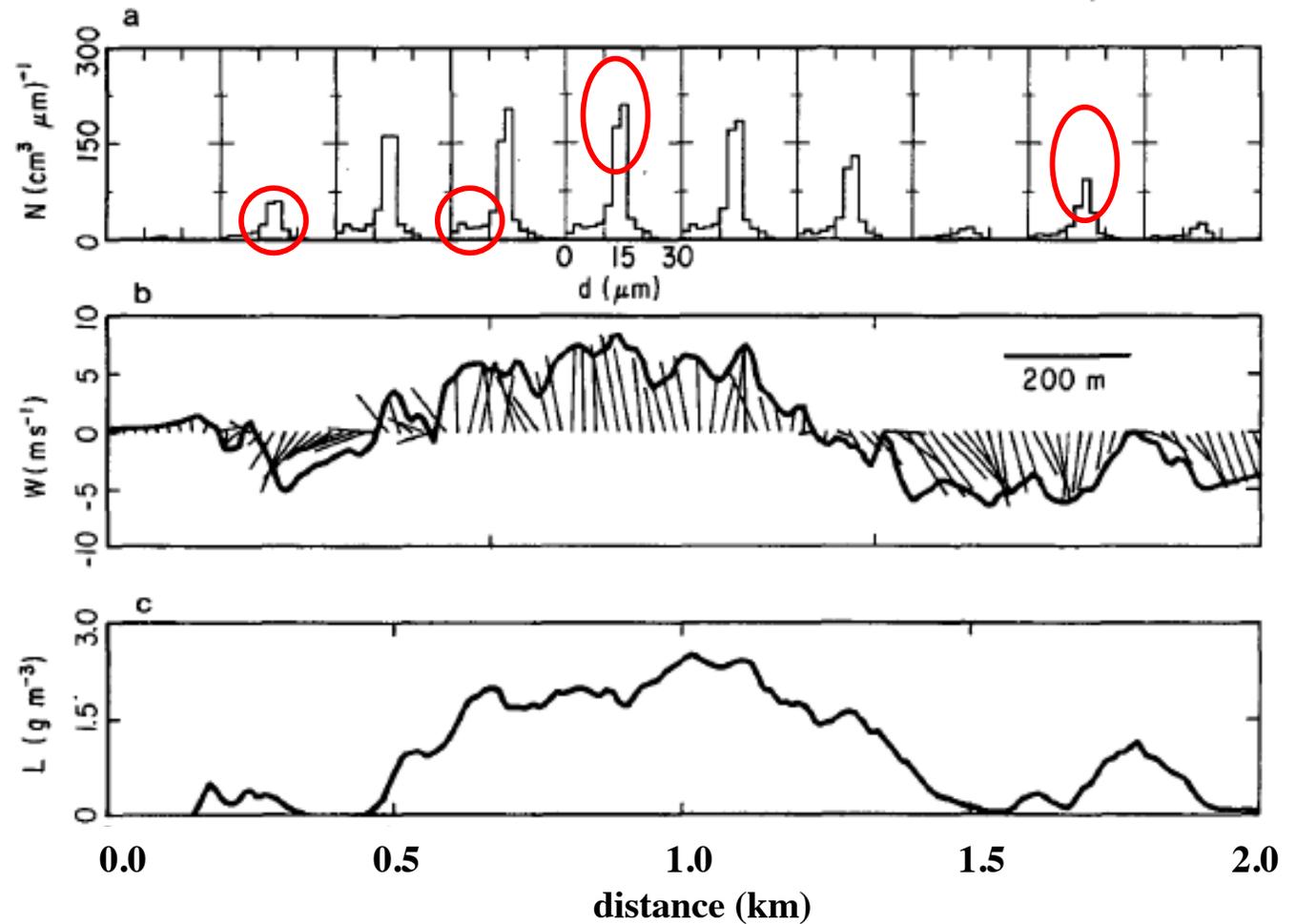


FIG. 3. Penetration at 600 mb, 6 June: (a) two-second averaged droplet spectra (sizes for diameter bins are those given by the manufacturer); (b) wind velocity, the lines represent wind vectors formed from the vertical wind and the wind along the flight path; (c) liquid water density measured by the Johnson-Williams device. All H-2 measurements.

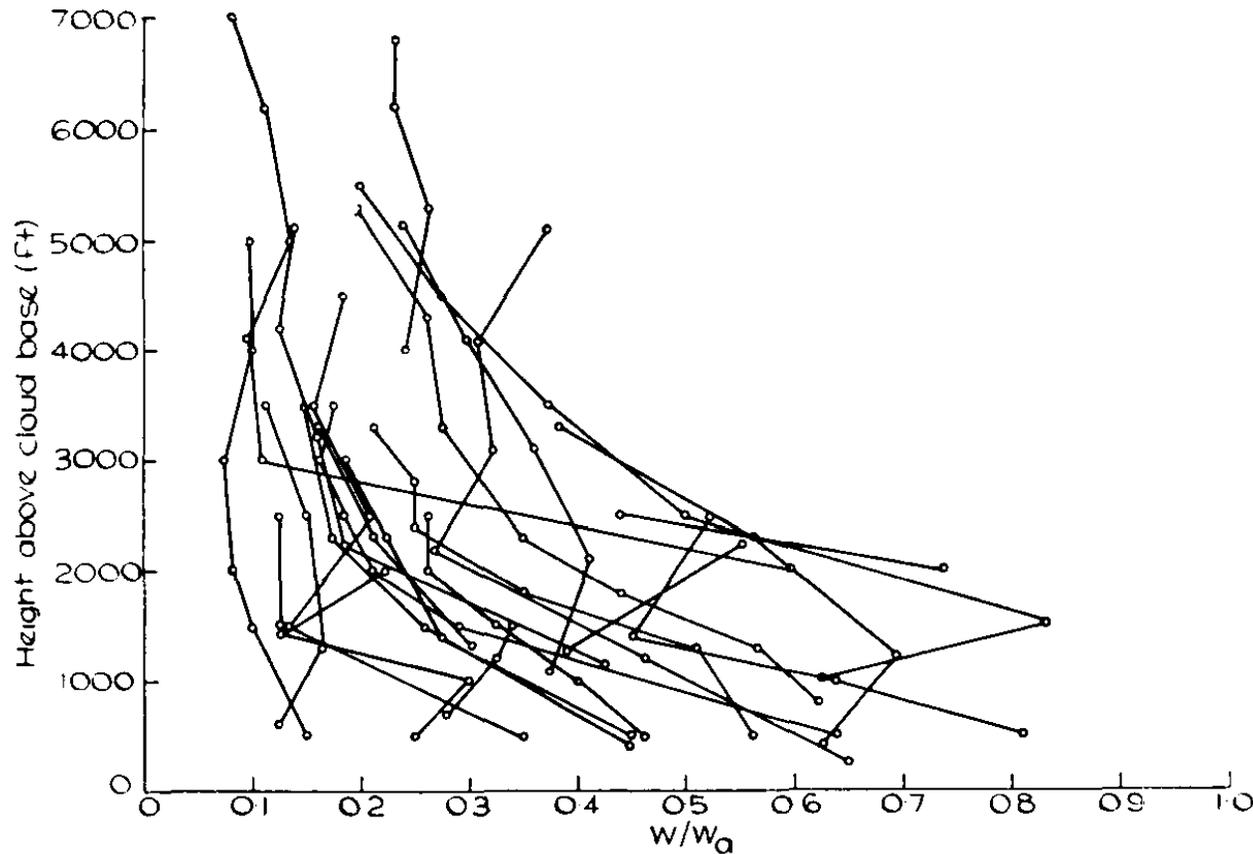


# The Water Content of Cumuliform Cloud

By J. WARNER, Radiophysics Laboratory, C.S.I.R.O., Sydney

*Tellus*, 1955

(Manuscript received April 5, 1955)

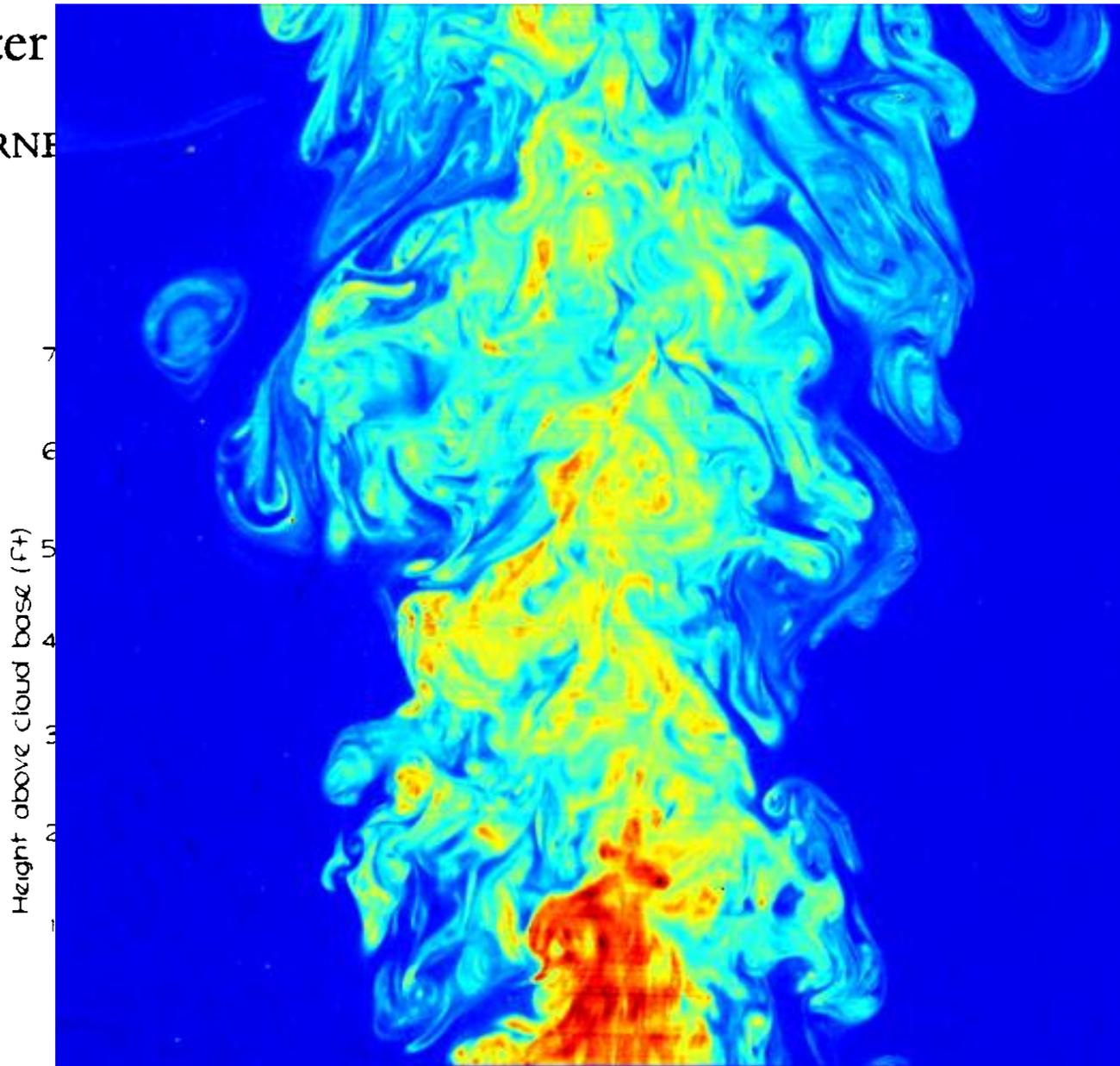


Cumulus clouds are heterogeneous and *on average* strongly diluted...

# The Water

By J. WARNE

1955



*turbulent laboratory jet*

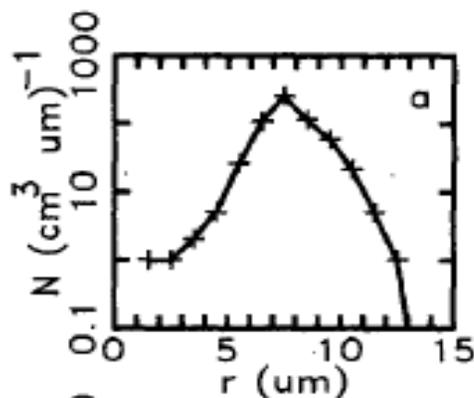
Cumulus clouds are heterogeneous and *on average* strongly diluted...

# **Growth of cloud droplets in turbulent clouds:**

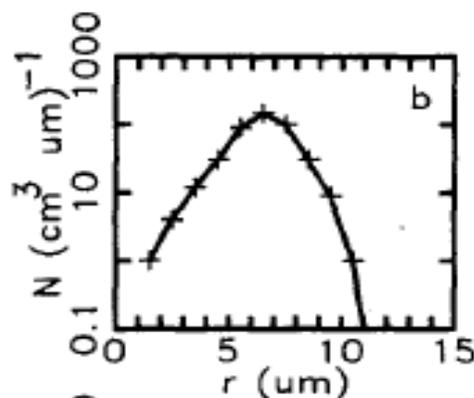
## **Part I: Growth by diffusion of water vapor**

Observed cloud droplet spectra averaged over ~100m:

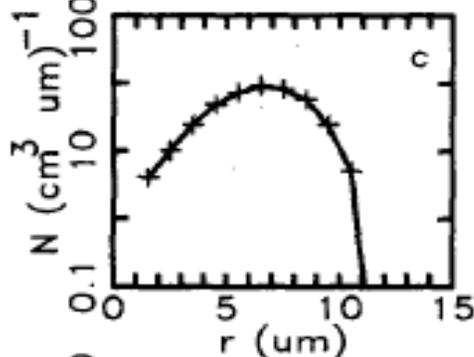
observed,  
adiabatic fraction  
 $AF \approx 1$ ;  $\sigma_r = 1.3 \mu\text{m}$



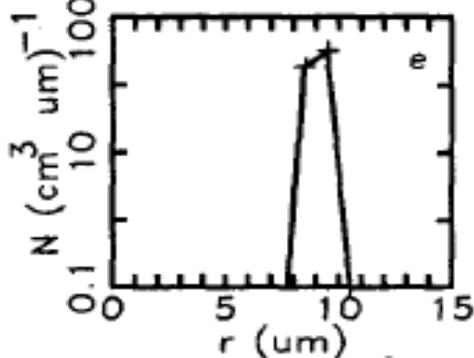
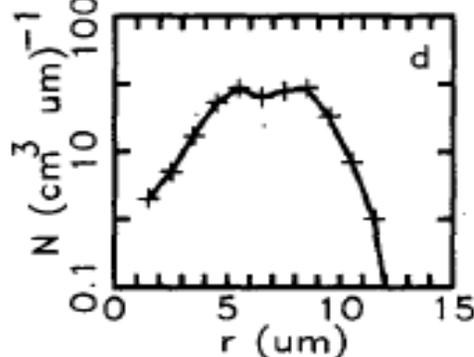
observed,  $AF \approx 0.8$ ;  
 $\sigma_r = 1.3 \mu\text{m}$



observed,  $AF \approx 0.8$ ;  
 $\sigma_r = 1.8 \mu\text{m}$



observed,  $AF \approx 1$ ;  
bimodal



calculated adiabatic  
spectrum;  $\sigma_r = 0.1 \mu\text{m}$

(Jensen et al. *JAS* 1985)

**Effects of Variable Droplet Growth Histories on Droplet Size Distributions.  
Part I: Theory**

WILLIAM A. COOPER

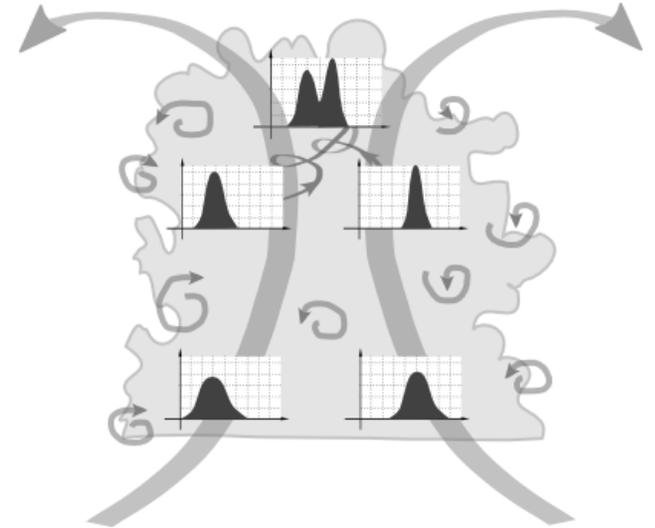
*National Center for Atmospheric Research,\* Boulder, Colorado*

(Manuscript received 4 April 1988, in final form 14 November 1988)

**The key idea: droplets observed in a single location within a turbulent cloud arrive along variety of air trajectories...**

## ***Eddy-hopping mechanism*** (Grabowski and Wang ARFM 2013)

**Droplets observed in a single location within a turbulent cloud arrive along variety of air trajectories:**



- *large scales are needed to provide different droplet activation/growth histories;*
- *small scales needed to allow hopping from one large eddy to another.*

[see also Sidin et al. (*Phys. Fluids* 2009) for idealized 2D synthetic turbulence simulations]

## Broadening of droplet size distributions from entrainment and mixing in a cumulus cloud

By SONIA G. LASHER-TRAPP<sup>†1</sup>, WILLIAM A. COOPER<sup>2</sup> and ALAN M. BLYTH<sup>3</sup>

<sup>1</sup>*New Mexico Institute of Mining and Technology, Socorro, USA*

<sup>2</sup>*National Center for Atmospheric Research, Boulder, USA*

<sup>3</sup>*University of Leeds, Leeds, UK*

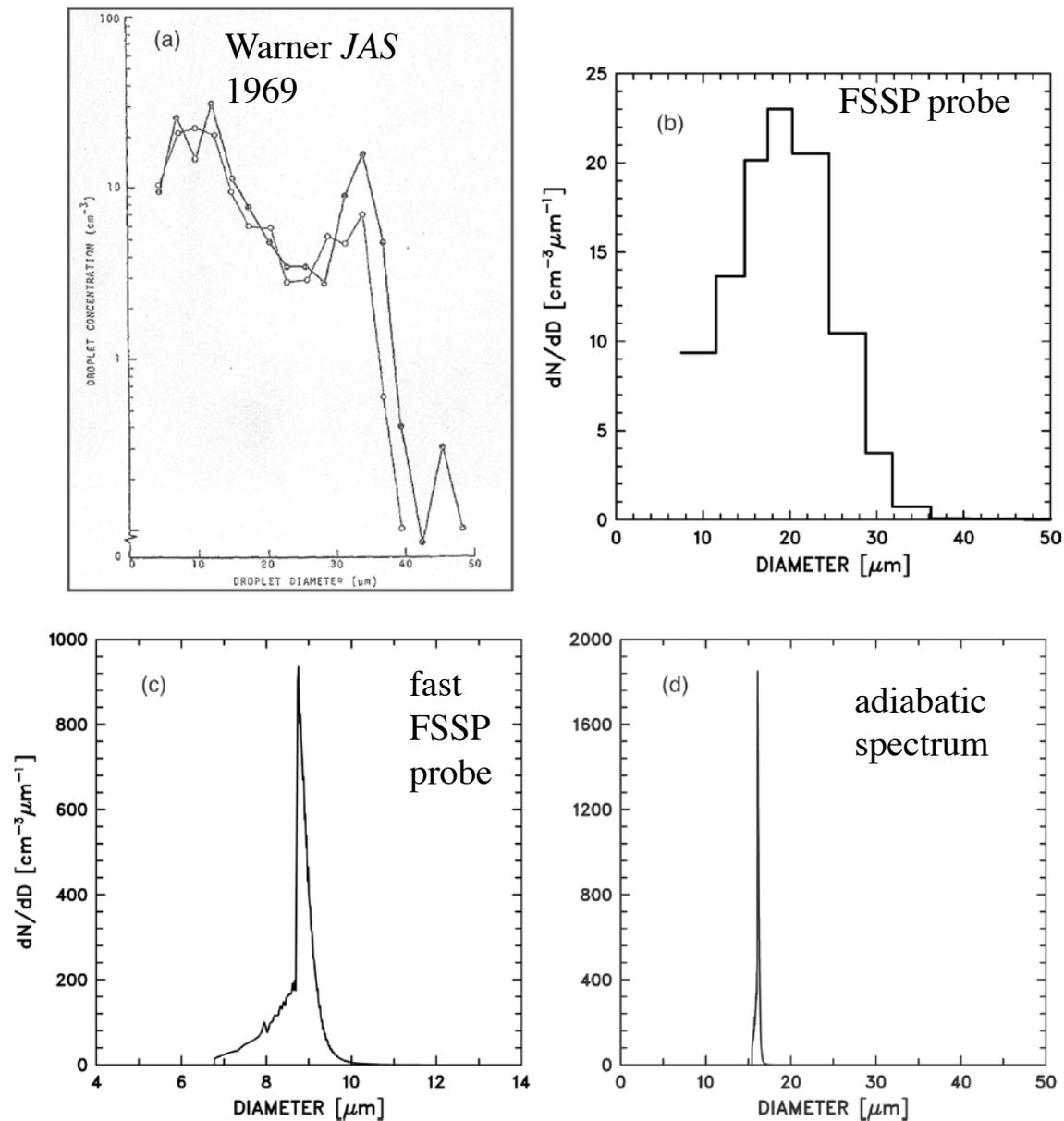
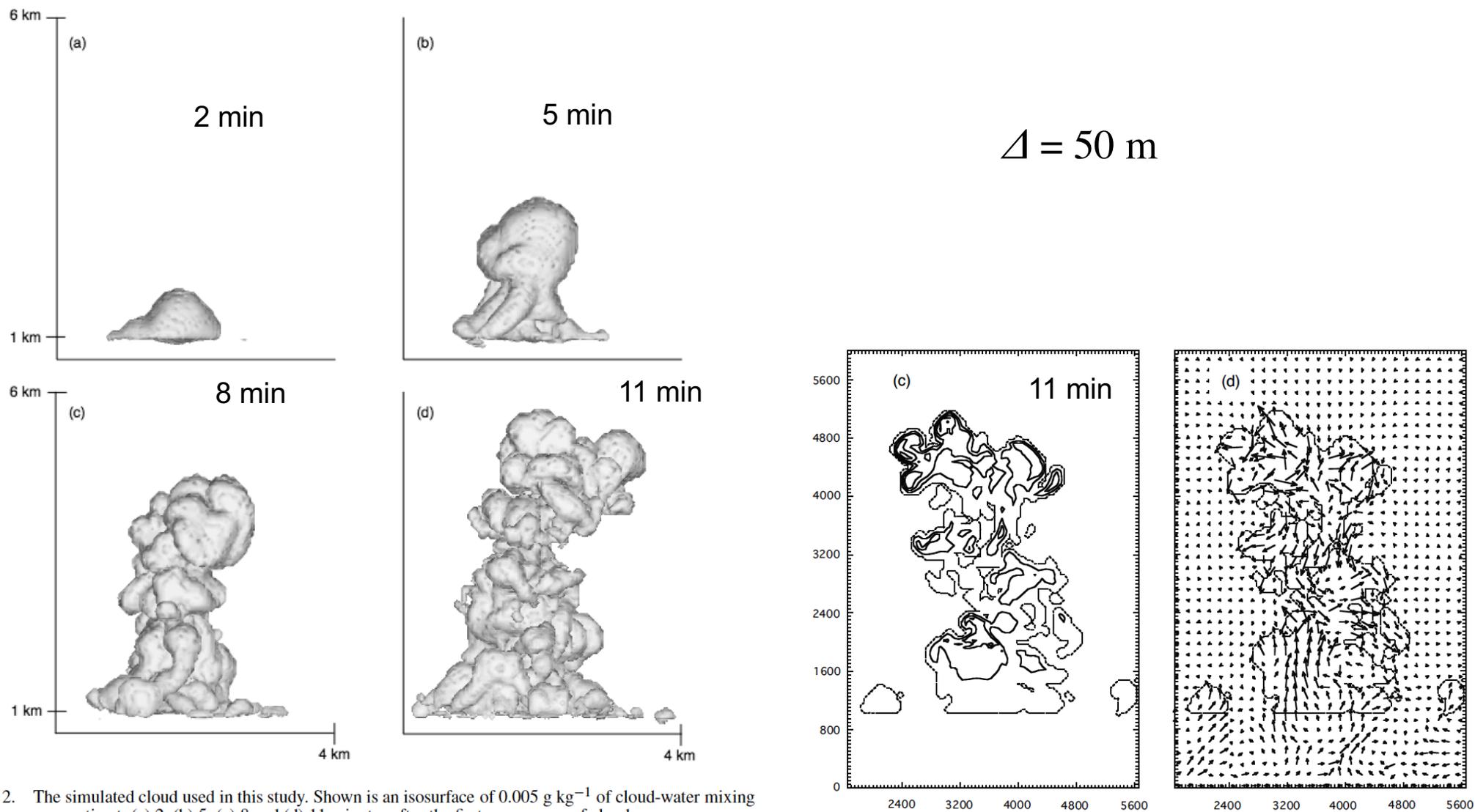


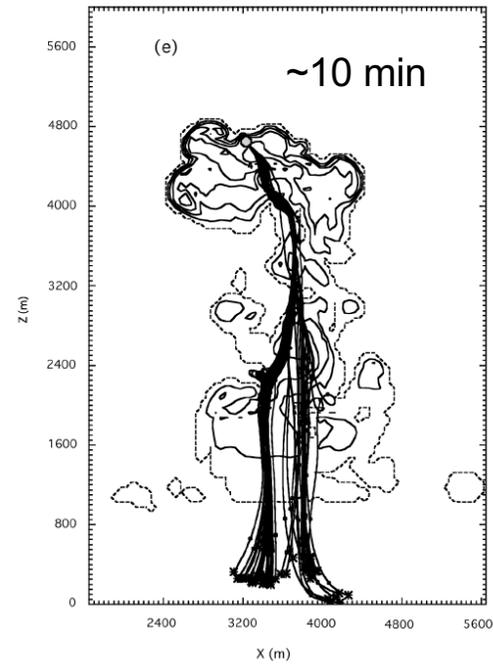
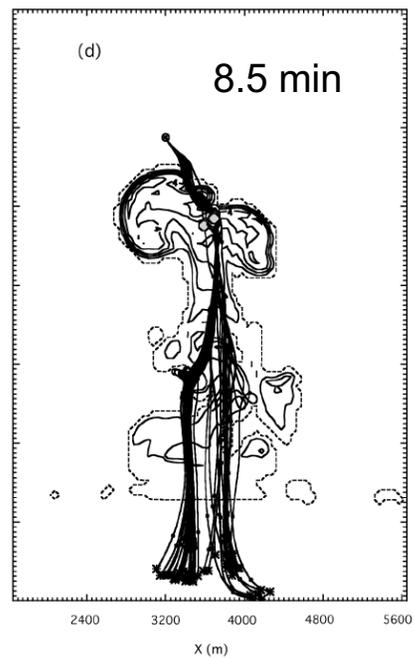
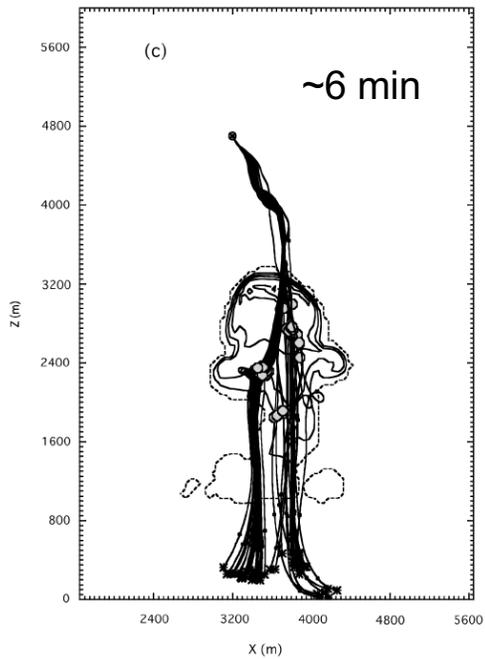
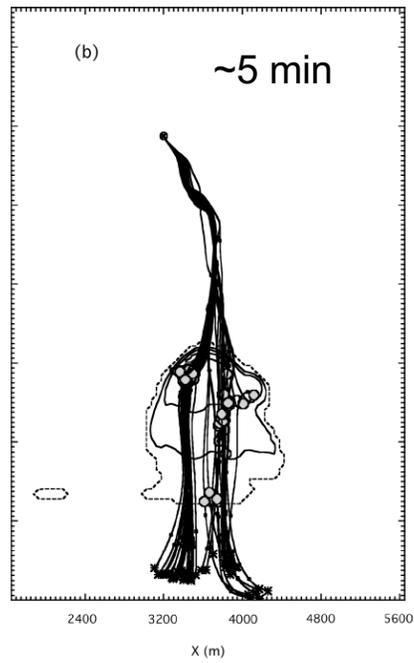
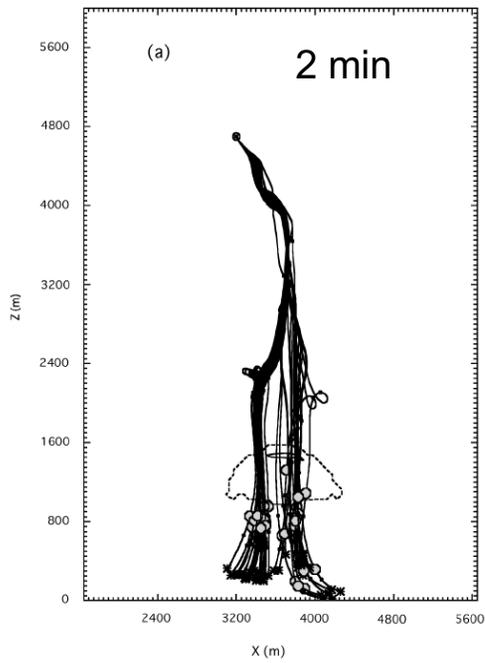
Figure 1. Observed cloud droplet size distributions collected *in situ* using: (a) exposed glass slides from Warner (1969a), (b) a Forward Scattering Spectrometer Probe (FSSP; manufactured by PMS Inc.), and (c) the fast-FSSP probe (Brennguier *et al.* 1998); (d) shows a typical droplet size distribution calculated using an adiabatic parcel model. All observed droplet size distributions are wider than that calculated.

*First, run a traditional Eulerian fluid dynamics cloud model...*



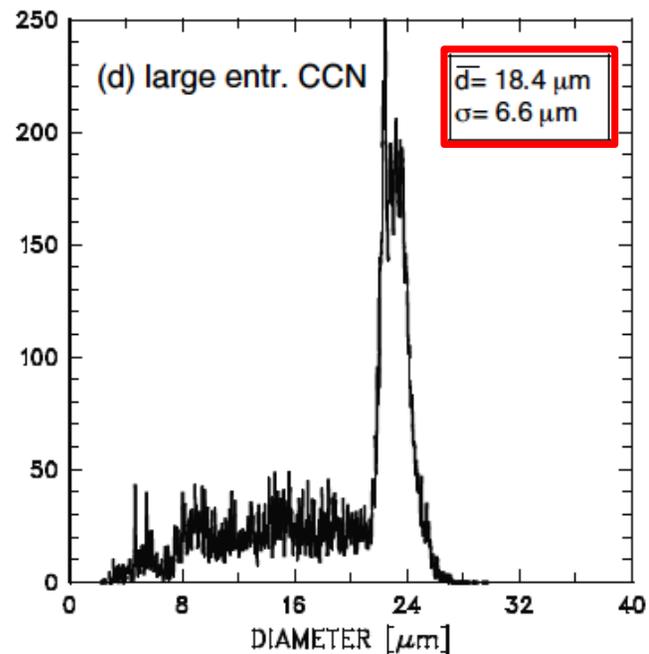
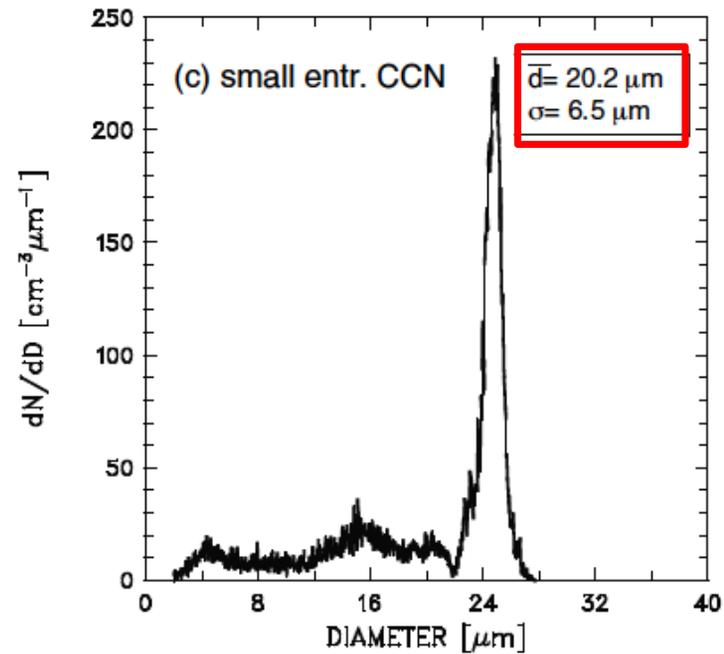
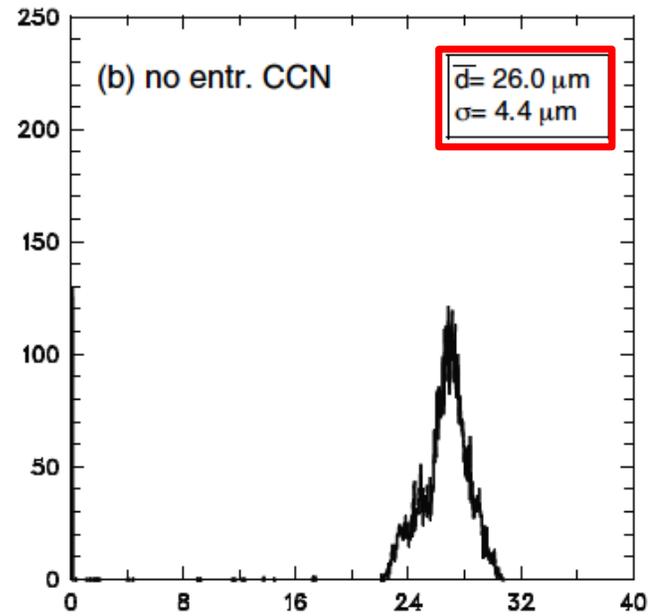
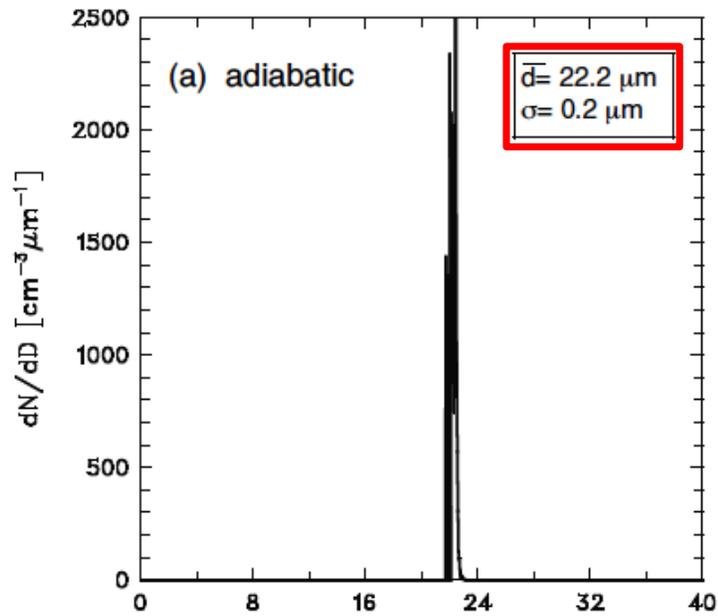
2. The simulated cloud used in this study. Shown is an isosurface of  $0.005 \text{ g kg}^{-1}$  of cloud-water mixing ratio at: (a) 2, (b) 5, (c) 8 and (d) 11 minutes after the first appearance of cloud.

*...second, run backward ensemble of trajectories from a selected point...*



**Lasher-Trapp et al. QJRMS  
2005**

*...third, calculate activation and growth of cloud droplets along trajectories.*



This is really nice to illustrate the role of eddy hopping for the spectral broadening, but the method is cumbersome and thus not practical.

Is there any other methodology that would work better?

# Lagrangian treatment of the condensed phase! aka “Lagrangian Cloud Model”, “Super-droplet method”...

**The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model**

S. Shima,<sup>a\*</sup> K. Kusano,<sup>c</sup> A. Kawano,<sup>a</sup> T. Sugiyama<sup>a</sup> and S. Kawahara<sup>b</sup>

**Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model**

M. Andrejczuk,<sup>1</sup> W. W. Grabowski,<sup>2</sup> J. Reisner,<sup>3</sup> and A. Gadian<sup>1</sup>

**Large-Eddy Simulations of Trade Wind Cumuli Using Particle-Based Microphysics with Monte Carlo Coalescence**

SYLWESTER ARABAS

*Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland*

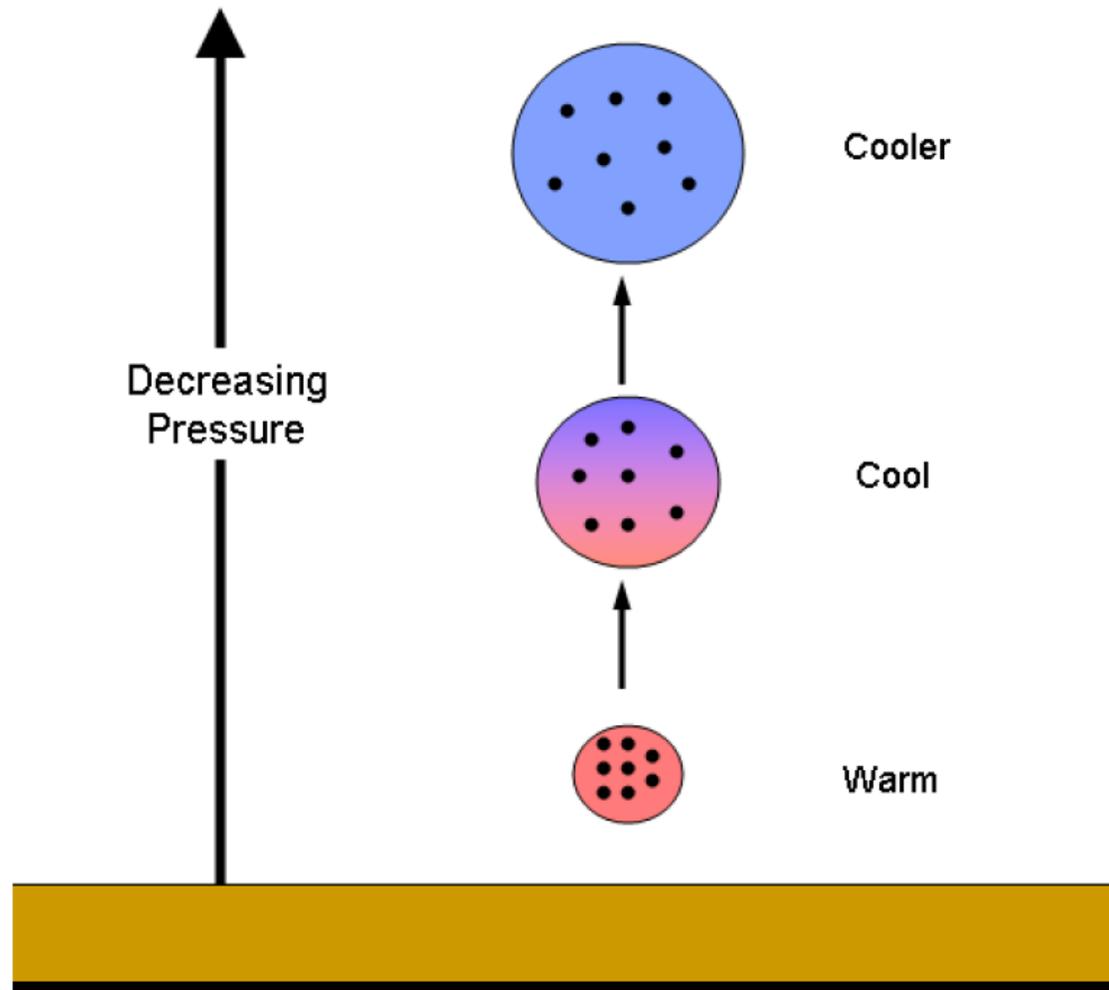
SHIN-ICHIRO SHIMA

*Graduate School of Simulation Studies, University of Hyogo, Kobe, and Japan Agency for Marine-Earth Science and Technology, Kanagawa, Japan*

**A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision**

T Riechelmann<sup>1,3</sup>, Y Noh<sup>2</sup> and S Raasch<sup>1</sup>

# The simplest model of cloud processes: the adiabatic parcel



Grabowski and Abade, 2017: Broadening of cloud droplet spectra through eddy hopping: Turbulent adiabatic parcel simulations. *J. Atmos. Sci.* **74**, 1485-1493.

$$c_p \frac{dT}{dt} = -gw + L_v C,$$

$$\frac{dq_v}{dt} = -C,$$

$$\frac{dp}{dt} = -\rho_0 w g,$$

$T$  – temperature

$q_v$  – water vapor mixing ratio

$w$  – updraft speed (1 m s<sup>-1</sup>)

$C$  – condensation rate  $C = \frac{d}{dt} \sum_i \frac{4}{3} \pi r_i^3 N_i \frac{\rho_w}{\rho_0}$

$g = 9.81 \text{ ms}^{-2}$  – gravitational acceleration

$L_v = 2.5 \times 10^6 \text{ J/kg}$  - latent heat of condensation

$p$  – environmental pressure

$\rho_0$  – environmental density (1 kg m<sup>-3</sup>)

## Cloud droplets (super-droplets; a sample of real droplets)

$$\frac{dr}{dt} = \frac{1}{r + r_0} AS$$

$r$  – droplet radius

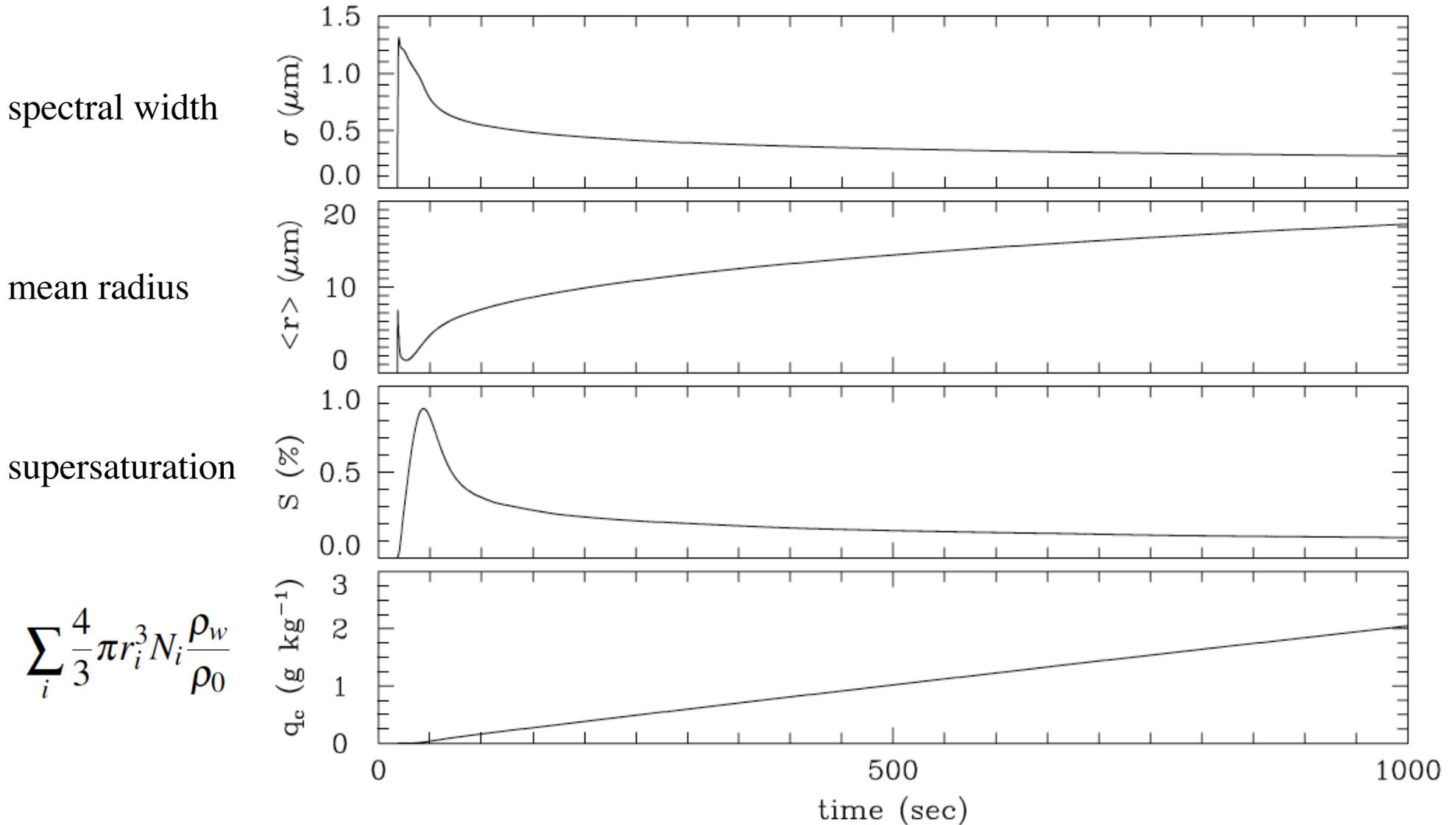
$S$  – supersaturation ( $S = q_v/q_{vs} - 1$ )

$A = 0.9152 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$

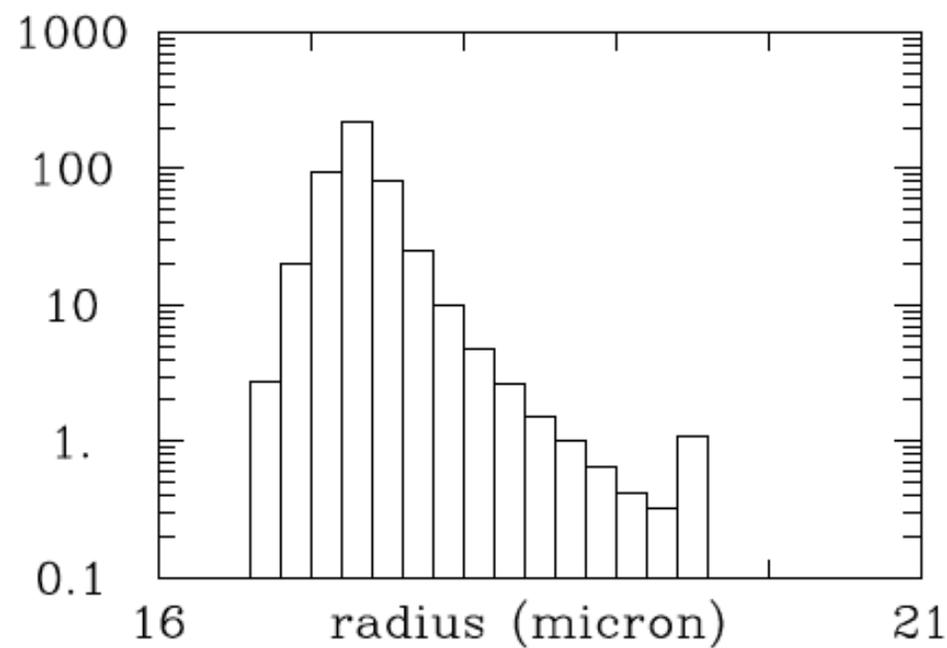
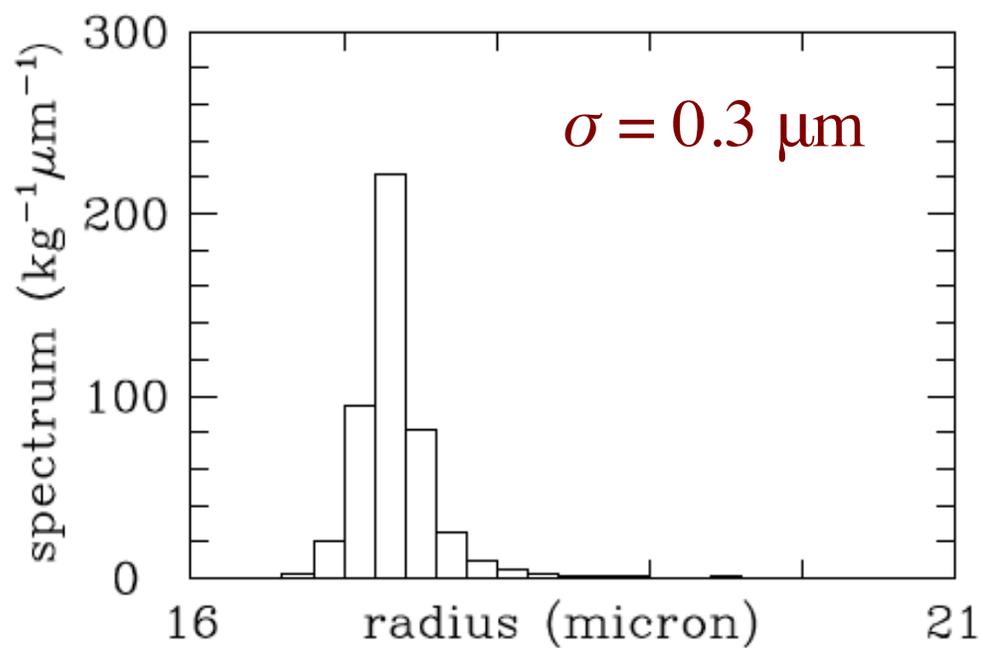
$r_0 = 1.86 \text{ } \mu\text{m}$

$T(t = 0) = 288.16 \text{ K}$ ,  $p(t = 0) = 900 \text{ hPa}$ , and relative humidity (RH) of 99%

$$w = 1 \text{ m s}^{-1}$$



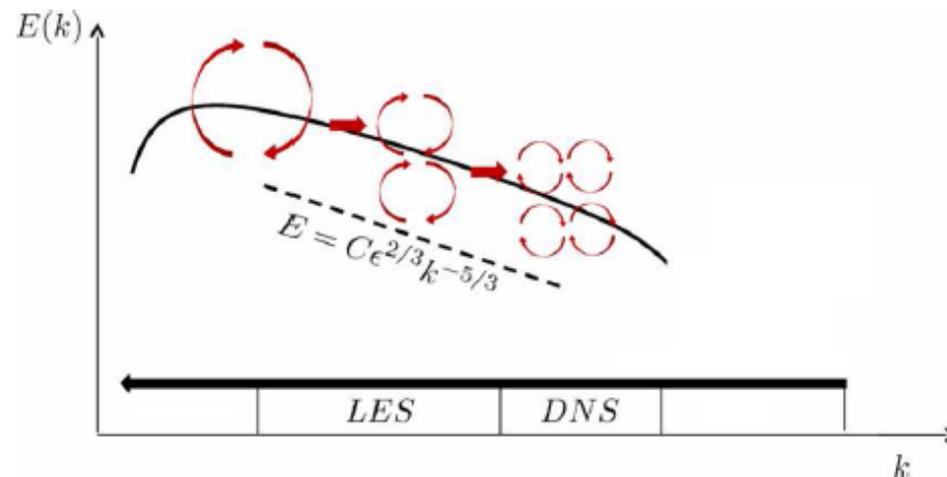
# Spectrum of cloud droplets at $t = 1000$ s:



*Turbulent adiabatic parcel model:* adiabatic parcel as before, but now assumed to be filled with homogeneous isotropic turbulence.

Two parameters determining the turbulence:

- 1) dissipation rate of TKE,  $\varepsilon$
- 2) scale (extent) of the parcel,  $L$



turbulent kinetic  
energy,  $E$

$$E = \left( \frac{L\varepsilon}{C_E} \right)^{2/3}$$

integral time  
scale,  $\tau$

$$\tau = \frac{L}{(2\pi)^{1/3}} \left( \frac{C_\tau}{E} \right)^{1/2}$$

$$C_E = 0.845$$

$$C_\tau = 1.5$$

Supersaturation fluctuation  $S'$ , on top of the mean supersaturation  $S$ , experienced by each super-droplet:

$$\frac{dS'_i}{dt} = a_1 w' - \frac{S'_i}{\tau_{\text{relax}}} \quad i - \text{superdroplet index}$$

$$\tau_{\text{relax}} = \left( a_2 \sum r_i N_i \right)^{-1} \quad a_2 = 2.8 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$
$$a_1 = 3 \times 10^{-4} \text{ m}^{-1}$$

**Important note:** phase relaxation time is the same for all droplets.

Hence, additional factors that may increase the impact (e.g., droplet concentration heterogeneities) are excluded...

Vertical velocity perturbation  $w'$  is assumed to be a random stationary processes and it is evolved in time as:

$$w'(t + \delta t) = w'(t)e^{-\delta t/\tau} + \sqrt{1 - e^{-2\delta t/\tau}} \sigma_{w'} \psi$$

$$\sigma_{w'}^2 = \frac{2}{3}E$$

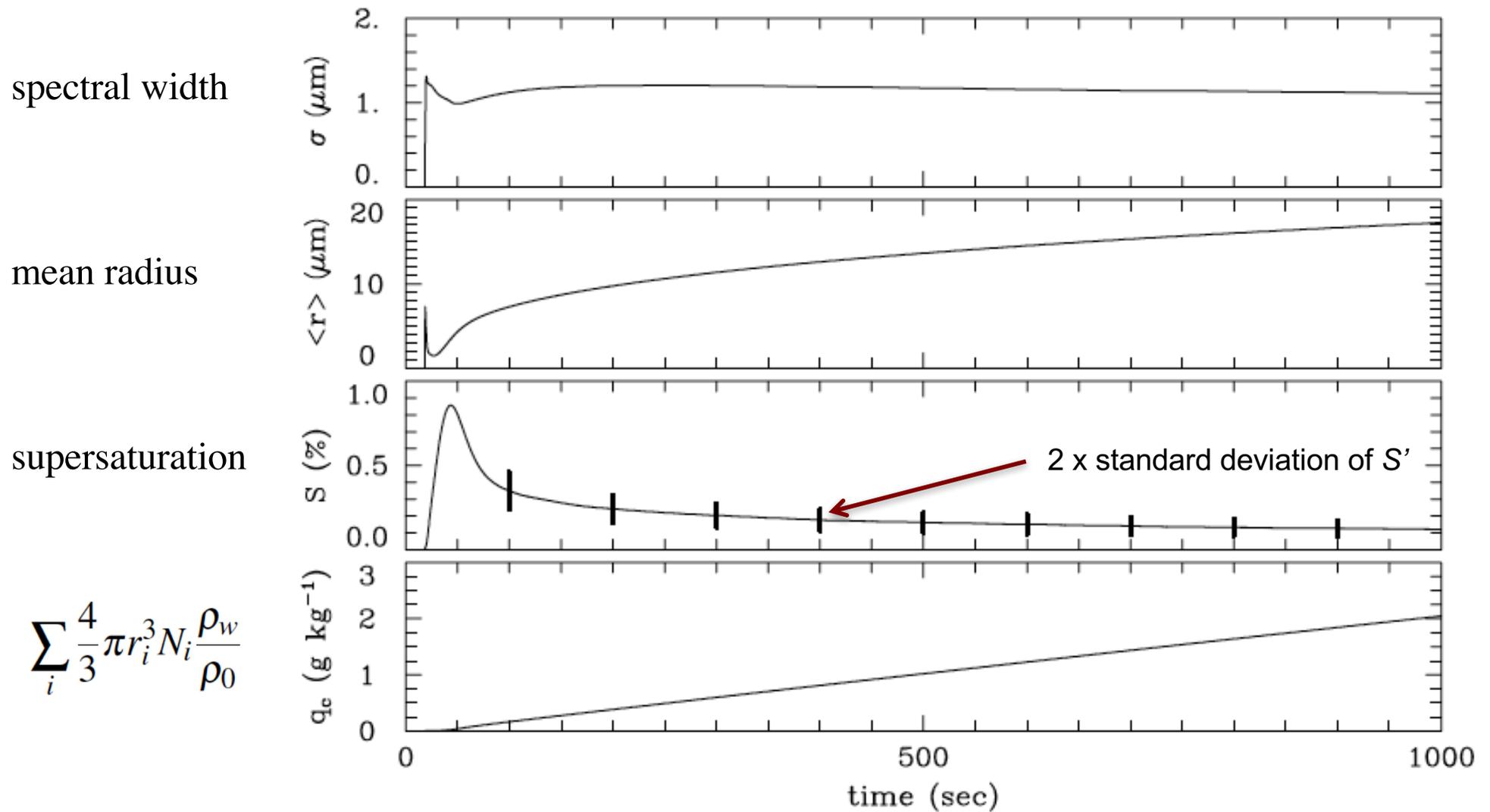
$E$  - turbulent kinetic energy

$\tau$  - turbulence integral time scale

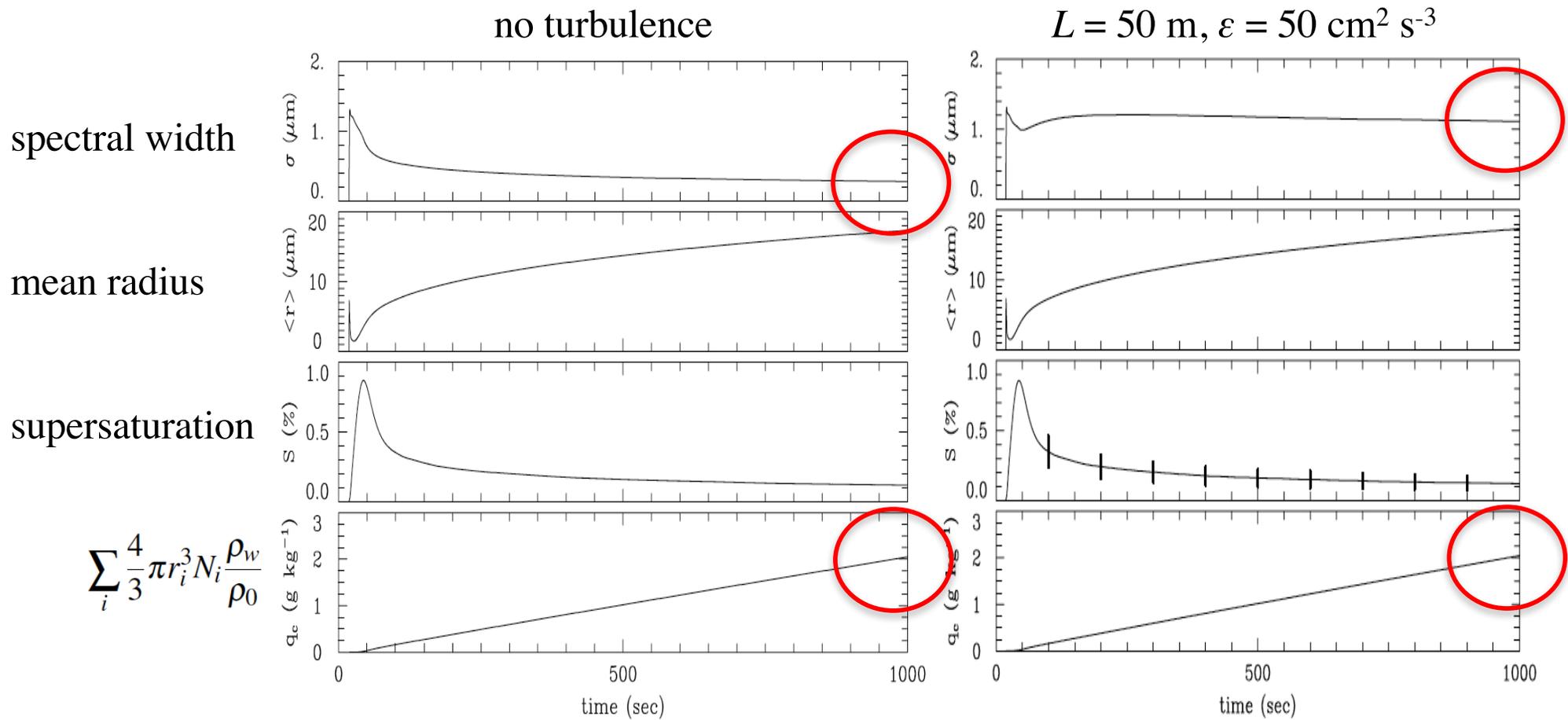
$\psi$  - Gaussian random number drawn every time step

$\delta t$  - model time step

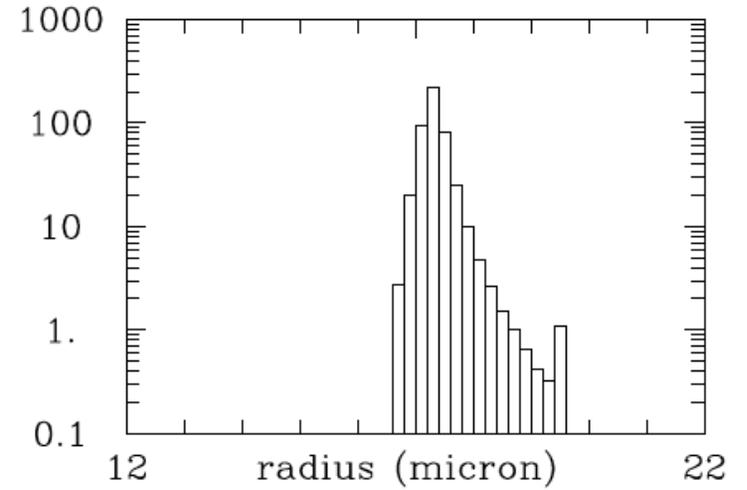
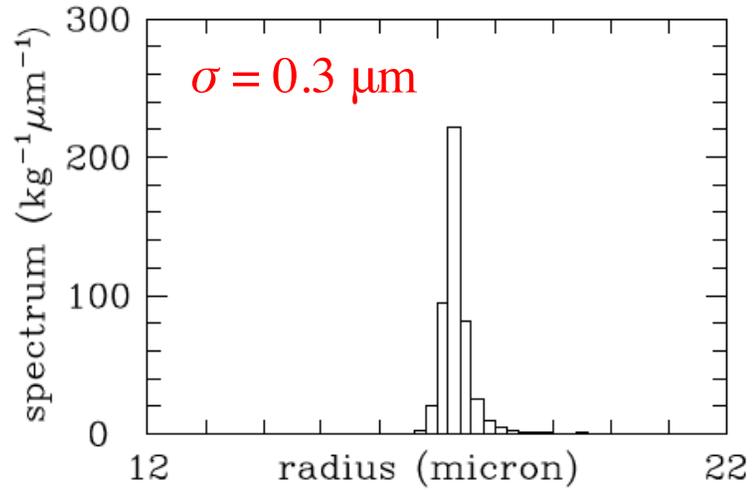
$$L = 50 \text{ m}, \varepsilon = 50 \text{ cm}^2 \text{ s}^{-3}$$



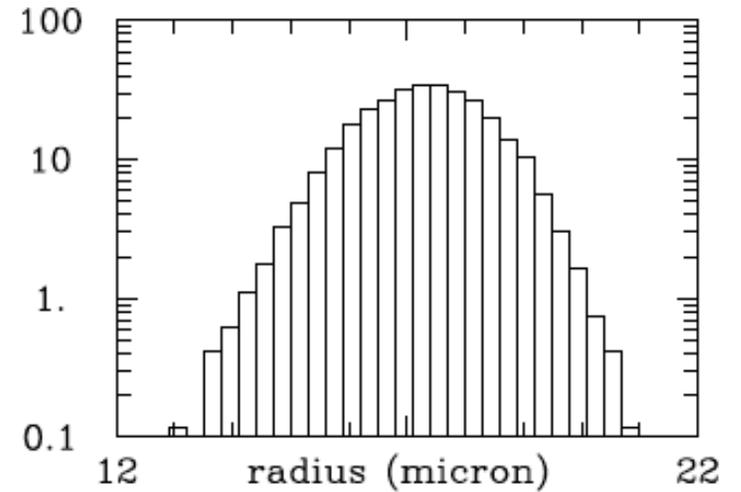
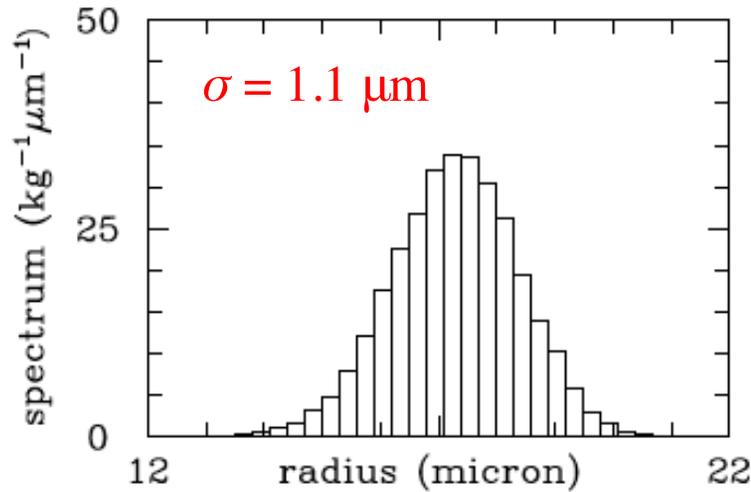
$$\sum_i \frac{4}{3} \pi r_i^3 N_i \frac{\rho_w}{\rho_0}$$



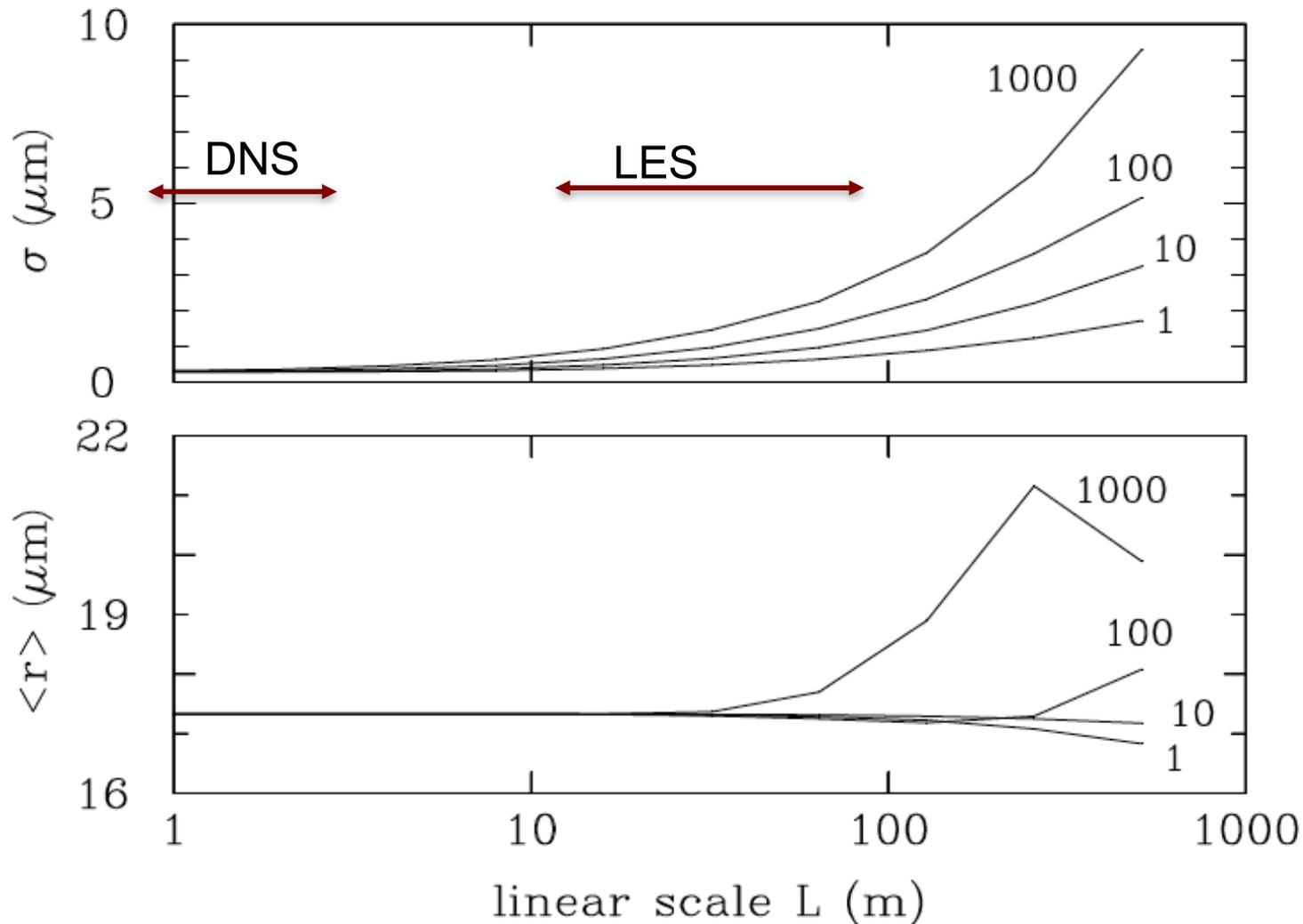
# No turbulence



$L = 50 \text{ m}, \varepsilon = 50 \text{ cm}^2 \text{ s}^{-3}$

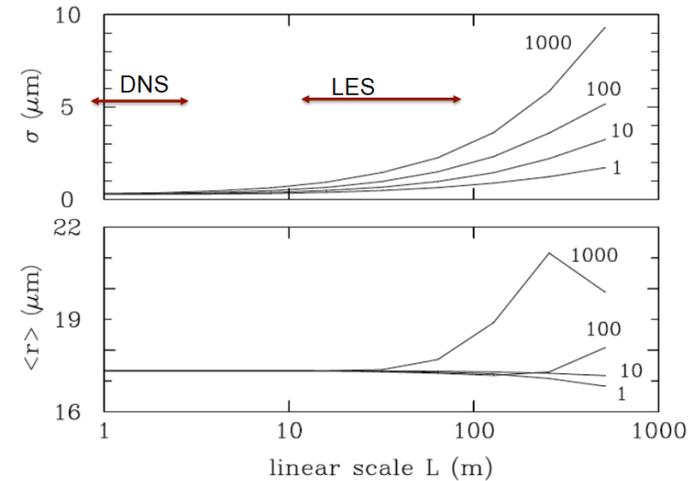


Spectral width and mean radius at  $t = 1000$ s as a function of  $L$  (m) for various dissipation rates ( $\text{cm}^2 \text{s}^{-3}$ )



## Summary for the diffusional growth:

Eddy-hopping mechanism plays a significant role in widening the droplet size distribution even for a homogeneous turbulent volume provided the the volume is large enough (i.e.,  $L$  larger than several meters).



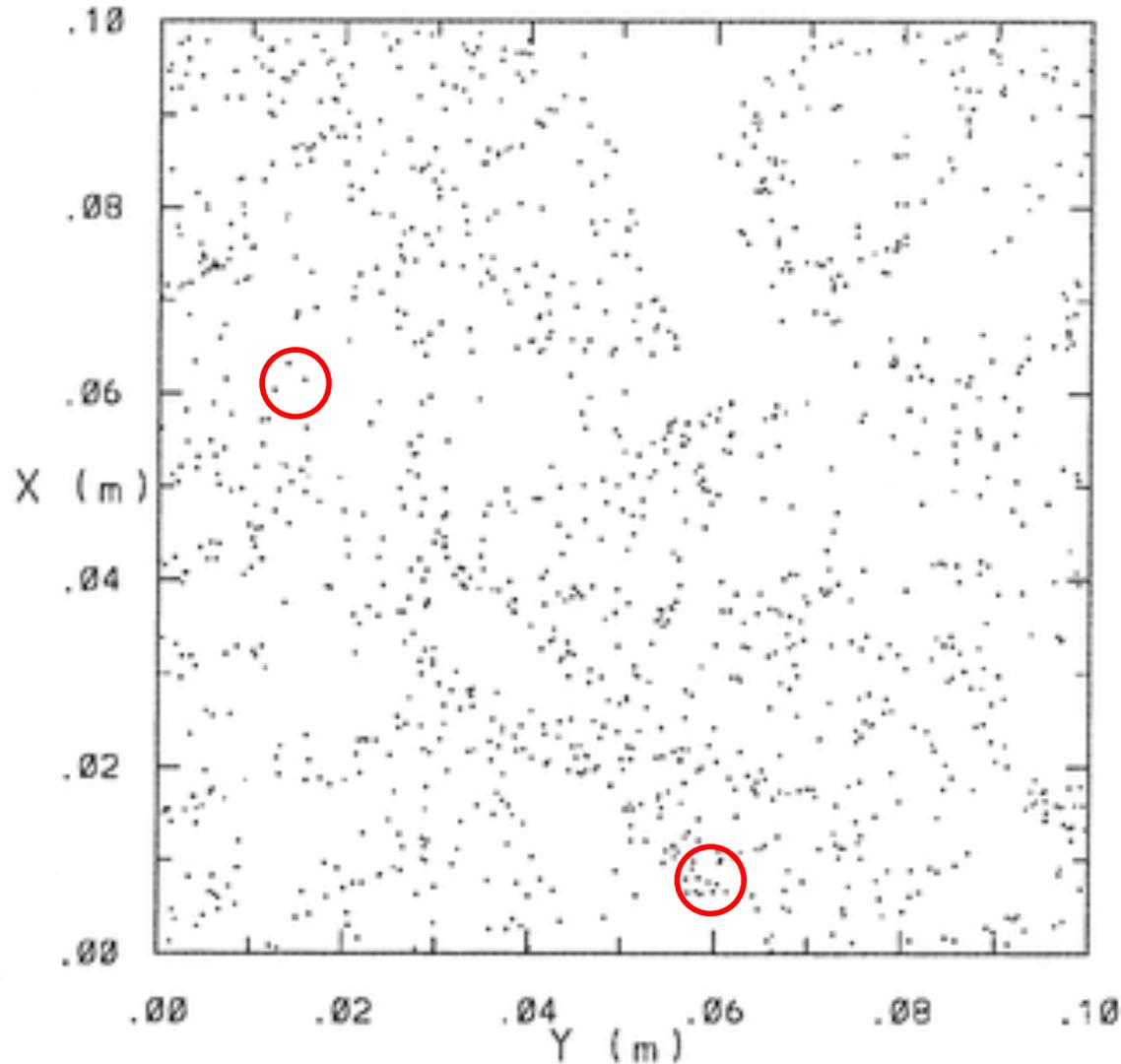
Since typical grid lengths in LES cloud simulations are a few 10s of meters, the impact of eddy hopping on the droplet spectrum needs to be included. This is straightforward when the super-droplet method is used, but difficult (impossible?) for traditional Eulerian LES models.

The eddy-hopping mechanism is especially important for rain development through collision/coalescence where the width of the spectrum is the key parameter. Quantification of this effect will be a subject of future work...

# **Growth of cloud droplets in turbulent clouds:**

## **Part I: Growth by collision-coalescence**

**Growth by collision/coalescence: nonuniform distribution of droplets in space affects droplet collisions...**



NB: insignificant impact on growth by the diffusion of water vapor at these scales; reversible vs irreversible growth (Grabowski and Wang; ARFM 2013).

## Three basic mechanisms of turbulent enhancement of gravitational collision/coalescence:

*-Turbulence modifies local droplet concentration (preferential concentration effect)*

*-Turbulence modifies relative velocity between colliding droplets (e.g., small-scale shears, fluid accelerations)*

*- Turbulence modifies hydrodynamic interactions when two droplets approach each other*

## Three basic mechanisms of turbulent enhancement of gravitational collision/coalescence:

geometric collisions  
(no hydrodynamic interactions)

*-Turbulence modifies local droplet concentration (preferential concentration effect)*

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collision efficiency

# Collision efficiency $E_c$ for the gravitational case:



Grazing trajectory

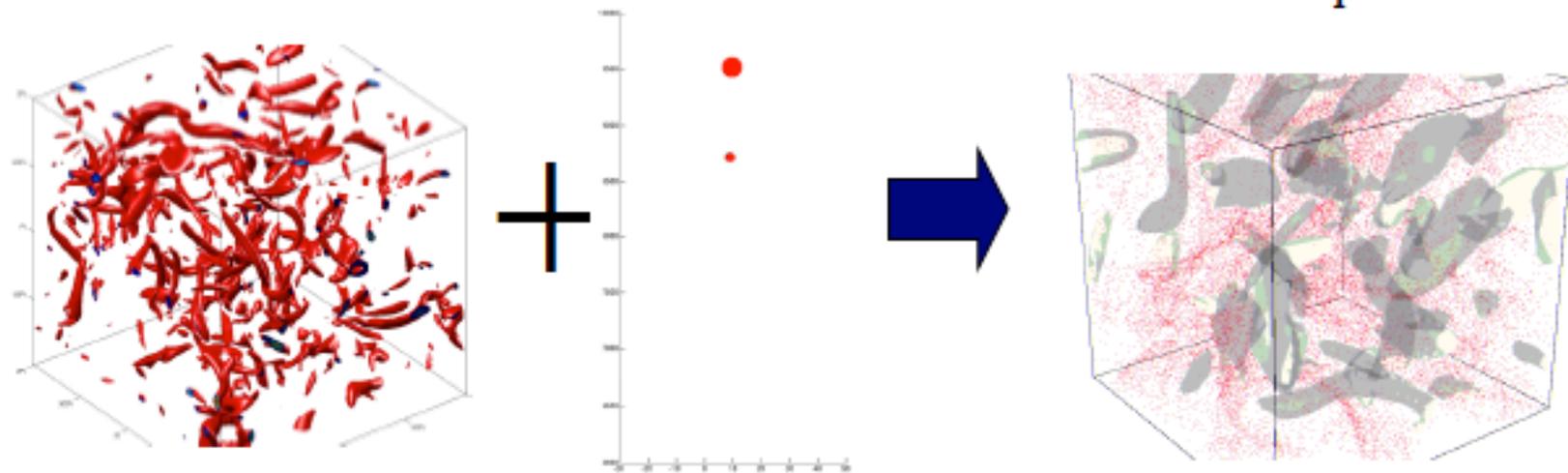
$$E_c = \frac{y_c^2}{(a_1 + a_2)^2}$$

**The hybrid DNS approach: including disturbance flows due to droplets**

$$\vec{U}(\vec{x}, t) + \sum_{k=1}^{N_p} \vec{u}_s(\vec{r}_k; a_k, \vec{V}_k - \vec{U}(\vec{Y}_k, t) - \vec{u}_k)$$

Background turbulent flow

Disturbance flows due to droplets

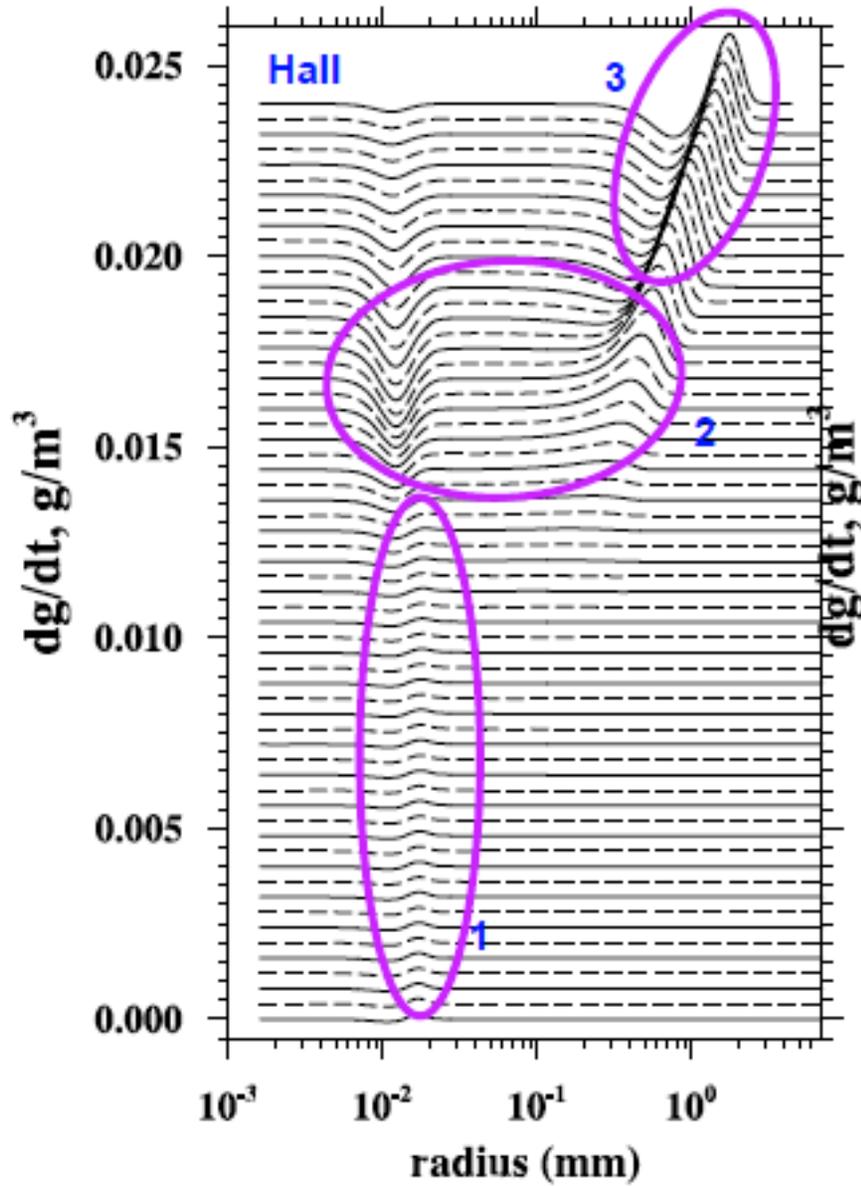


Features: Background turbulent flow can affect the disturbance flows;  
No-slip condition on the surface of each droplet is satisfied on average;  
Both near-field and far-field interactions are considered.

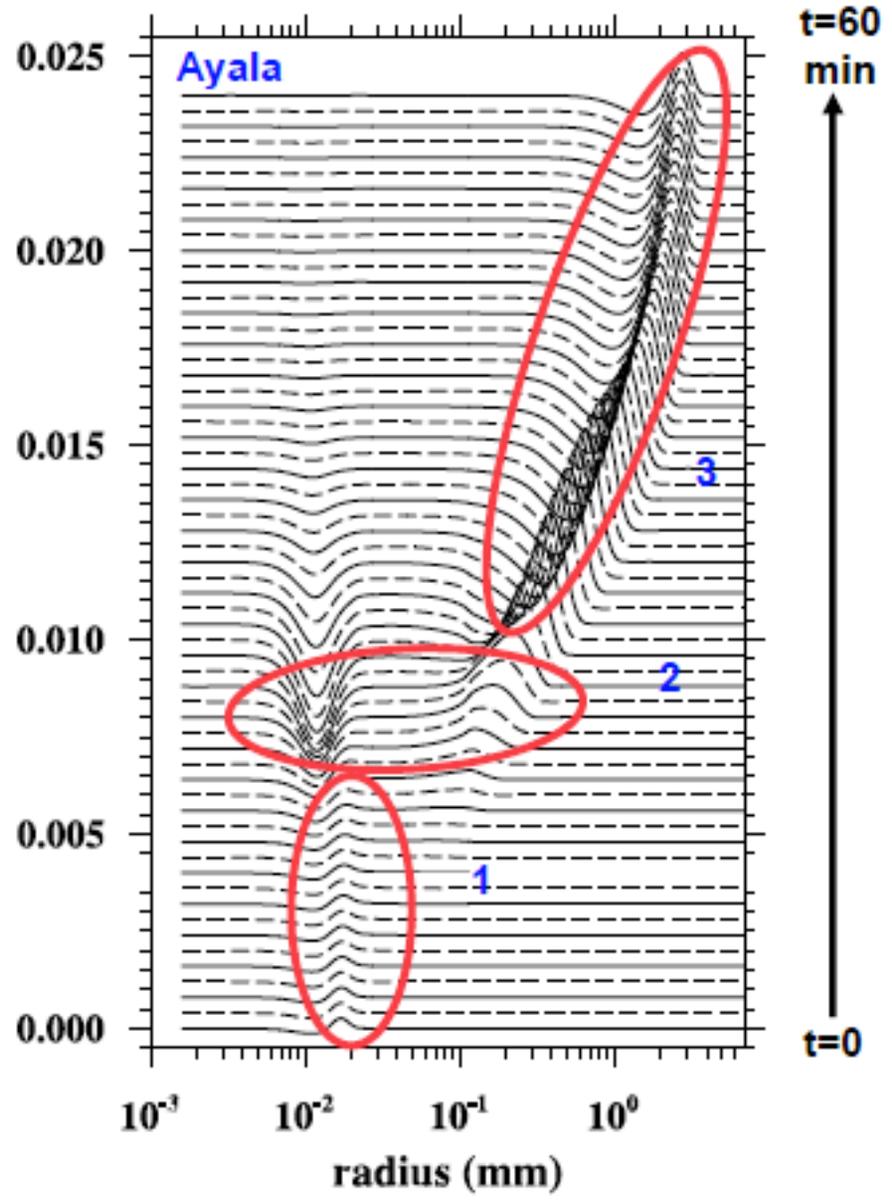
Wang, Ayala, and Grabowski, *J. Atmos. Sci.* **62**: 1255-1266 (2005).  
Ayala, Wang, and Grabowski, *J. Comp. Phys.* **225**: 51-73 (2007).



1. Autoconversion; 2. Accretion; 3. Hydrometeor self-collection  
 (Berry and Reinhardt, 1974)



without turbulence



with turbulence,  $\epsilon = 400 \text{ cm}^2\text{s}^{-3}$



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## Turbulence effects on warm-rain autoconversion in precipitating shallow convection

Axel Seifert,<sup>a\*</sup> Louise Nuijens<sup>b</sup> and Bjorn Stevens<sup>b,c</sup>

<sup>a</sup>*Deutscher Wetterdienst, Offenbach, Germany*

<sup>b</sup>*Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles, USA*

<sup>c</sup>*Max-Planck-Institut für Meteorologie, Hamburg, Germany*

\*Correspondence to: A. Seifert, Deutscher Wetterdienst, GB Forschung und Entwicklung, Frankfurterstrasse 139, 63067 Offenbach, Germany. E-mail: axel.seifert@dwd.de

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A parametrization of the rain formation in warm clouds is developed which includes the effects of turbulence on the collision rate of droplets in a cloud. It is shown that already moderate turbulence with dissipation rates of  $400 \text{ cm}^2 \text{ s}^{-3}$  can lead to a significant speed-up of the rain formation corresponding to an increase in the autoconversion rate by a factor of 4–6 depending on the size of the droplets. Large-eddy simulations of trade wind cumuli also produce a significant enhancement of rain formation through turbulence effects on cloud microphysics. In small cumulus clouds, the highest turbulent dissipation rates occur near cloud-top, which is also the decisive region of the cloud for the initial rain formation. Copyright © 2010 Royal Meteorological Society

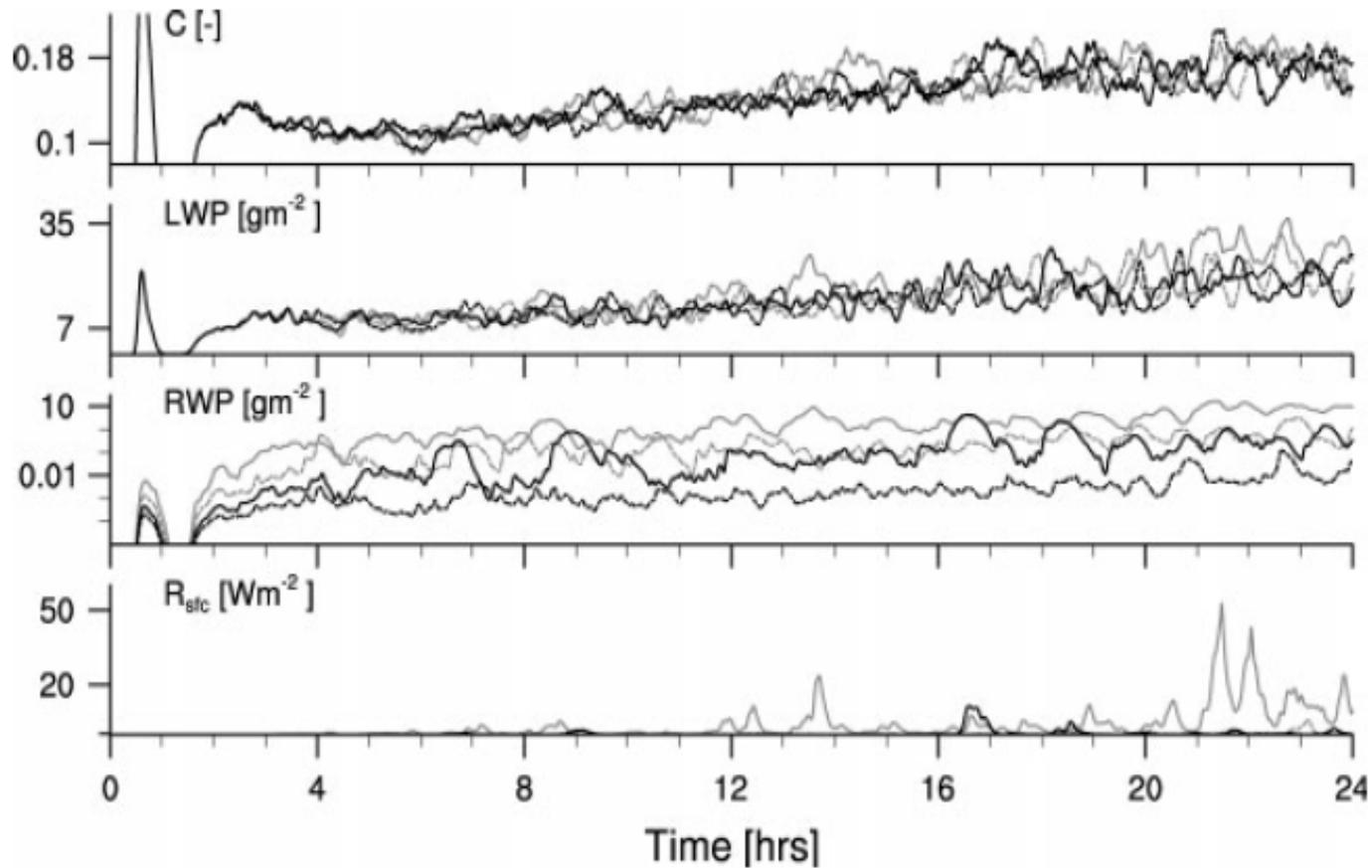


Figure 5. Time series of the number of cloudy columns (cloud cover)  $C$ , liquid (cloud + rain) water path  $L$ , rain water path  $R$ , and surface rain-rate  $R_{sfc}$  for simulations with cloud droplet number concentrations  $N_c$  of  $140 \text{ cm}^{-3}$  (grey) and  $300 \text{ cm}^{-3}$  (black), with (solid line) and without (dashed line) turbulence-enhanced coalescence.

Table II. Sensitivity to turbulence-enhanced coalescence (T), versus no turbulence enhancement (NT), for cloud droplet number concentrations  $N_c = 70, 140$  and  $300 \text{ mg}^{-1}$ . NT-140-hr and T-140-hr represent simulations with doubled horizontal resolution (grid spacing of 50 m).

Run	$L$ ( $\text{gm}^{-2}$ )	$R$ ( $\text{gm}^{-2}$ )	$z_i$ (m)	$C$	$R_{\text{sfc}}$ ( $\text{W m}^{-2}$ )	$R_{\text{max}}$ ( $\text{W m}^{-2}$ )	$N_r$ ( $\text{dm}^{-3}$ )
NT-70	18.6	7.0	2418	0.17	8.6	16.6	19.7
T-70	19.3	22.2	2358	0.15	43.3	51.6	26.6
NT-140	18.9	0.8	2449	0.17	0.8	2.0	8.7
T-140	19.7	8.3	2422	0.17	13.2	18.8	14.9
NT-140-hr	21.1	1.0	2422	0.21	1.1	2.6	8.9
T-140-hr	21.9	3.9	2399	0.21	4.9	9.9	10.9
NT-300	20.2	0.0	2442	0.17	0.0	0.0	4.7
T-300	18.3	0.4	2438	0.16	0.4	0.9	6.4

Variables are cloud (liquid) water path  $L$ , rain water path  $R$ , inversion height  $z_i$ , fraction of cloudy columns  $C$ , rain-drop number concentrations averaged over raining regions  $N_r$ , surface rain rate  $R_{\text{sfc}}$ , and the maximum rain-rate  $R_{\text{max}}$  within the (domain-averaged) profile of rain-rate.

All variables are averaged over the last four hours of each simulation.

A rain rate of  $29 \text{ W m}^{-2}$  corresponds to  $1 \text{ mm day}^{-1}$ .

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## Turbulent collision-coalescence in maritime shallow convection

A. A. Wyszogrodzki<sup>1</sup>, W. W. Grabowski<sup>1</sup>, L.-P. Wang<sup>2</sup>, and O. Ayala<sup>2,3</sup>

<sup>1</sup>National Center for Atmospheric Research, Boulder, Colorado, USA

<sup>2</sup>Department of Mechanical Engineering, University of Delaware, Newark, Delaware, USA

<sup>3</sup>Department of Engineering Technology, Old Dominion University, Norfolk, Virginia, USA

*Correspondence to:* W. W. Grabowski (grabow@ncar.ucar.edu)

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# A Large Eddy Simulation Intercomparison Study of Shallow Cumulus Convection

JAS  
2003

A. PIER SIEBESMA,<sup>a</sup> CHRISTOPHER S. BRETHERTON,<sup>b</sup> ANDREW BROWN,<sup>c</sup> ANDREAS CHLOND,<sup>d</sup> JOAN CUXART,<sup>e</sup>  
PETER G. DUYNKERKE,<sup>f\*</sup> HONGLI JIANG,<sup>g</sup> MARAT KHAIROUTDINOV,<sup>h</sup> DAVID LEWELLEN,<sup>i</sup> CHIN-HOH MOENG,<sup>j</sup>  
ENRIQUE SANCHEZ,<sup>k</sup> BJORN STEVENS,<sup>l</sup> AND DAVID E. STEVENS<sup>m</sup>

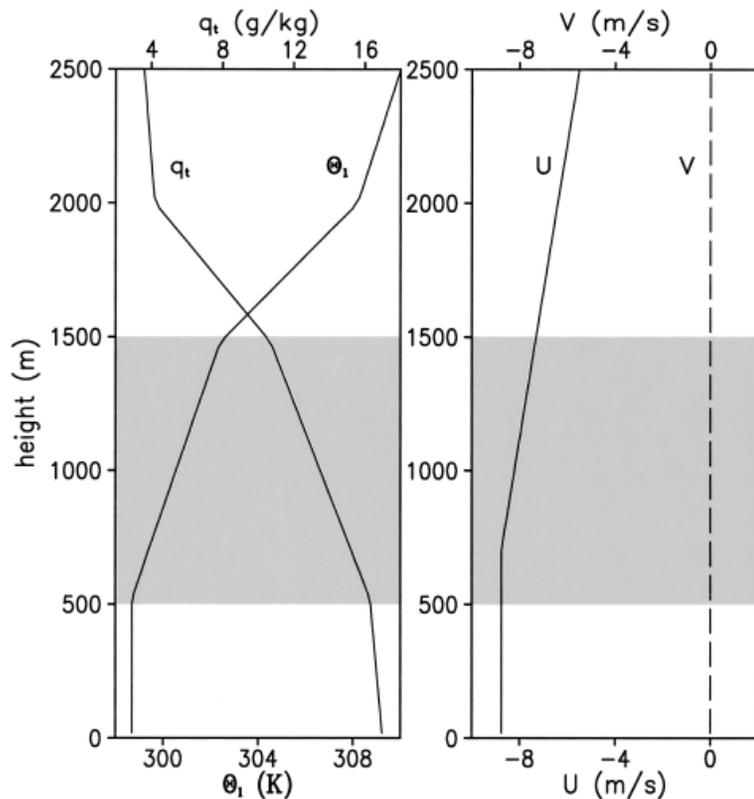
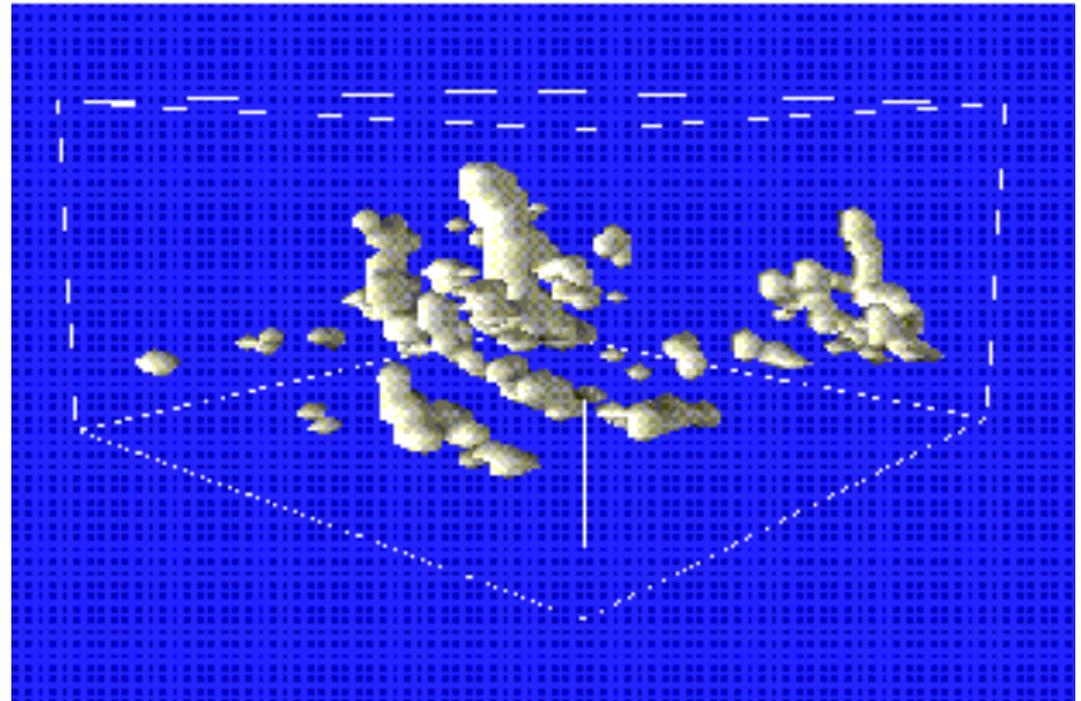


FIG. 1. Initial profiles of the total water specific humidity  $q_t$ , the liquid water potential temperature  $\theta_l$ , and the horizontal wind components  $u$  and  $v$ . The shaded area denotes the conditionally unstable cloud layer.



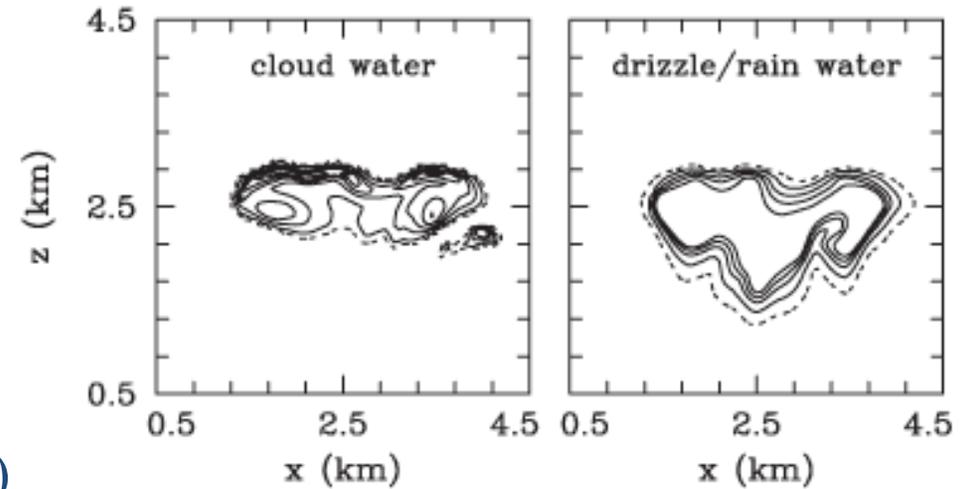
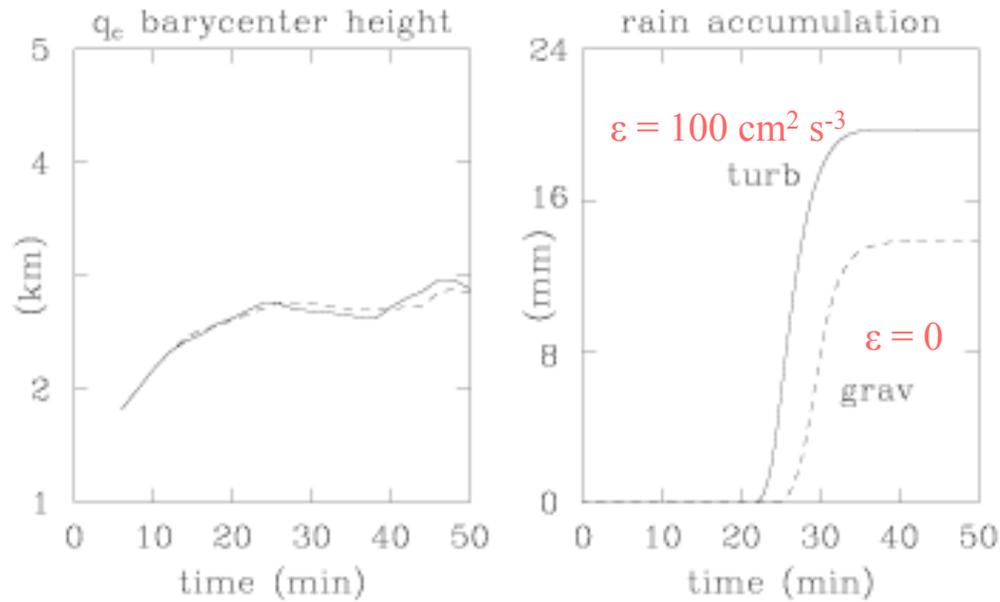
The Barbados Oceanographic and Meteorological Experiment (BOMEX) case (Holland and Rasmusson 1973)

Rain formation depends critically on the CCN concentration, so we consider a range ...

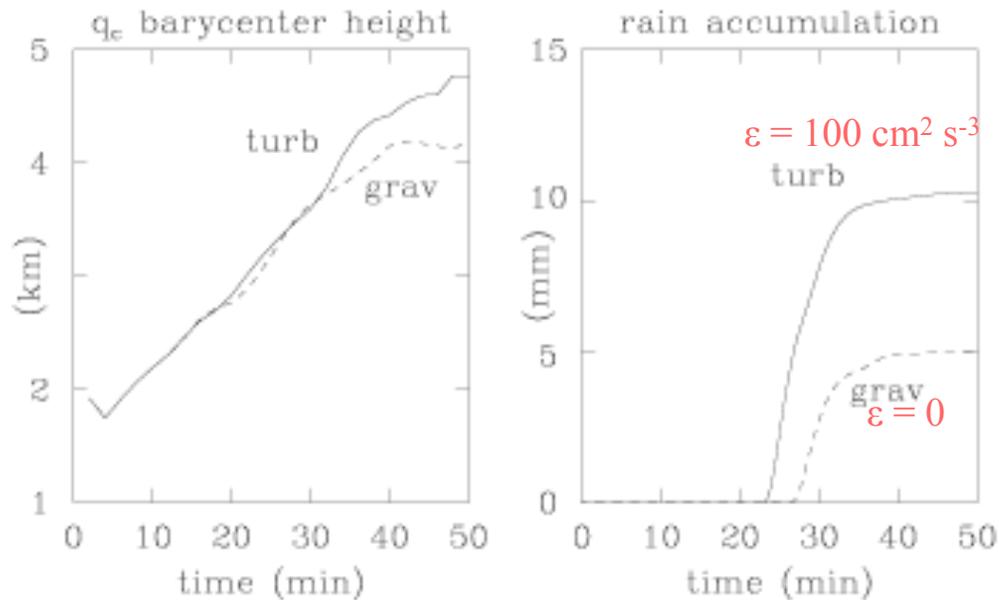
$$N_{\text{CCN}} = \begin{cases} N_{\text{CCN}}^0 & \text{for } S > 1 \\ N_{\text{CCN}}^0 S^{0.4} & \text{for } 0.1 < S < 1 \\ N_{\text{CCN}}^0 (0.1)^{-3.6} S^4 & \text{for } S < 0.1 \end{cases}$$

$N_{\text{CCN}}^0$ : 30, 60, 120, 240  $\text{mg}^{-1}$

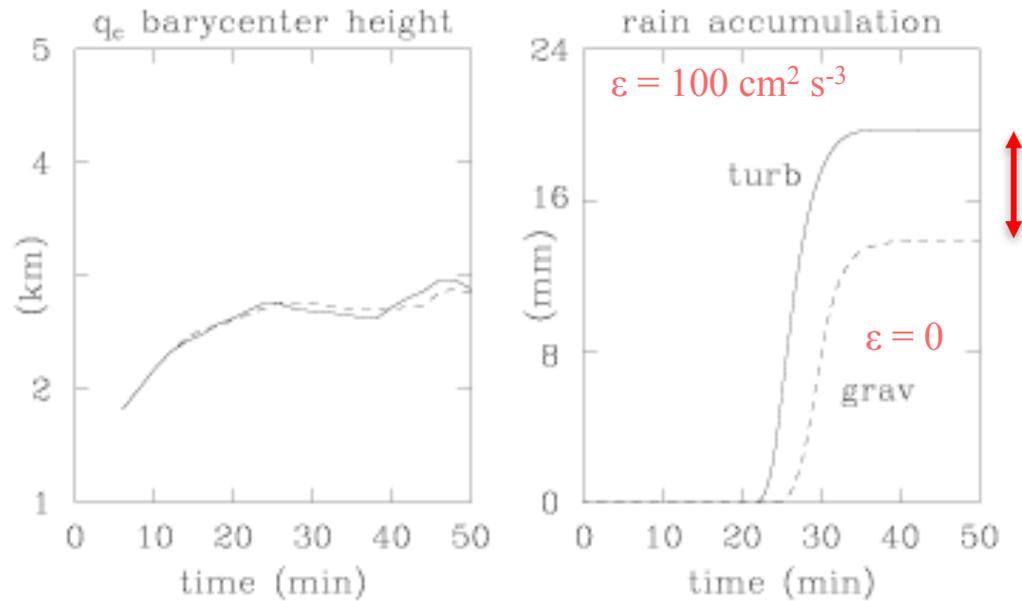
## Simulation with an inversion at 2.5 km (2 layers)



## Simulation without an inversion (1 layer)

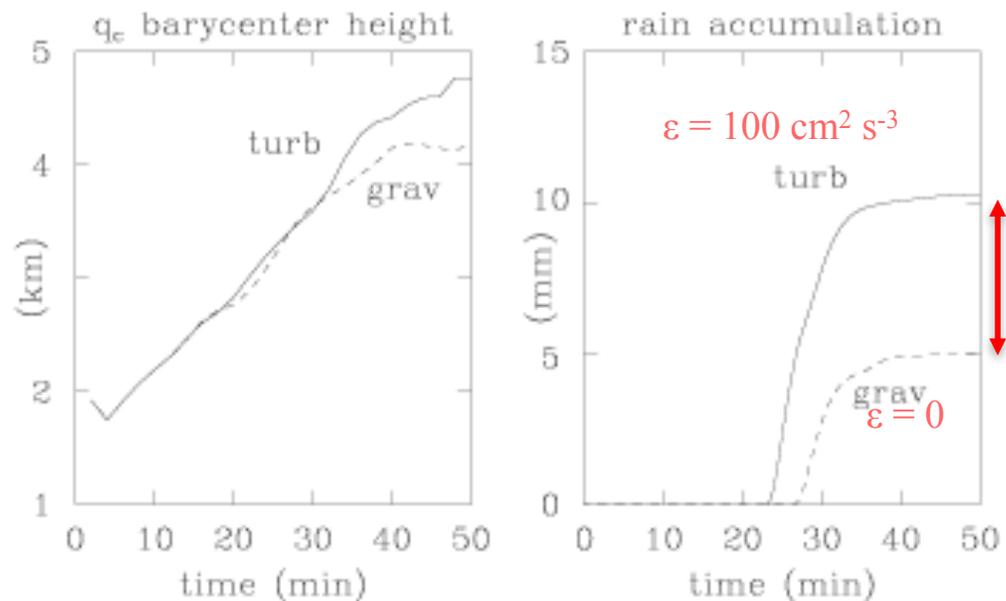


## Simulation with an inversion at 2.5 km (2 layers)

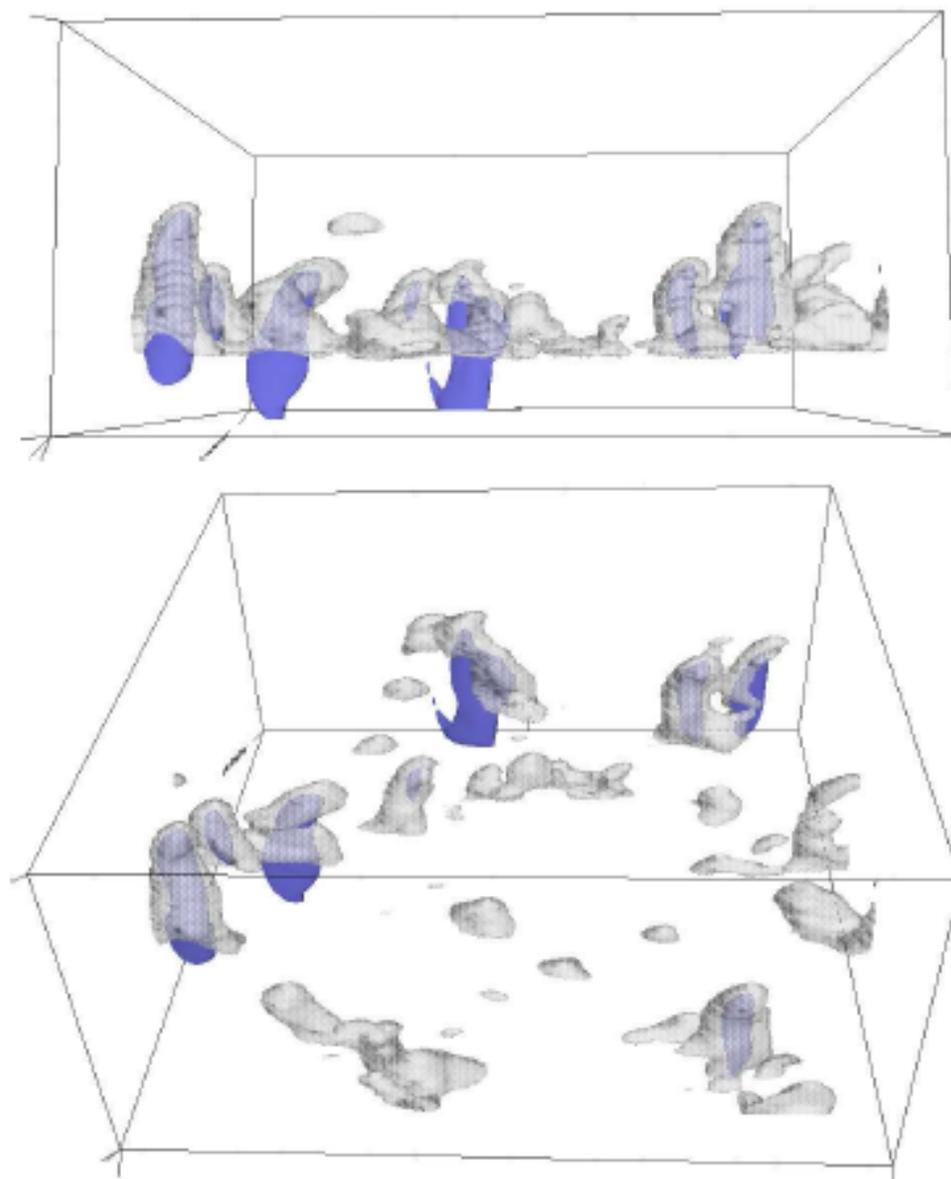


Microphysical  
enhancement

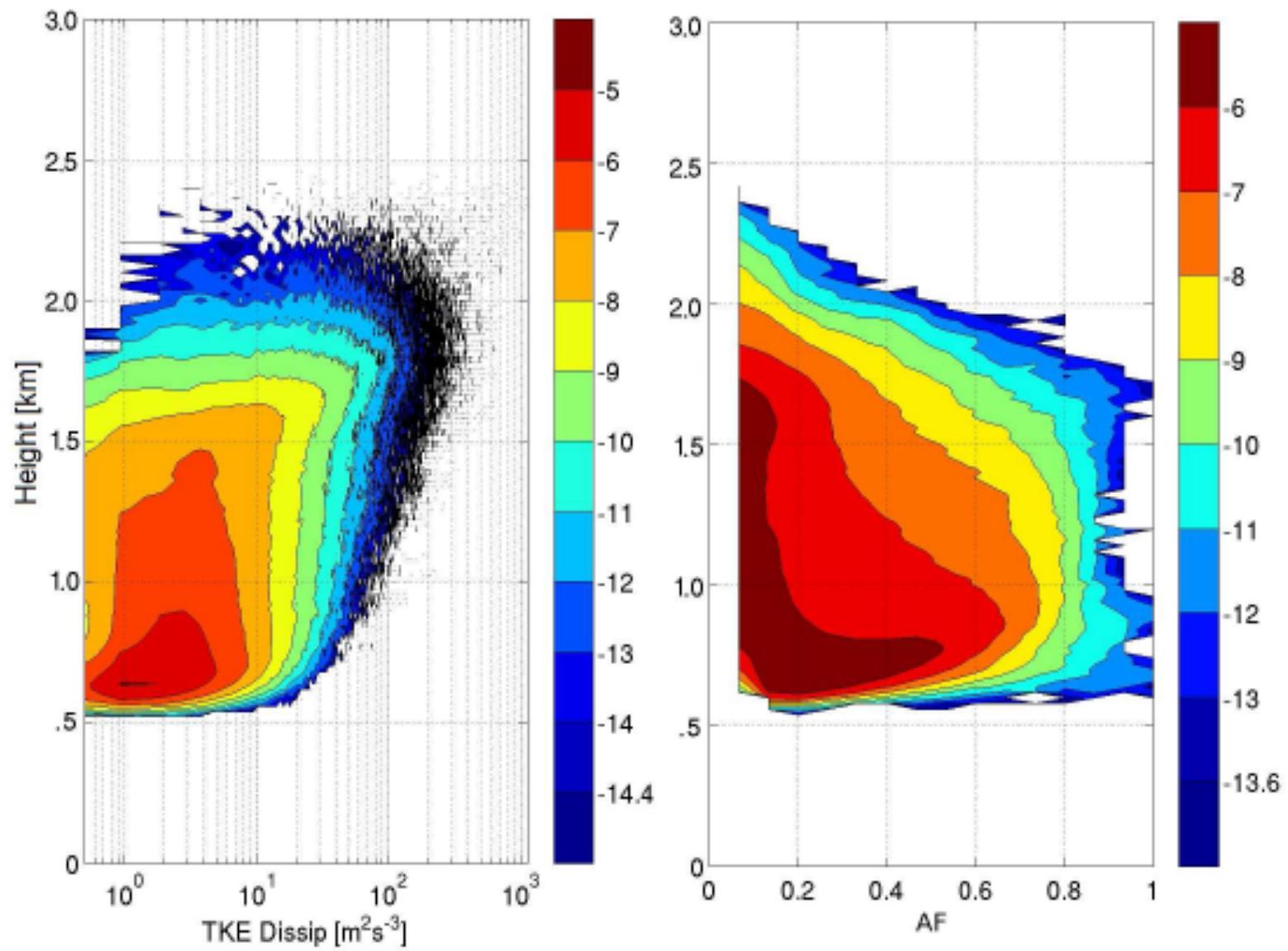
## Simulation without an inversion (1 layer)



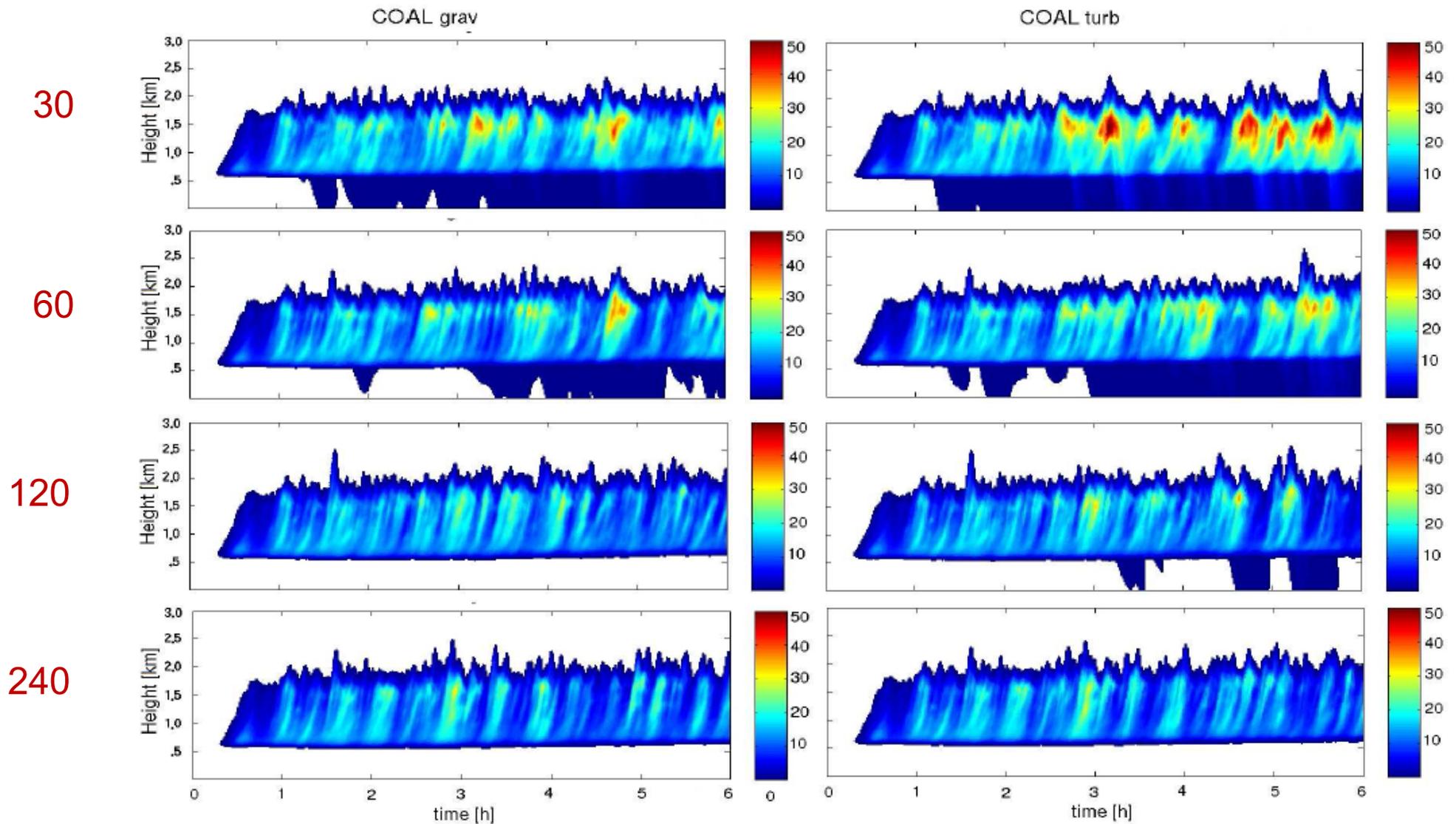
Dynamical and  
microphysical  
enhancement



**Fig. 4.** Snapshots of cloud water mixing ratio (transparent gray) and rain water mixing ratio (solid blue) at the 6th hour of the simulation. The isosurfaces show values  $q_c = 0.05 \text{ g kg}^{-1}$  and  $q_r = 0.02 \text{ g kg}^{-1}$ .



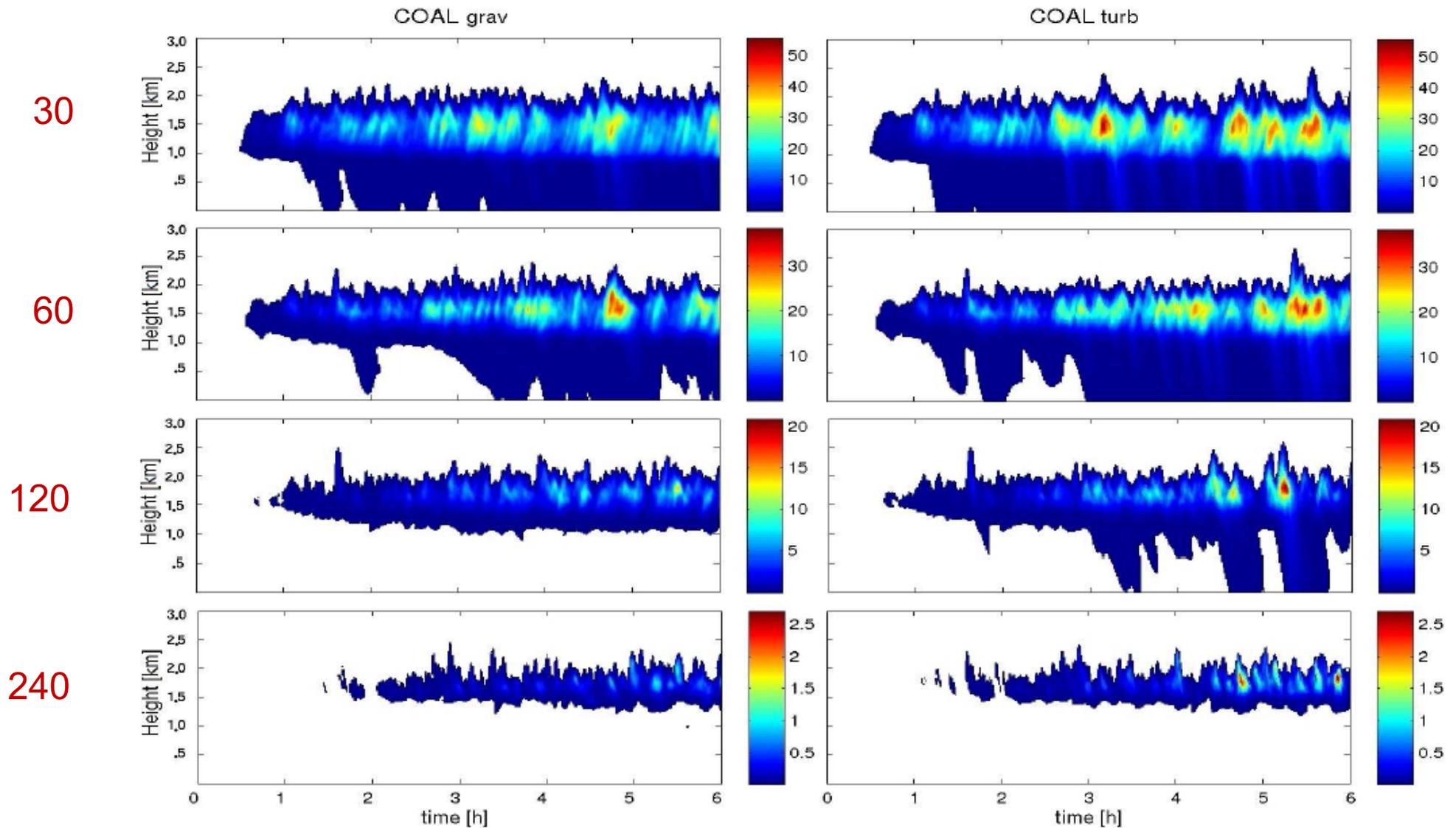
# Domain-averaged cloud water mixing ratio



Gravitational kernel

Turbulent kernel

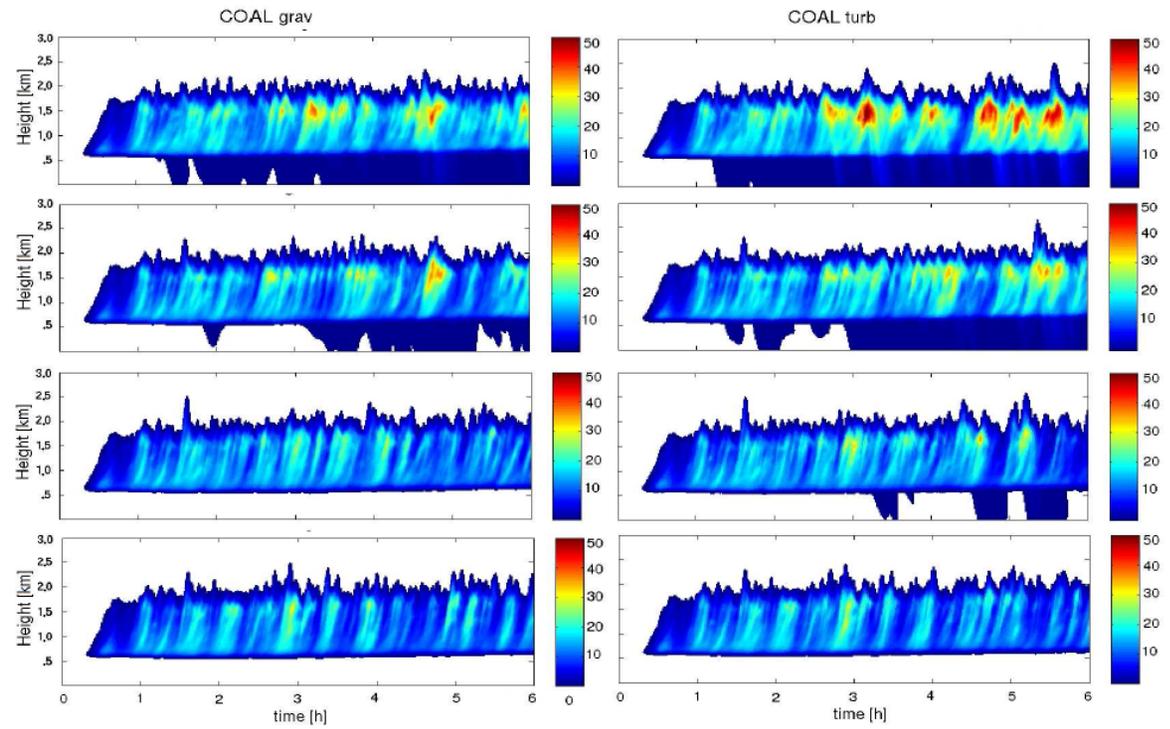
# Domain-averaged rain water mixing ratio



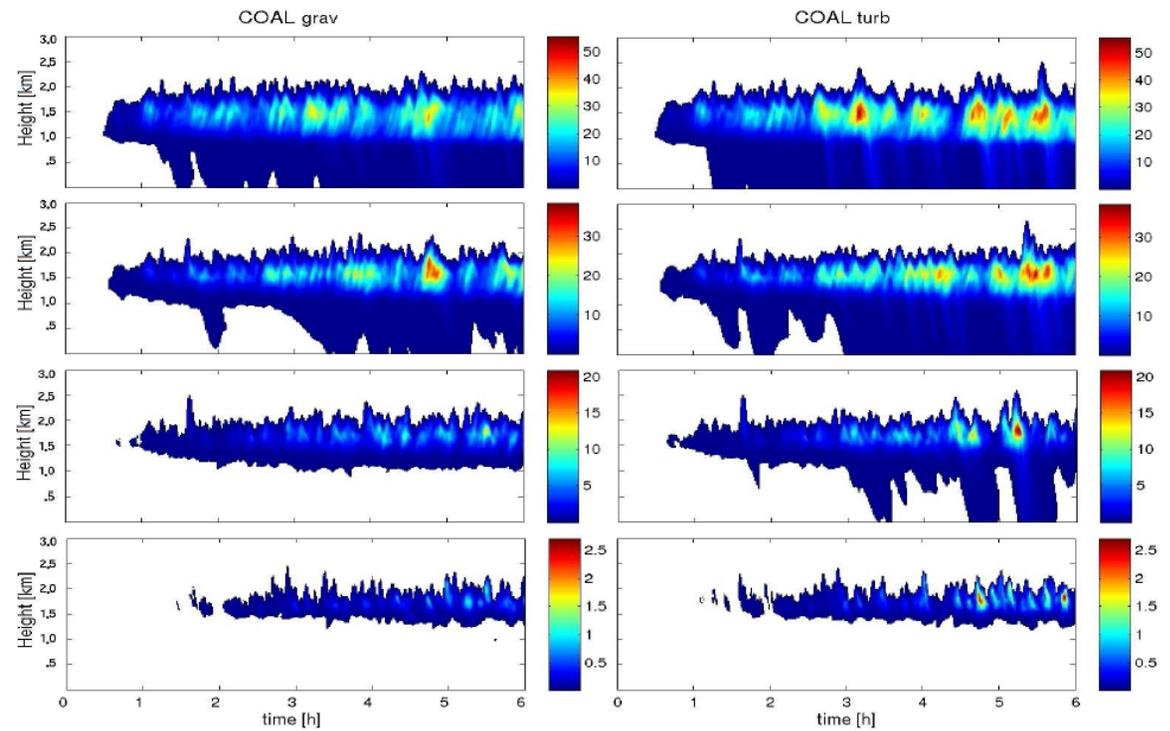
Gravitational kernel

Turbulent kernel

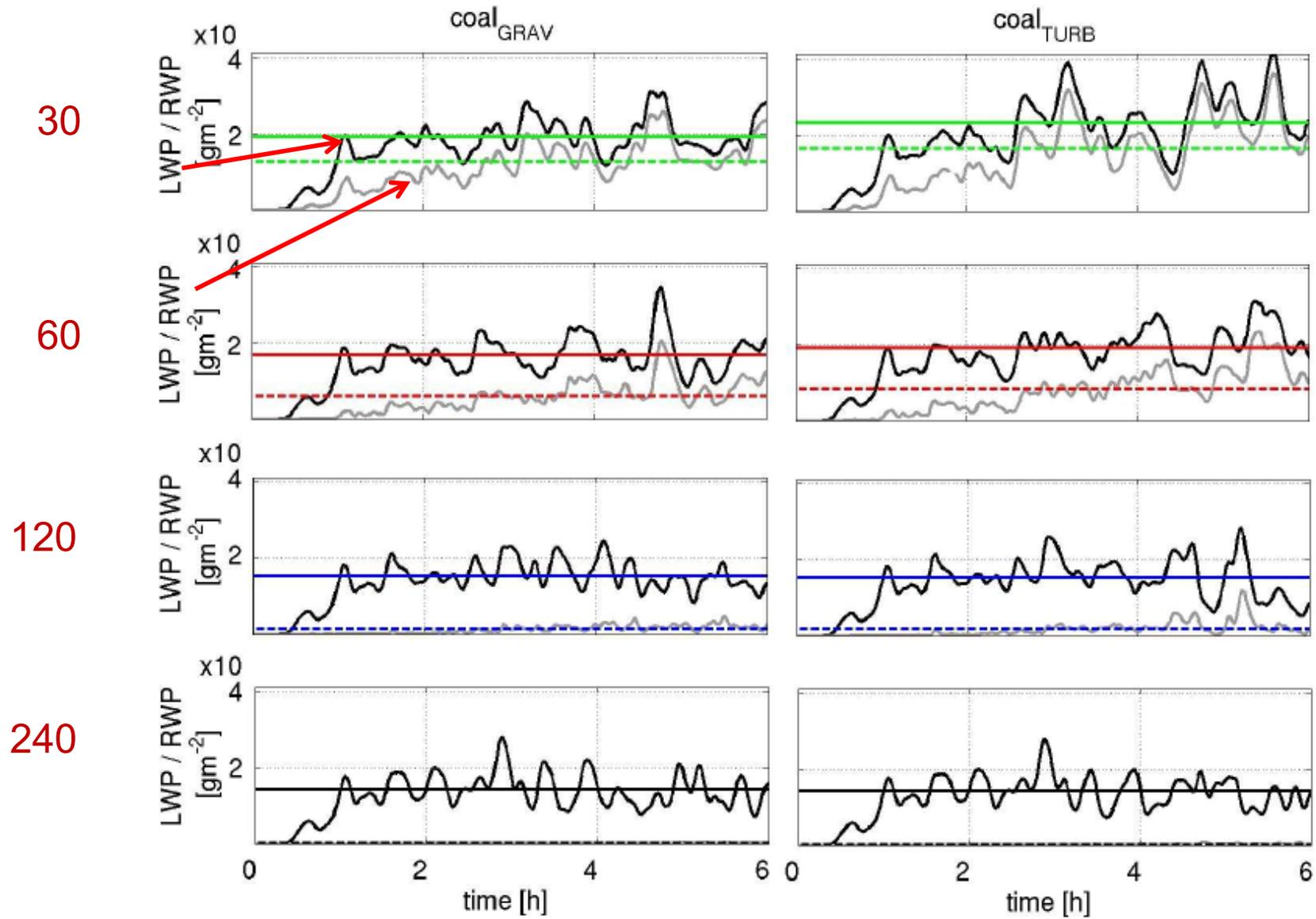
Cloud water



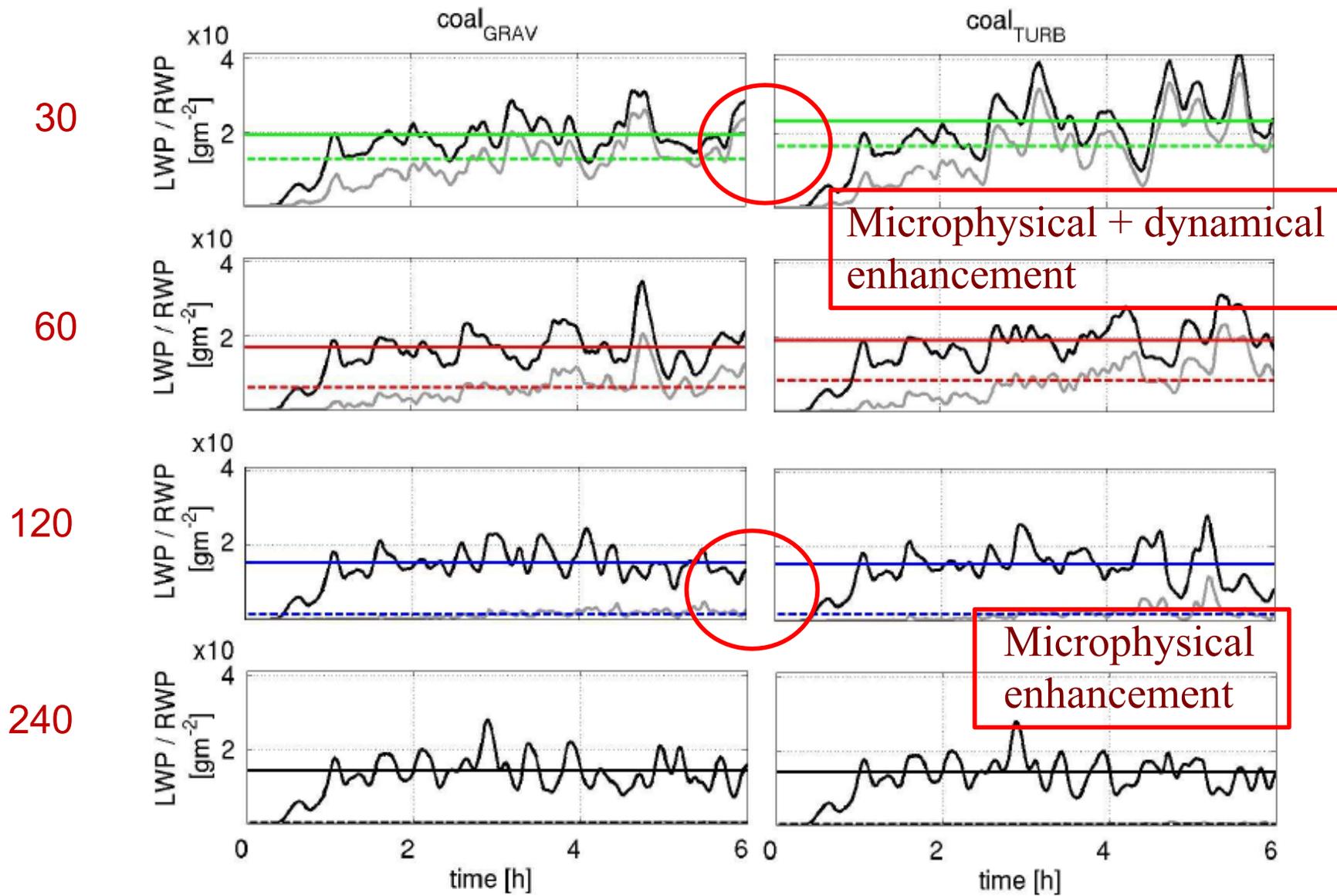
Rain water



# Domain-averaged liquid water path(LWP, cloud water only) and rain water path (RWP)

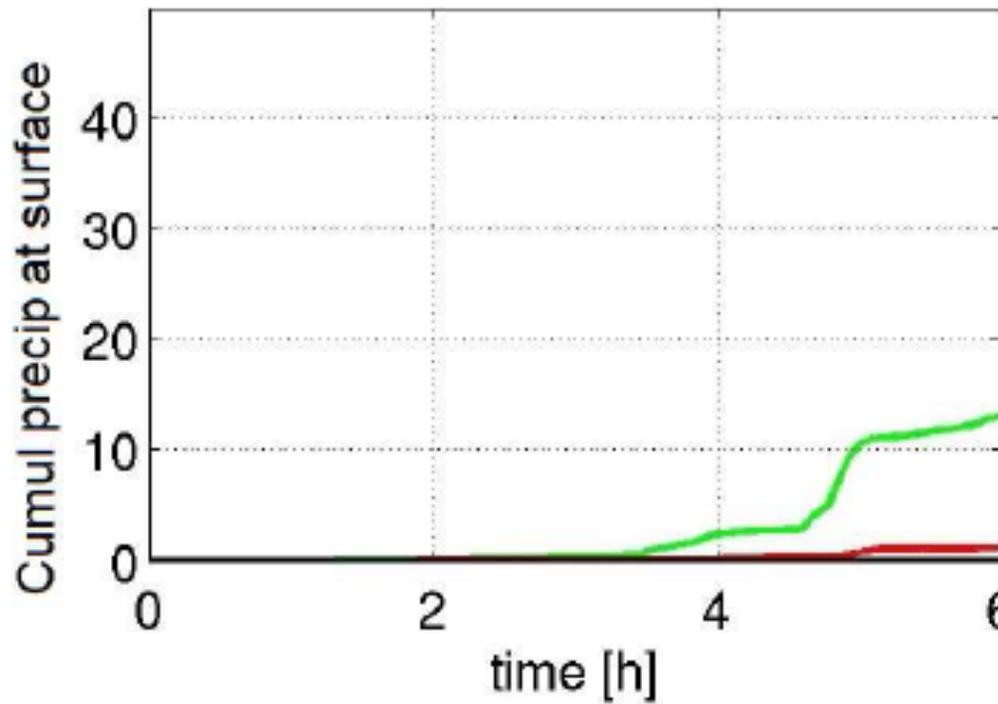


# Domain-averaged liquid water path(LWP, cloud water only) and rain water path (RWP)

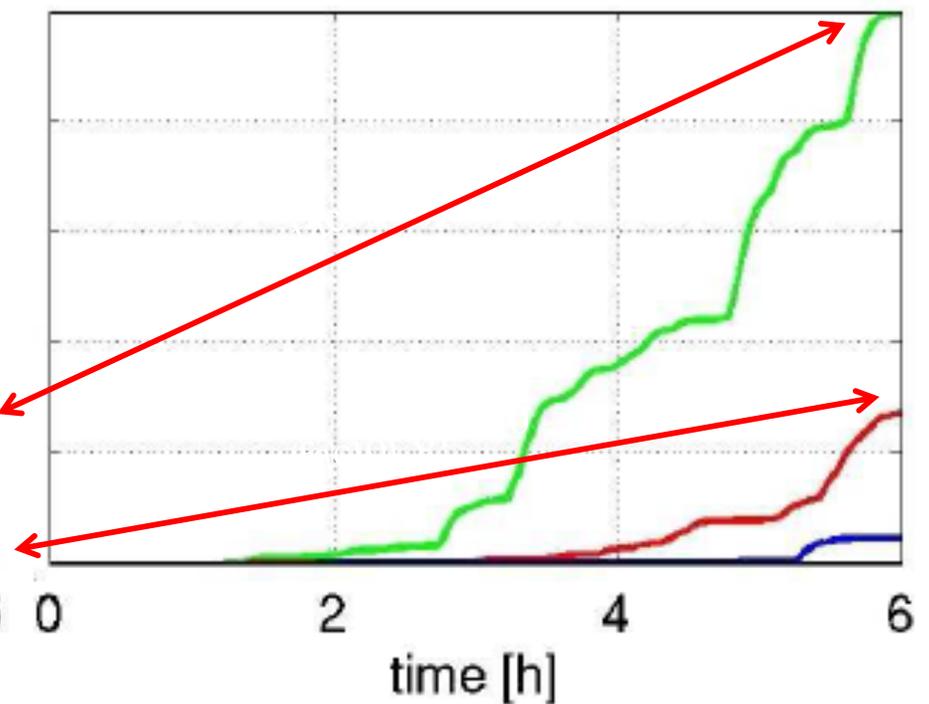


# Surface rain accumulation from cloud field:

## Gravitational kernel



## Turbulent kernel



## Summary for the collision-coalescence :

Small-scale turbulence appears to have a significant effect on collisional growth of cloud droplets and development of warm rain in shallow cumuli. Not only rain tends to form earlier in a single cloud, but also turbulent clouds seem to rain more. This is a combination of microphysical and dynamical effects. The dynamical effect is due to clouds getting deeper due to cloud condensate off-loading that increases cloud buoyancy.

The (perhaps surprising) magnitude of this effect calls for further observational and modeling studies to provide more support for these findings, perhaps applying the super-droplet method.