# Modeling of cloud processes. Part I: warm-rain microphysics

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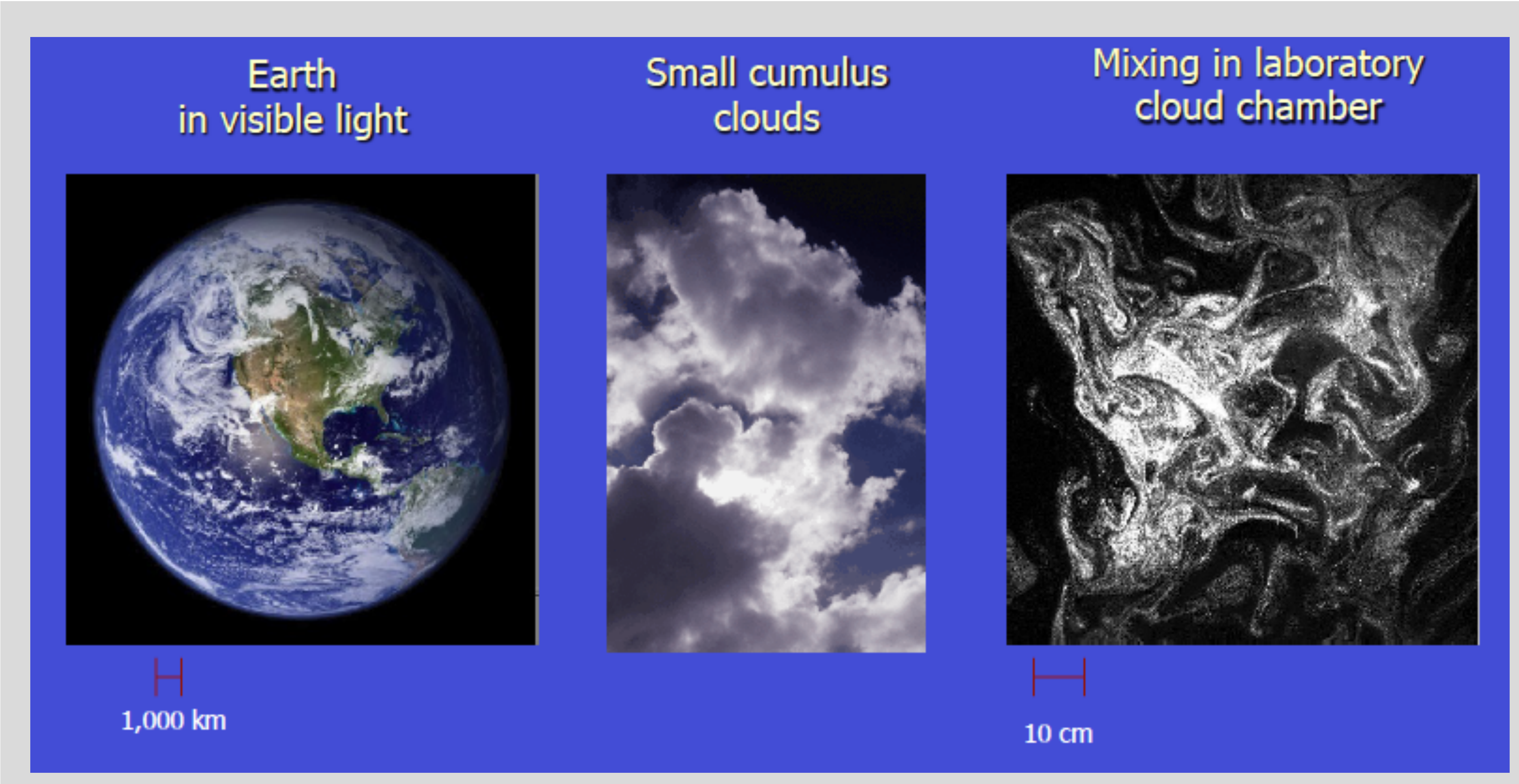
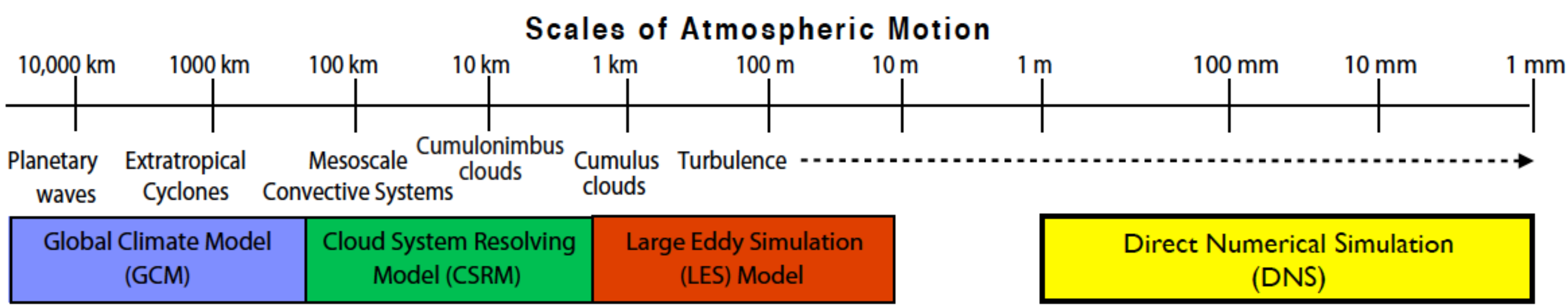
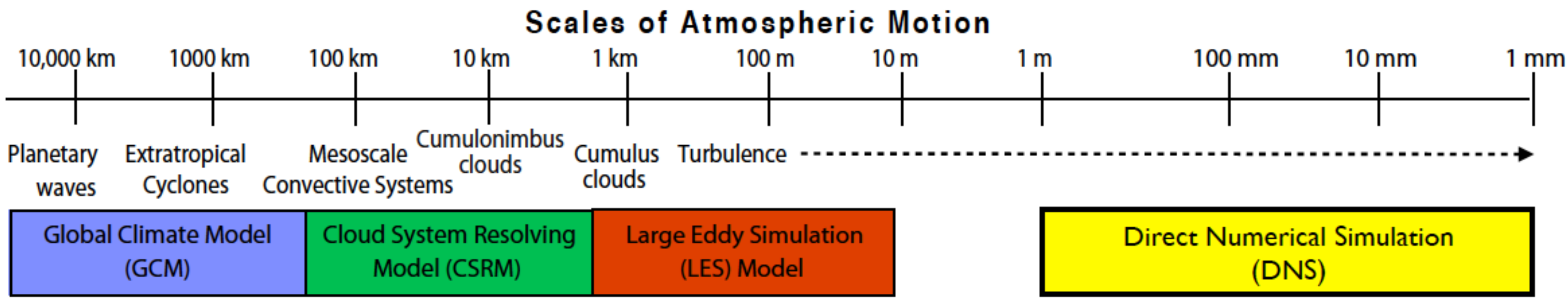
Mesoscale & Microscale Meteorology
MMM
LABORATORY


Earth
in visible light
Small cumulus
clouds
Mixing in laboratory
cloud chamber
1,000 km
10 cm


Scales of Atmospheric Motion
10,000 km
1000 km
100 km
10 km
1 km
100 m
10 m
1 m
100 mm
10 mm
1 mm
Planetary waves
Extratropical Cyclones
Mesoscale Convective Systems
Cumulonimbus clouds
Cumulus clouds
Turbulence
Global Climate Model (GCM)
Cloud System Resolving Model (CSRM)
Large Eddy Simulation (LES) Model
Direct Numerical Simulation (DNS)




*parameterization problem*: parameterized microphysics in (under)resolved clouds

*parameterization$^2$ problem*: parameterized microphysics in parameterized clouds

***microphysics at its native scale***

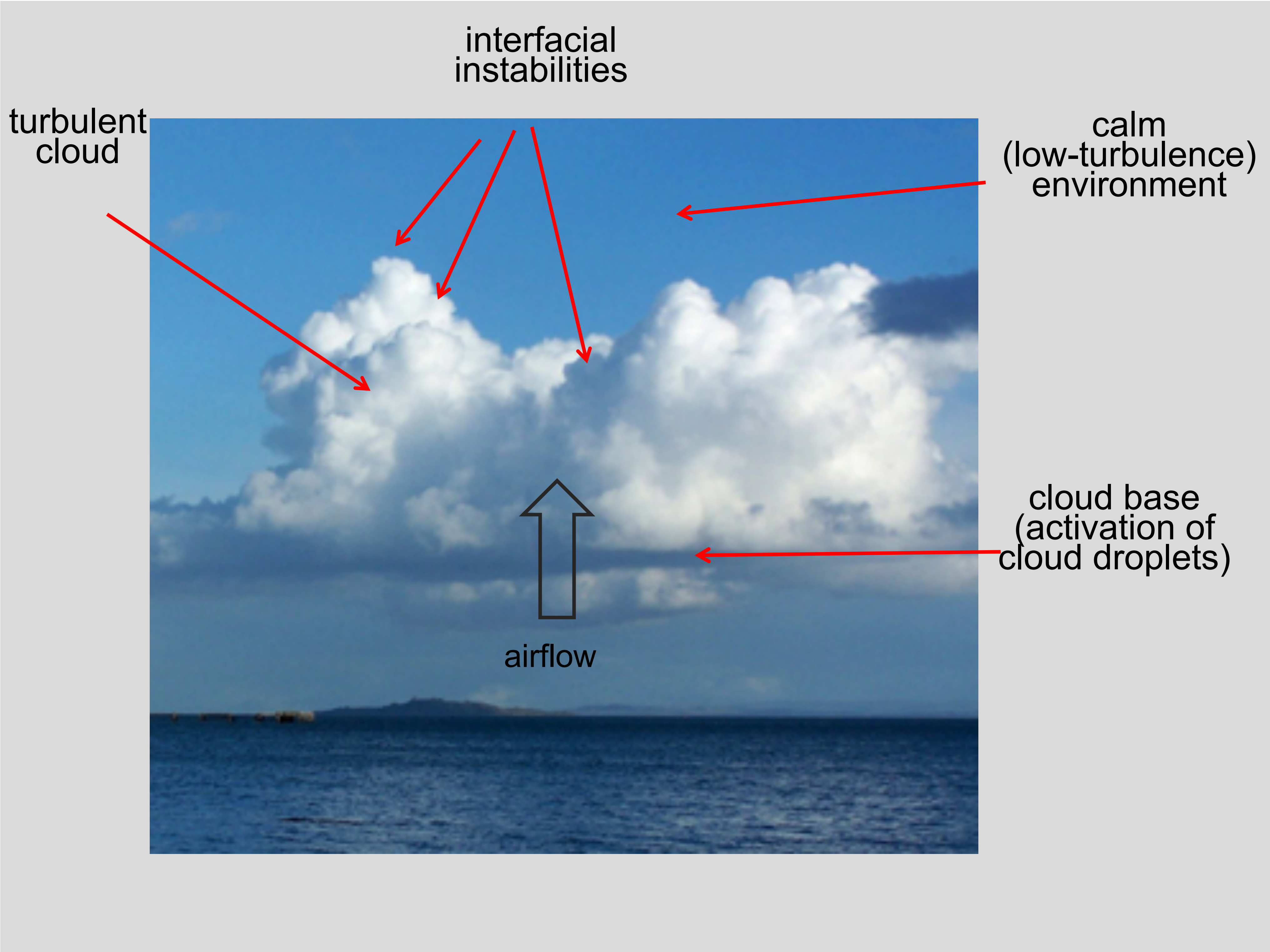
# Cloud microphysics across scales

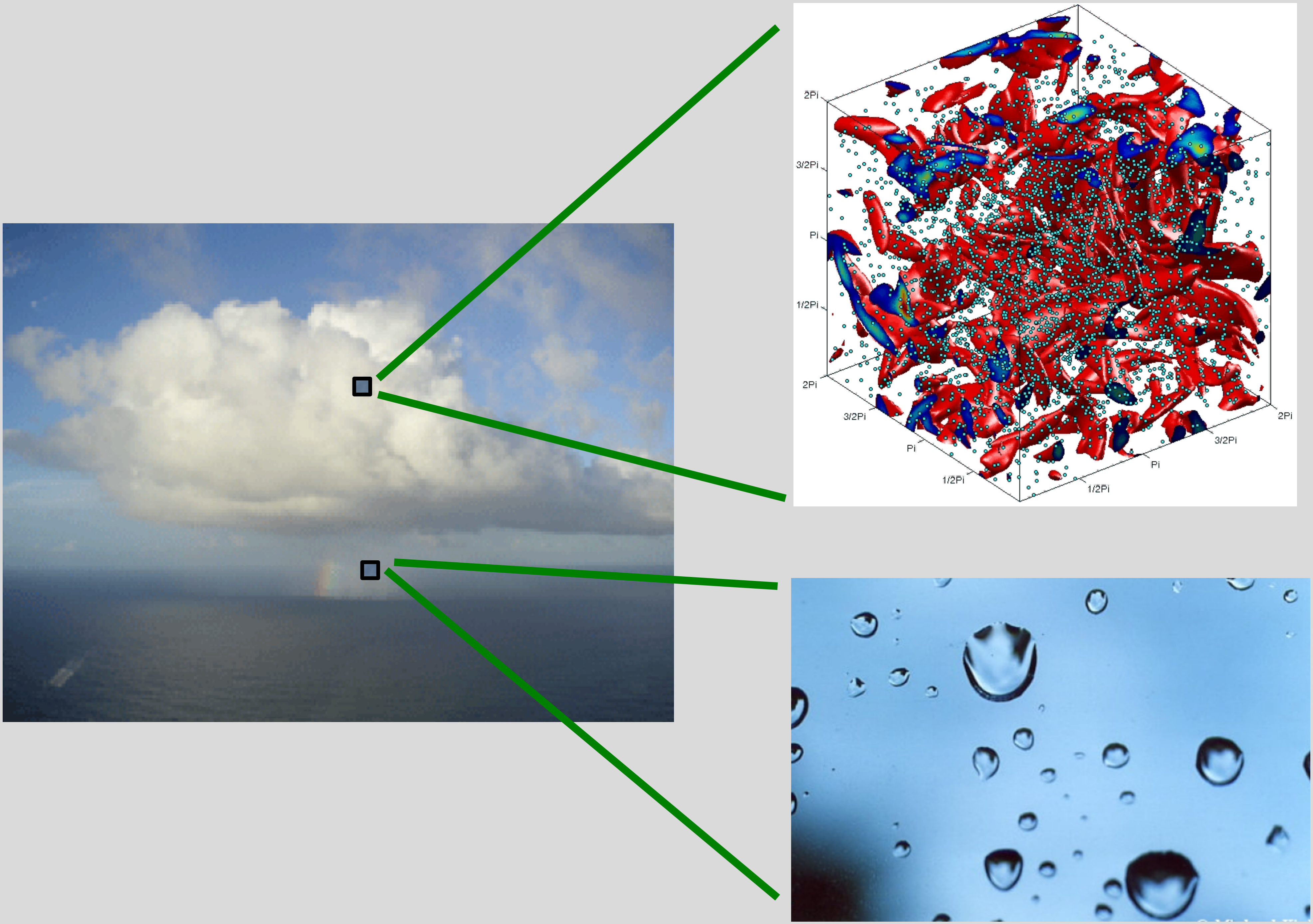
Eulerian versus Lagrangian methodology
(continuous medium versus particle-based)

Warm (no-ice) versus ice-bearing clouds

Understanding of the physics
versus numerical implementation

Precise and complex
versus approximate and easy to apply


interfacial instabilities
turbulent cloud
calm (low-turbulence) environment
cloud base (activation of cloud droplets)
airflow


2Pi
3/2Pi
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Pi
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2Pi

# *Fundamentals of cloud physics*

## ELEMENTARY CLOUD PHYSICS:

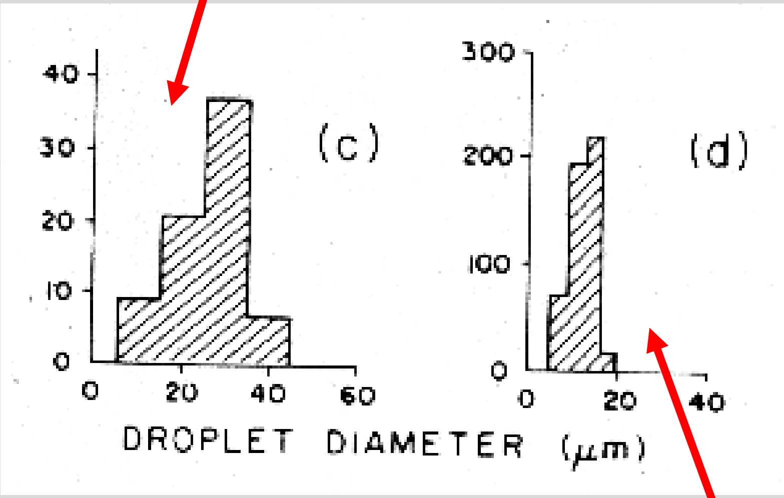
**clouds form due to cooling of air (e.g., adiabatic expansion of a parcel of air rising in the atmosphere)**

- *condensation:* water vapor ⟶ cloud droplets

*heterogeneous nucleation* on atmospheric aerosols called Cloud Condensation Nuclei (CCN); typically highly soluble salts (sea salt, sulfates, ammonium salts, nitrates)

typically, only a small percentage of CCN used by clouds (i.e., water clouds form just above saturation)

Maritime cumulus




Continental cumulus

**From cloud droplets and ice crystals to precipitation:**

*WARM RAIN:*

⟶ gravitational collision and coalescence between cloud droplets

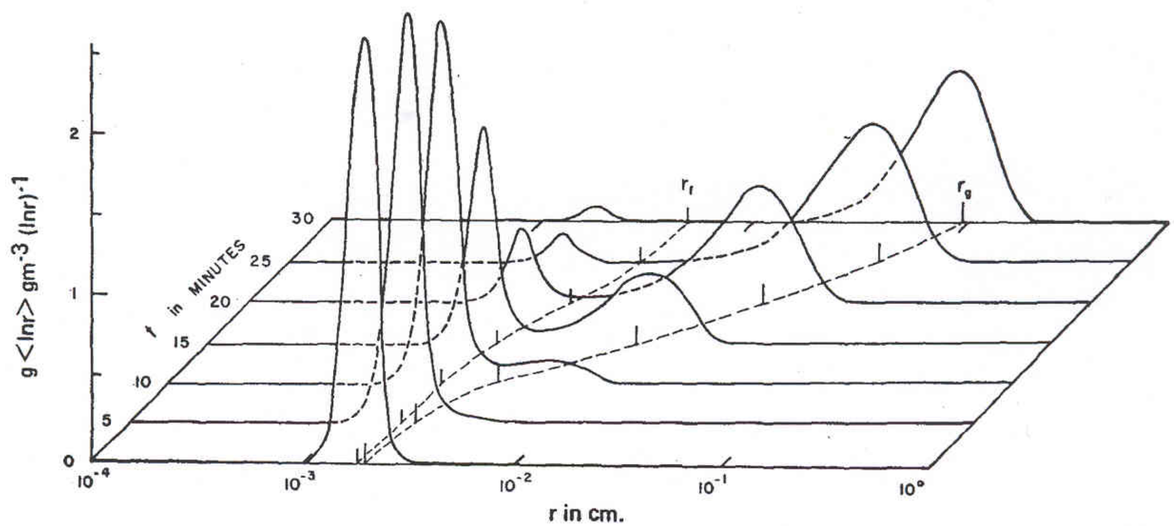



FIG. 5. Time evolution of the initial spectrum for $r_f{}^0 = 18\ \mu m$, var $x = 0.25$.

Berry and Reinhardt JAS 1974

# THE DISTRIBUTION OF RAINDROPS WITH SIZE

*By J. S. Marshall and W. McK. Palmer*[1]

McGill University, Montreal

(Manuscript received 26 January 1948)

$$N_D = N_0 e^{-\Lambda D}$$

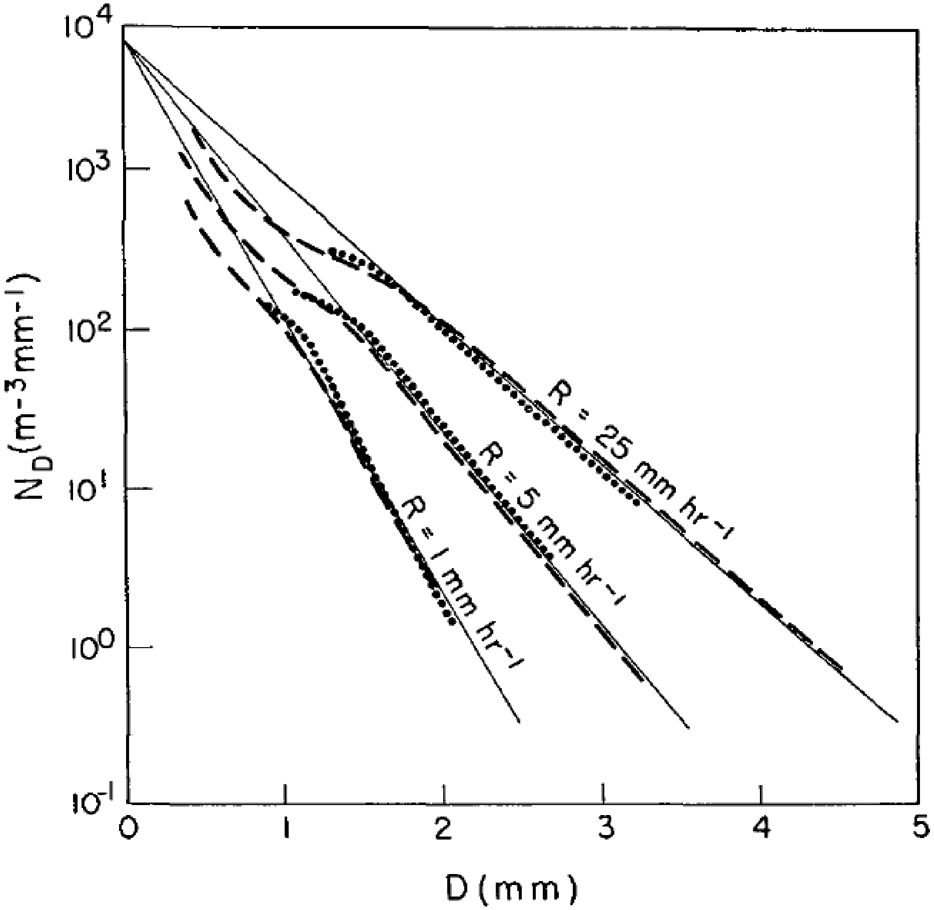



FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

# *Fundamentals of cloud dynamics and thermodynamics modeling*

**Water vapor is a minor constituent:**

mass loading is typically smaller than 1%; thermodynamic properties (e.g., specific heats etc.) only slightly modified;

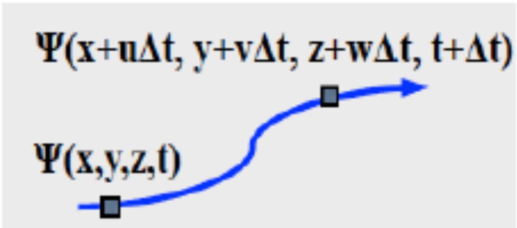
**Suspended small particles (cloud droplets, cloud ice):**

mass loading is typically smaller than a few tenths of 1%, particles are much smaller than the smallest scale of the flow; multiphase approach is not required, but sometimes used (e.g., DNS with suspended droplets, Lagrangian Cloud Model)

**Precipitation (raindrops, snowflakes, graupel, hail):**

mass loading can reach a few %, particles are larger than the smallest scale the flow; multiphase approach needed only for very-small-scale modeling

# Lagrangian versus Eulerian formulation



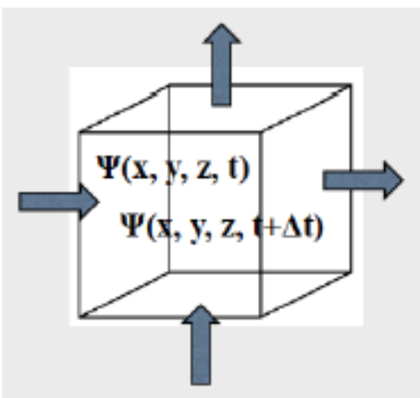

$$\frac{D\Psi}{Dt} = S$$

or

$$\frac{\partial \Psi}{\partial t} + \mathbf{u} \cdot \nabla \Psi = S$$

combined with dry air continuity equation:

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0$$




gives:

$$\frac{\partial \rho_a \Psi}{\partial t} + \nabla(\rho_a \mathbf{u} \Psi) = \rho_a S$$

For the anelastic system:

$$\frac{\partial \Psi}{\partial t} + \frac{1}{\rho_o} \nabla(\rho_o \mathbf{u} \Psi) = S$$

$$\rho_o = \rho_o(z)$$

# SMALL-SCALE DYNAMICS:

## Navier-Stokes equations

In the spirit of the Boussinesq approximation, moisture and condensate affect gas dynamics equations only though the buoyancy term

$p = \rho R T$
$p'/p = \rho'/\rho + T'/T$

$p \sim 1000$ hPa
$p' \sim \rho u^2 \sim 1$ hPa

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k} + \ldots.(\text{Coriolis, turbulence, etc})$$

$$\rho = \rho_o(z) + \rho'$$
$$p = p_o(z) + p'$$

$$(\rho_o + \rho')\frac{dw}{dt} = -\frac{\partial p_o}{\partial z} - \rho_o g - \frac{\partial p'}{\partial z} - \rho' g + \ldots.$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho_o}\nabla p' - g\mathbf{k}\frac{\rho'}{\rho_o} + \ldots.$$

For small-Mach number flows ($|\mathbf{u}| \ll c_s$; $c_s$ - speed of sound):

$$\frac{\rho'}{\rho_o} \approx -\frac{T'}{T_o}$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho_o}\nabla p' + g\mathbf{k}\frac{T'}{T_o} + \ldots.$$

*Density temperature* $T_d$: the temperature dry air has to have to yield the same density as moist cloudy air

$$T_d = T\frac{1 + q/\epsilon}{1 + q + Q}$$

$T$ - air temperature

$q$ - water vapor mixing ratio ($\sim 10^{-3}$)

$Q$ - condensate mixing ratio (cloud water, rain, ice, snow, etc.; $\sim 10^{-3}$)

$\epsilon = \frac{R_d}{R_v} \approx 0.622$

$$T_d \approx T\left[1 + \left(\frac{1}{\epsilon} - 1\right)q - Q\right]$$

$$T_d \approx T\,(1 + 0.61q - Q)$$

***T*, *q* and *Q* – thermodynamics (and much more!)**

# And we also need equation for the temperature:

*First Law of Thermodynamics:*

$$dq = du + p\,dv \tag{1}$$

$dq$ - heat (per unit mass) added to the system
$du$ - increase of internal energy (per unit mass)
$p\,dv$ - work (per unit mass) performed by the system

$$du = c_v\,dT, \quad pv = RT, \quad v = 1/\rho, \quad c_v + R = c_p$$

$$dq = c_p\,dT - \frac{RT}{p}dp \tag{2}$$

Indtroducing *potential temperature* as:

$$\theta = T\left(\frac{p_{oo}}{p}\right)^{R/c_p} \tag{3}$$

where $p_{oo}$=const (typically 1000 mb), (1) can be written as:

$$d\theta = \frac{\theta}{c_p T}dq \tag{4}$$

## *And we also need equation for the temperature:*

$$\frac{d\theta}{dt} = \frac{\theta}{c_p T} S$$

where $S = \frac{dq}{dt}$ in the heat source per unit mass [in J kg$^{-1}$ s$^{-1}$]

$S = 0$ - adiabatic motions

$S \neq 0$ - motions with diabatic processes (heating due to radiative transfer, phases changes, chemical reactions, etc)

*For phase changes of water substance:*

$$S = L \frac{dQ}{dt}$$

where $L$ is the latent heat (of condensation, freezing, or sublimation), and $\frac{dQ}{dt}$ is the change of corresponding water mixing ratio

**Continuous medium approach:** density (i.e., mass of the unit volume) is the main field variable (density of water vapor, density of cloud water, density of rainwater, etc…)

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + u \cdot \nabla \psi$$

*In practice, mixing ratios are typically used. Mixing ratio is the ratio between the density (of water vapor, cloud water…) and the air density.*

## Mixing ratios versus specific humidities…

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_a}{dt} + \rho_a \nabla \mathbf{u} = 0$$

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

$$\boxed{\text{mixing ratio :} \quad q = \frac{\rho_v}{\rho_a}}$$

$$\frac{dq}{dt} = \frac{S}{\rho_a}$$

$$\boxed{\text{specific humidity :} \quad Q = \frac{\rho_v}{\rho_v + \rho_a}}$$

$$\frac{dQ}{dt} = \frac{\rho_a}{\rho_v + \rho_a} \frac{S}{\rho_v + \rho_a}$$

# *Modeling of warm-rain microphysics*

# BULK MODEL OF CONDENSATION:

$$\frac{d\theta}{dt} = \frac{L_v\theta}{c_pT}C_d$$
$$\frac{dq_v}{dt} = -C_d$$
$$\frac{dq_c}{dt} = \quad C_d$$

$\theta$ - potential temperature
$q_v$ - *water vapor* mixing ratio
$q_c$ - *cloud water* mixing ratio
$L_v$ - latent heat of condensation/evaporation
$C_d$ - condensation rate
Note: $\theta/T$ function of pressure only ($\approx \theta_o/T_o$)

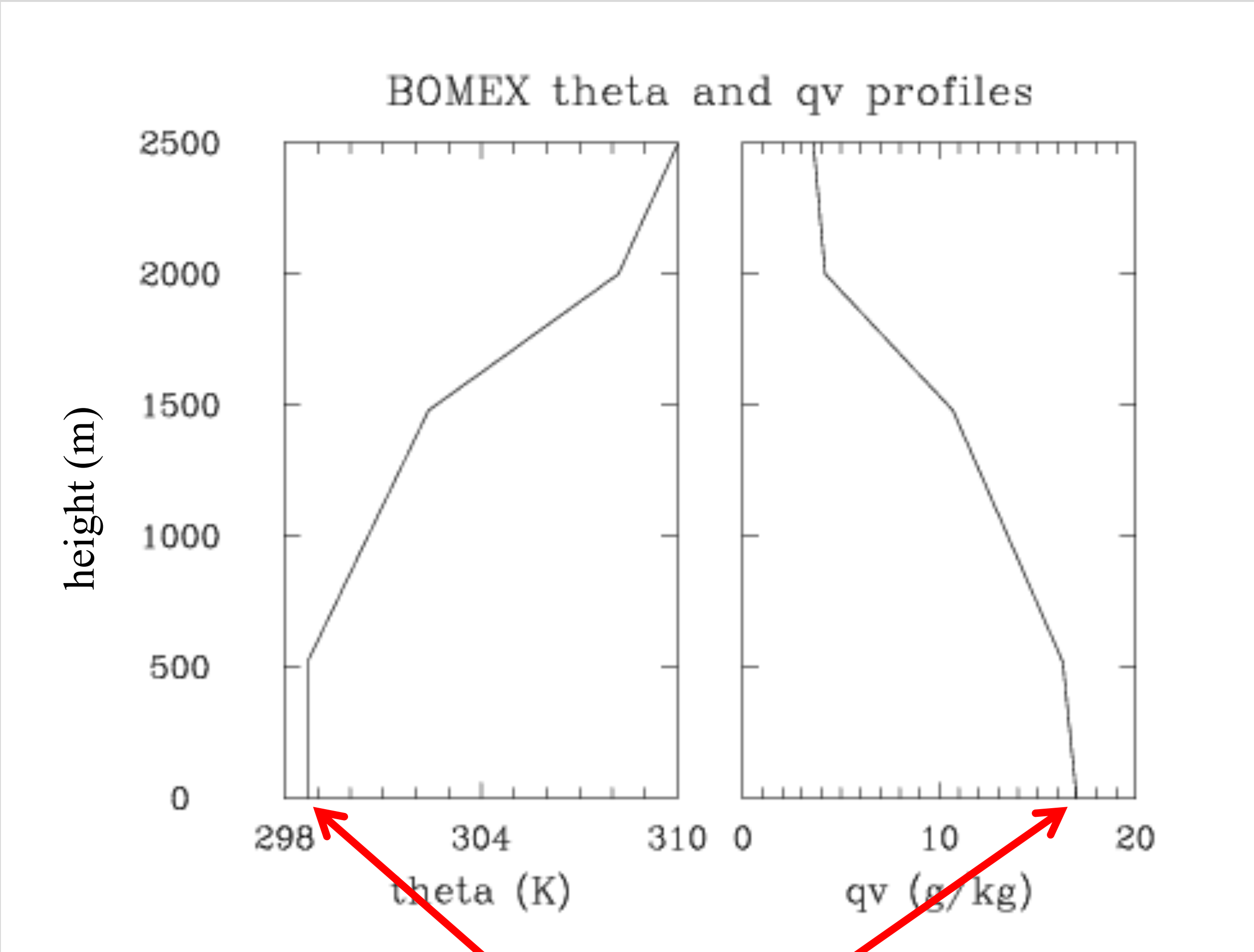
$$\frac{L_v}{c_p\Pi_e}$$

$C_d$ is defined such that cloud is always at saturation, which is a very good approximation:

$$q_c = 0 \quad \text{if} \quad q_v < q_{vs}$$
$$q_c > 0 \quad \text{only if} \quad q_v = q_{vs}$$

where $q_{vs}(p,T) \approx 0.622\dfrac{e_s(T)}{p}$ is the water vapor mixing ratio at saturation

## A very simple (but useful) model: rising adiabatic parcel…



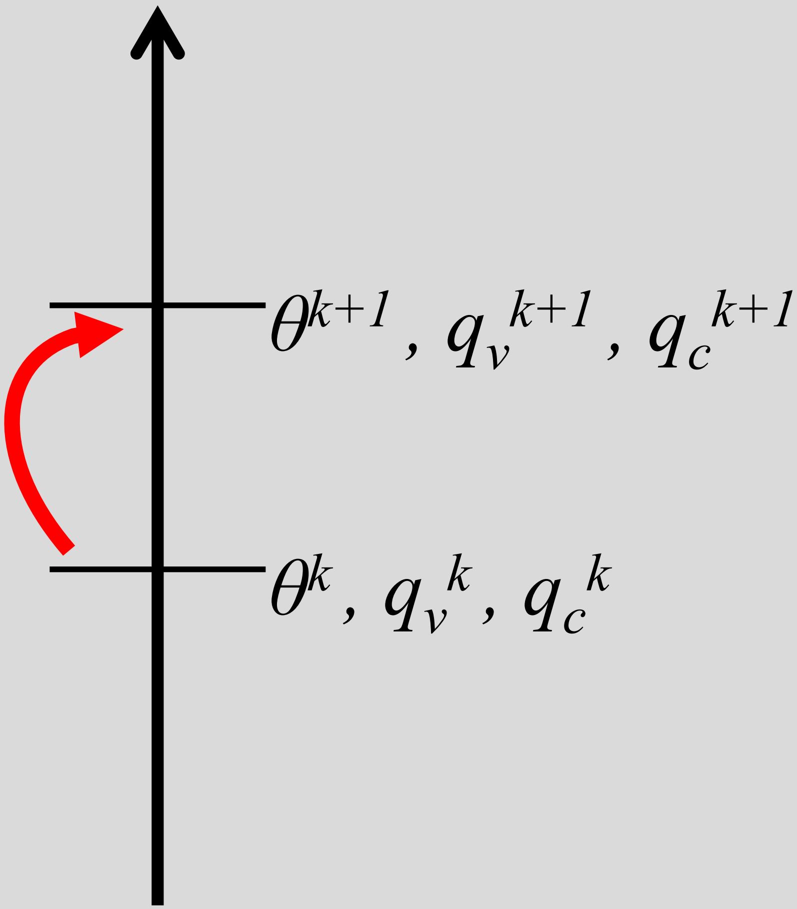

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$

… by solving these equations.

Take a parcel from the surface and move it up…




$$\theta^{k+1} = \theta^k + \frac{L_v}{c_p \Pi_e} \Delta q$$

$$q_v^{k+1} = q_v^k - \Delta q$$

$$q_c^{k+1} = q_c^k + \Delta q$$

$$\Delta q = ?$$

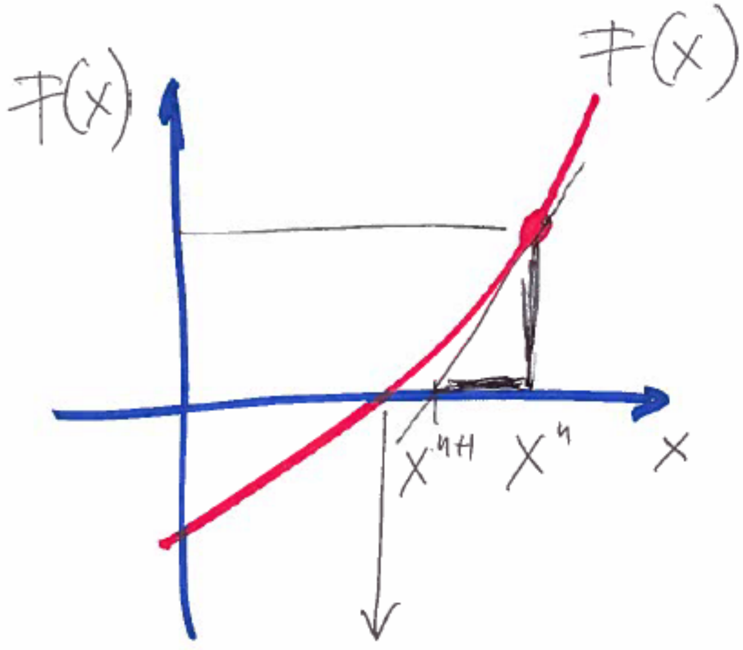
$$q_v^{k+1} = q_{vs}(\theta^{k+1})$$

$$q_v^k - \Delta q = q_{vs}(\theta^k + \frac{L_v}{c_p \Pi_e} \Delta q)$$

The nonlinear equation for $\Delta q$ can be solved using the Newton-Raphson method…

$F(x) = 0$

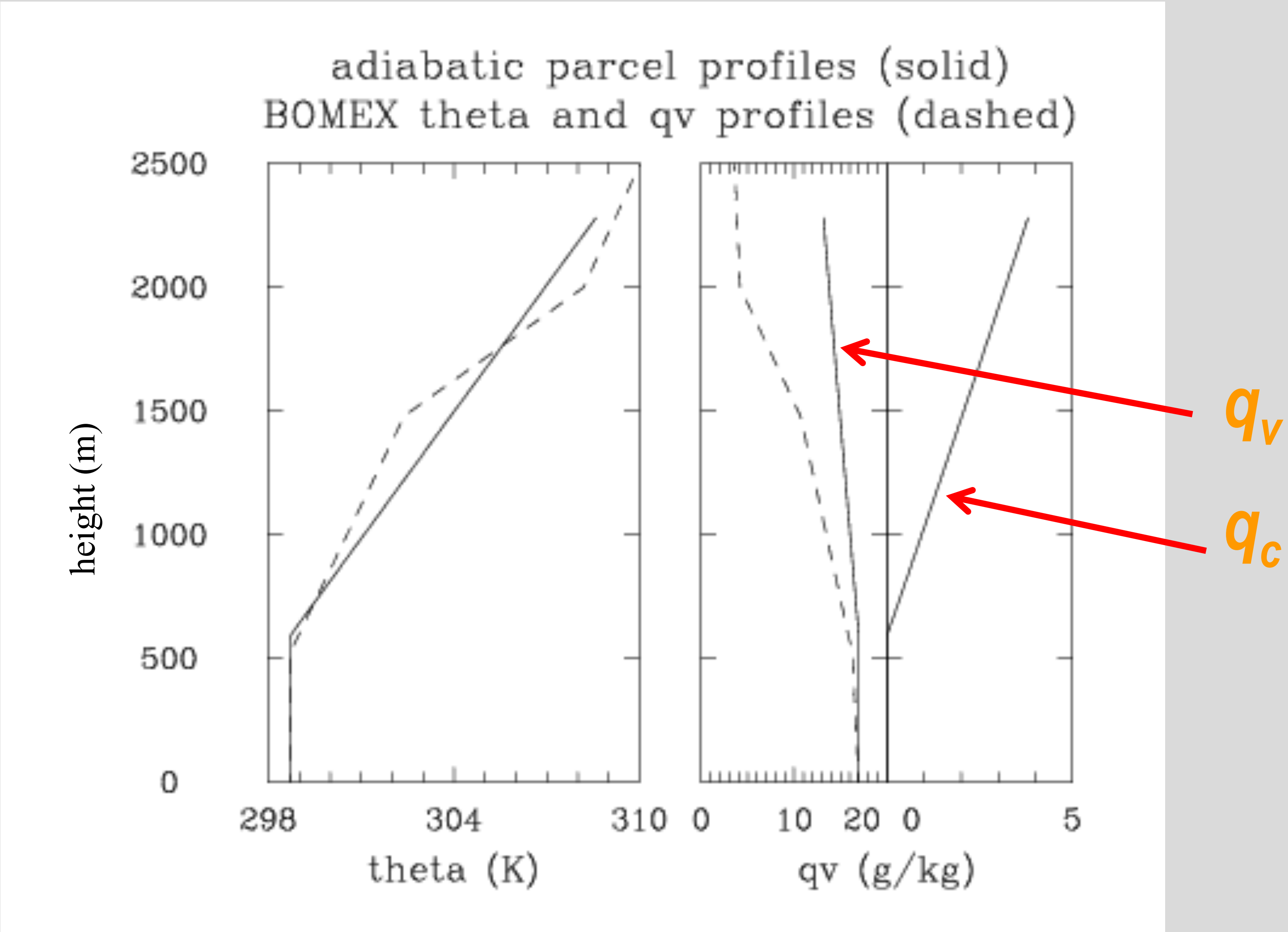
$x = ?$




$$F(x) = 0$$

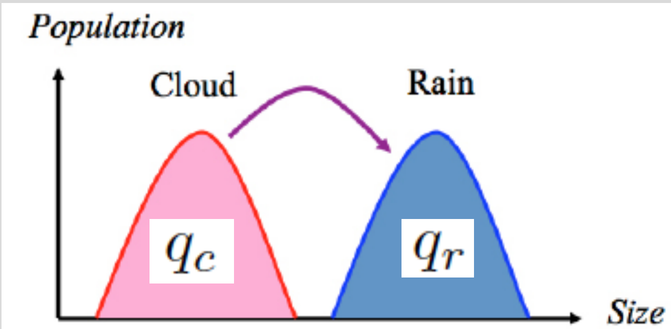
$$F'(x^n) = \frac{F(x^n)}{x^n - x^{n+1}}$$

$$\boxed{x^{n+1} = x^n - \frac{F(x^n)}{F'(x^n)}}$$




**Look not only on the patterns (i.e., processes), but also on specific numbers (e.g., temperature change, mixing ratios, etc).**

# Adding rain or drizzle:

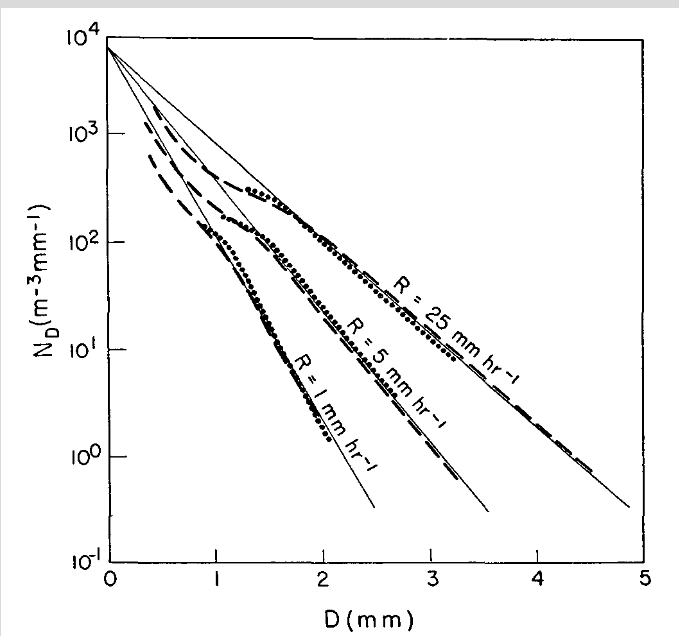



**THE DISTRIBUTION OF RAINDROPS WITH SIZE**

*By J. S. Marshall and W. McK. Palmer*[1]

McGill University, Montreal

(Manuscript received 26 January 1948)




## WARM RAIN BULK MODEL (Kessler 1969):

$$\frac{D\theta}{Dt} = \frac{L_v \theta}{c_p T}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

$$\frac{Dq_c}{Dt} = C_d - AUT - ACC$$

$$\frac{Dq_r}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

$\theta$ - potential temperature
$q_v$ - *water vapor* mixing ratio
$q_c$ - *cloud water* mixing ratio
$q_r$ - *rain water* mixing ratio
$C_d$ - condensation rate
$EVAP$ - rain evaporation rate
$AUT$ - "autoconversion" rate: $q_c \rightarrow q_r$
$ACC$ - accretion rate: $q_c, q_r \rightarrow q_r$

$v_t(q_r)$ - rain terminal velocity (typically derived by assuming a drop size distribution; e.g., the Marshall-Palmer distribution $N(D) = N_o exp(-\Lambda D)$, $N_o = 10^7$ m$^{-4}$).

# Parameterization of microphysical terms:

$$\frac{D\theta}{Dt} = \frac{L_v\theta}{c_pT}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

$$\frac{Dq_c}{Dt} = C_d - AUT - ACC$$

$$\frac{Dq_r}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

***$C_d$*** – condensation rate: already discussed in the case of the parcel model:

"return to saturation" through condensation or cloud water evaporation (infinitely fast).

# Parameterization of microphysical terms:

$$\frac{D\theta}{Dt} = \frac{L_v \theta}{c_p T}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

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$$\frac{Dq_r}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

*EVAP* – rain evaporation

*AUT* "autoconversion" (cloud water being converted into drizzle/rain)

*ACC* – accretion (falling drizzle/rain collecting cloud droplets)

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$$N_D = N_0 e^{-\Lambda D}$$

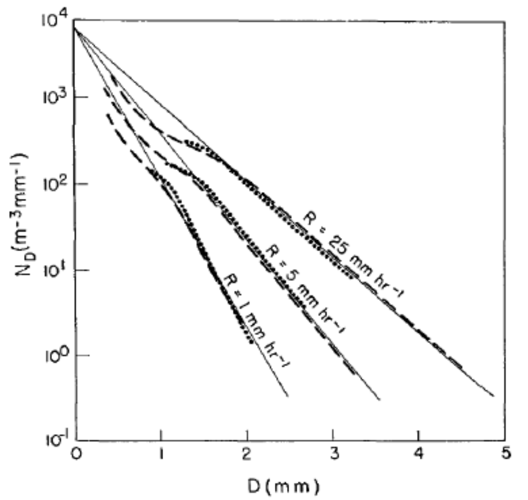



FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

threshold radius separating cloud droplets from drizzle/rain is often taken as 40 microns...

*AUT* autoconversion

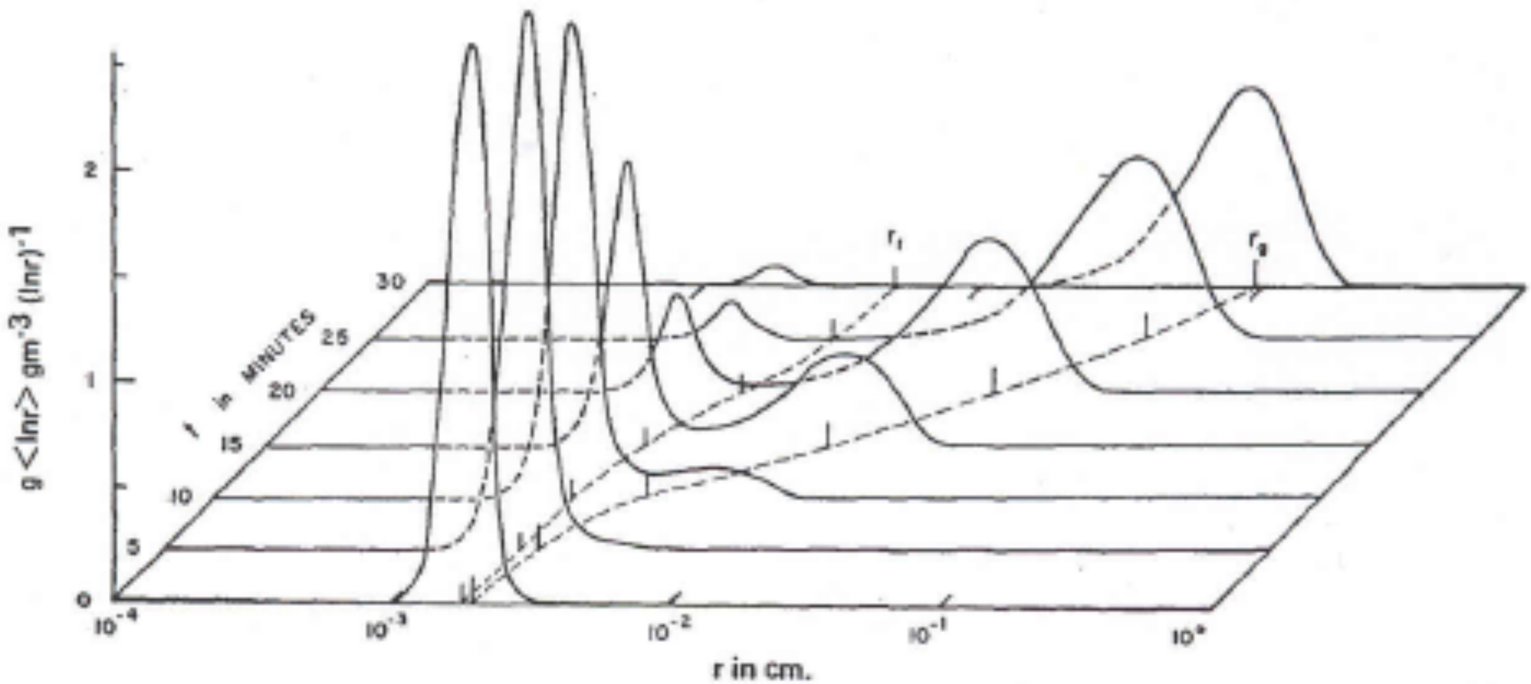



FIG. 5. Time evolution of the initial spectrum for $r_f{}^k = 18\ \mu m$, var $x = 0.25$.

Berry and Reinhardt JAS 1974

# The Simulation of Three-Dimensional Convective Storm Dynamics

JOSEPH B. KLEMP

*National Center for Atmospheric Research[1], Boulder, Colo. 80307*

ROBERT B. WILHELMSON

J. Atmos. Sci. 1978

rain evaporation *EVAP*

$$E_r = \frac{1}{\bar{\rho}} \frac{(1 - q_v/q_{vs}) C (\bar{\rho} q_r)^{0.525}}{5.4 \times 10^5 + 2.55 \times 10^6 / (\bar{\rho} q_{vs})},$$

$$C = 1.6 + 124.9 (\bar{\rho} q_r)^{0.2046}.$$

$$\frac{D\theta}{Dt} = \frac{L_v \theta}{c_p T}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

$$\frac{Dq_c}{Dt} = C_d - AUT - ACC$$

$$\frac{Dq_r}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

autoconversion

*AUT* $A_r = k_1 (q_c - a),$

*ACC* $C_r = k_2 q_c q_r^{0.875},$

accretion

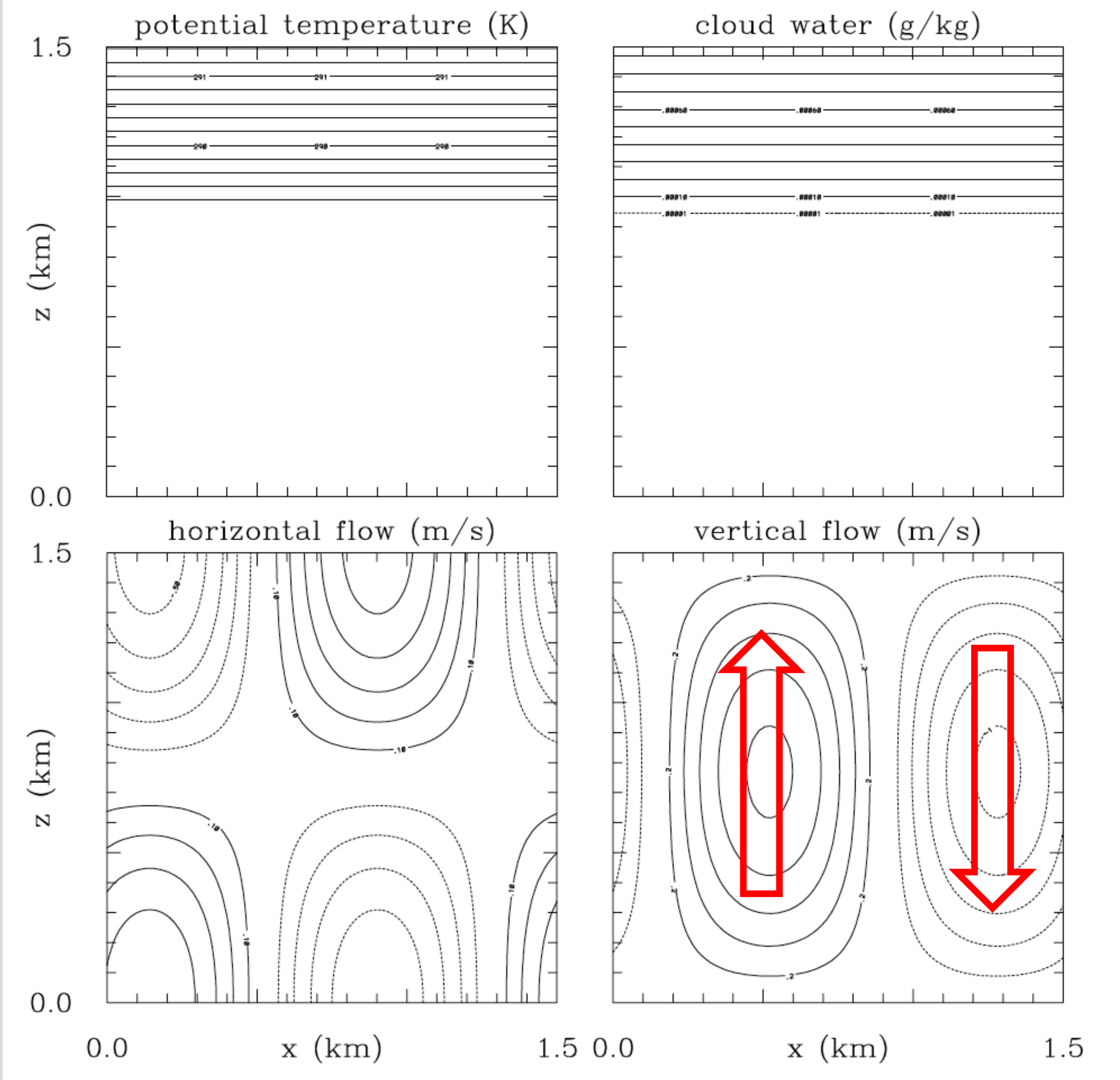
$$V = 3634 (\bar{\rho} q_r)^{0.1346} \left(\frac{\bar{\rho}}{\rho_0}\right)^{-\frac{1}{2}} \ [\text{cm s}^{-1}],$$

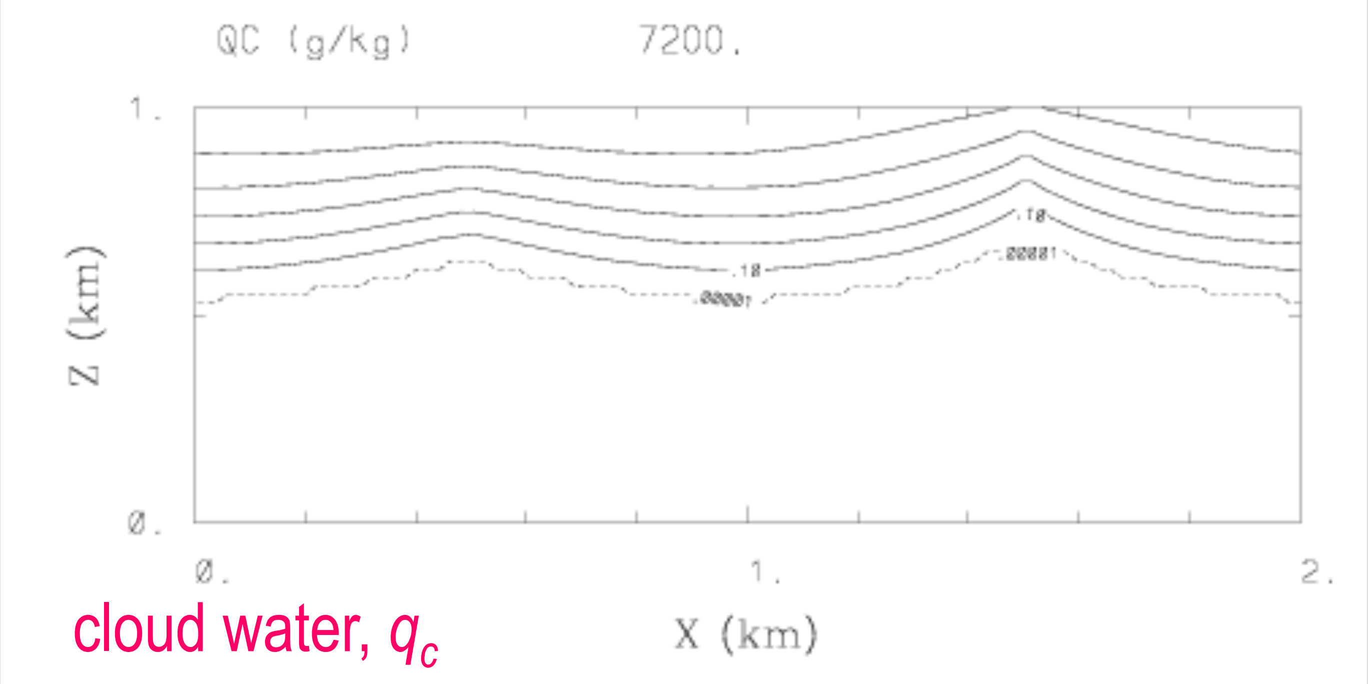
mass-weighted fall velocity

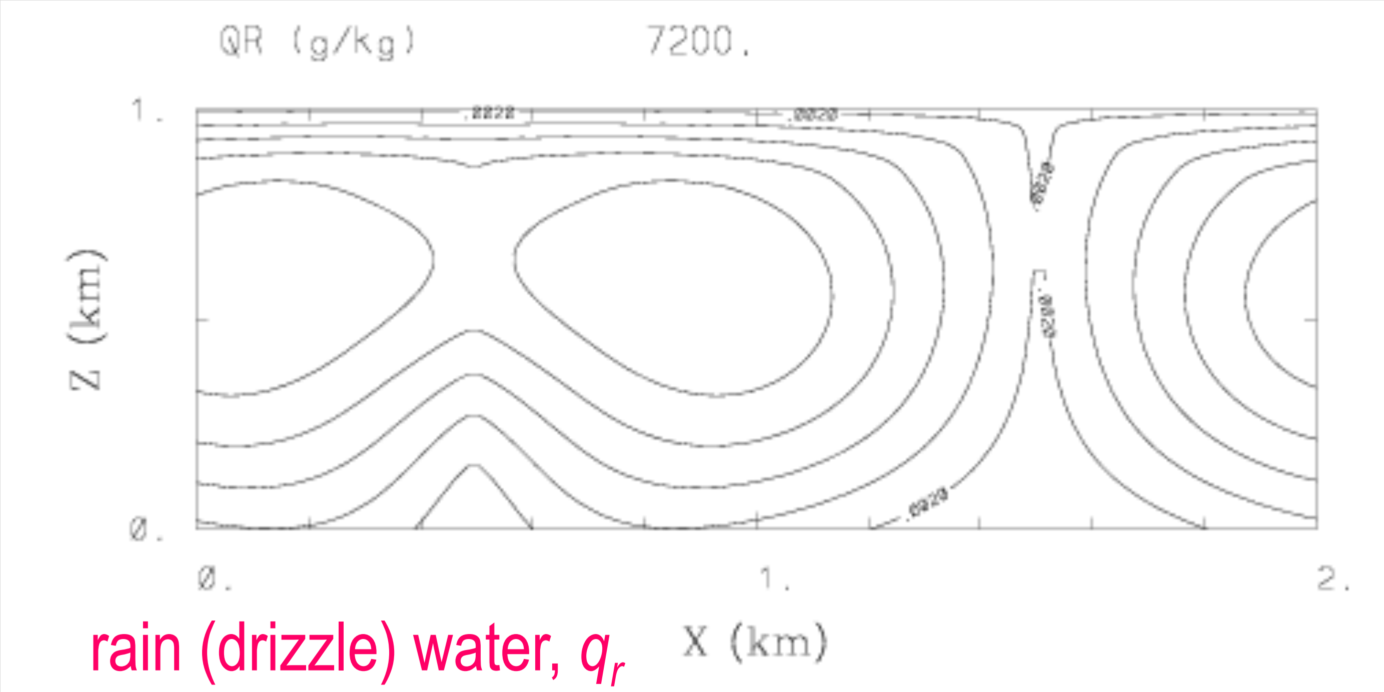
***We need something more complicated than a rising parcel as rain has to fall out. One possibility is to use the kinematic (prescribed flow) framework…***

```
cc initial theta_l and q_tot profiles:
        do k=1,npin
        zin(k)=float(k-1)*dz
          theta_l(k)=289.
          q_tot(k)=7.5e-3
        enddo
```

```
cc rho*ux velocity:
      do i=1,nxp
      do k=1,nz
      ux(i,k)=-(phi(i,k+1)-phi(i,k))/dz
      enddo
      enddo
cc rho*uz velocity
      do k=1,nzp
      do i=1,nx
      uz(i,k)=(phi(i+1,k)-phi(i,k))/dx
      enddo
      enddo
```

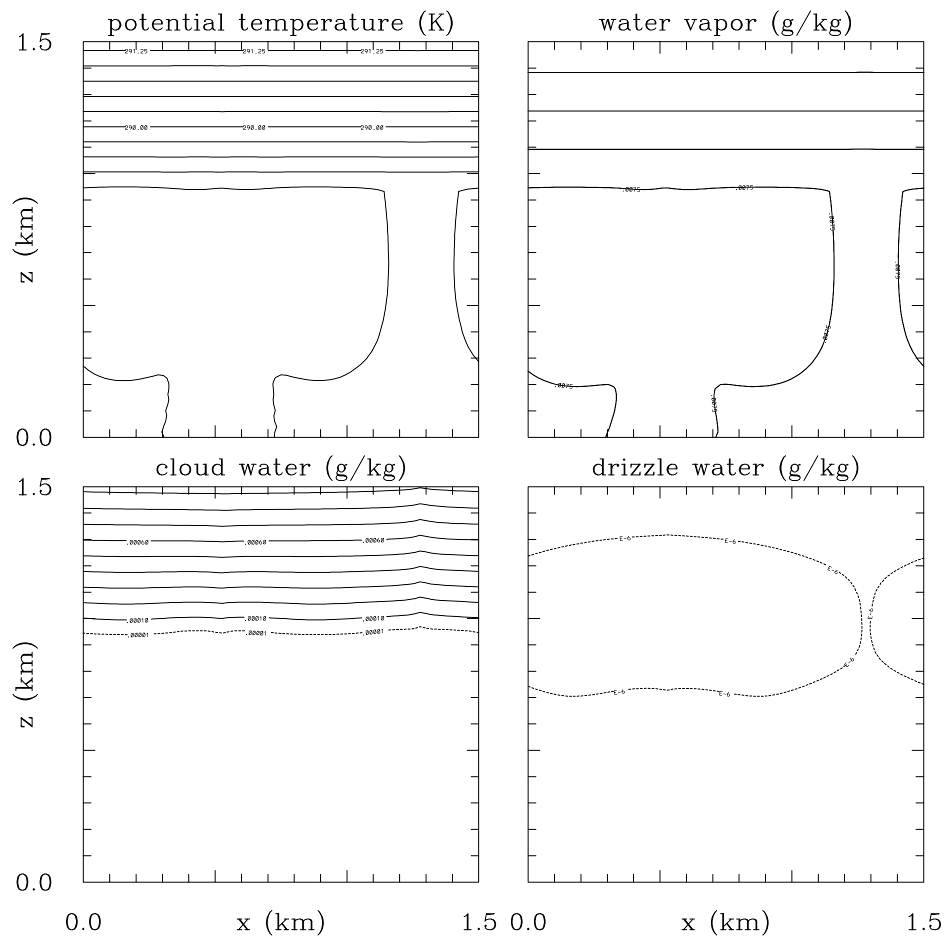




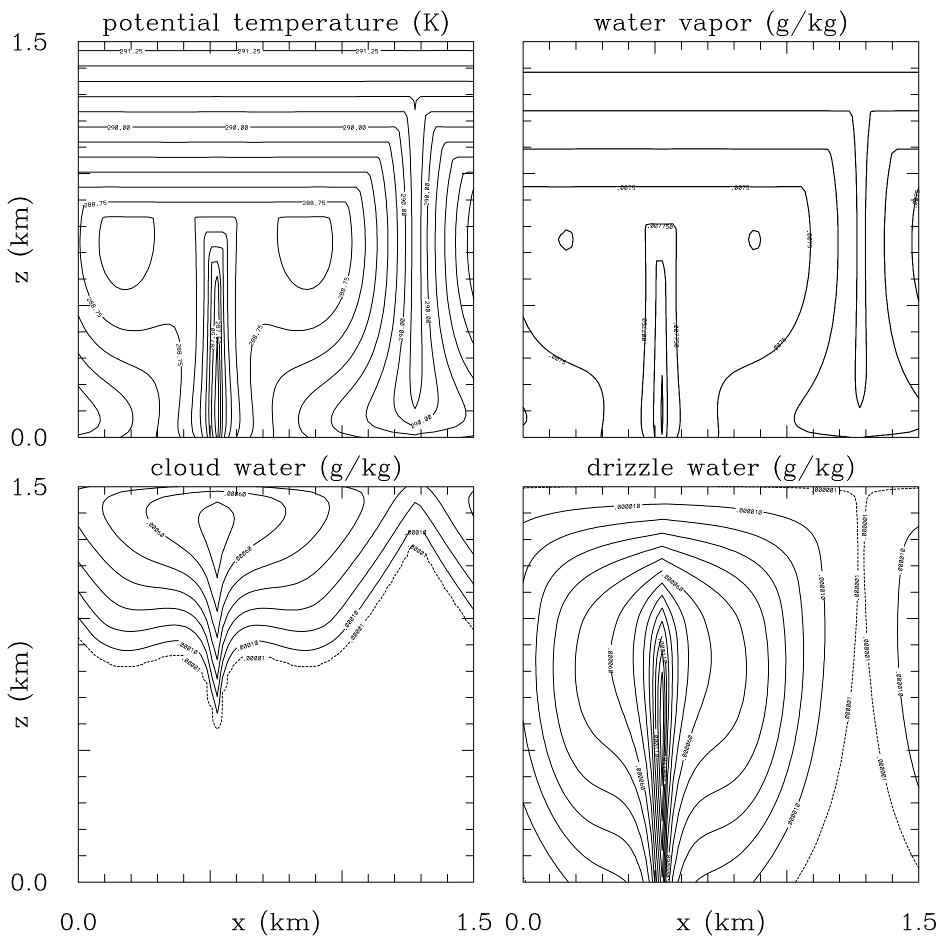




**Cloud water and rain (drizzle) fields after 2 hrs (almost quasi-equilibrium…)**

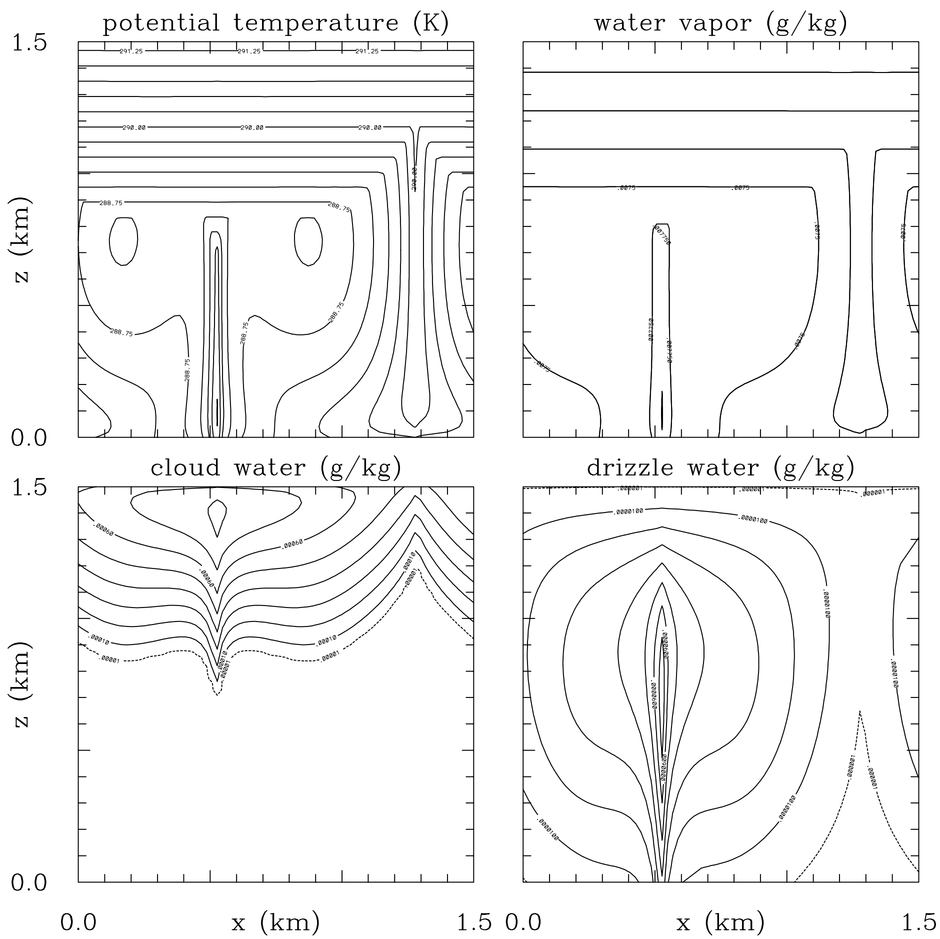
# Kogan JAS 2013



# Kessler


potential temperature (K)
water vapor (g/kg)
cloud water (g/kg)
drizzle water (g/kg)
z (km)
x (km)
0.0
1.5

# Berry 1968 / Grabowski 1998

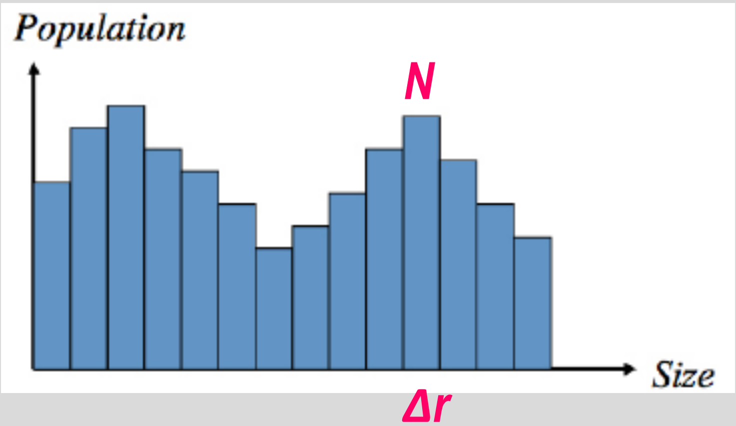


# BIN-RESOLVING WARM MICROPHYSICS:

Introducing *spectral density function* $f(r,t)$:

$$f(r,t) \equiv \frac{d\,N(r,t)}{d\,r}$$

$d\,N(r,t)$ is the concentration (per unit mass as mixing ratio) of droplets smaller than $r$ (cumulative concentration).




$f = N / \Delta r$

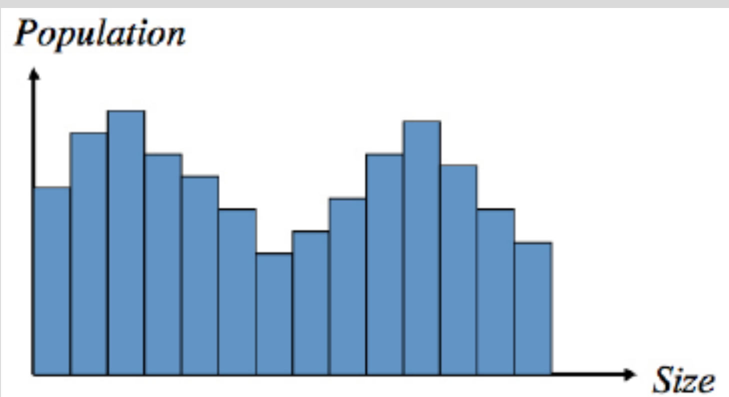
Continuity equation for the growth by condensation:

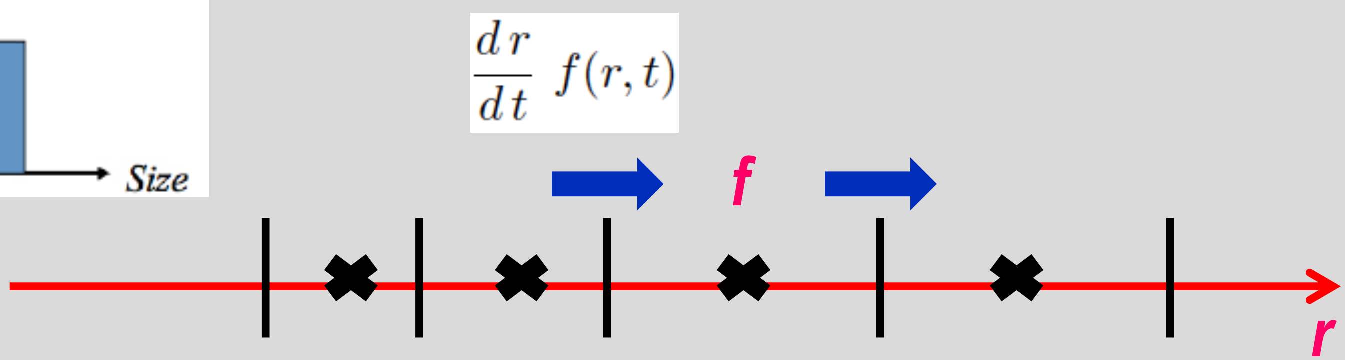
$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{d\,r}{d\,t}\; f(r,t)\right) = 0$$

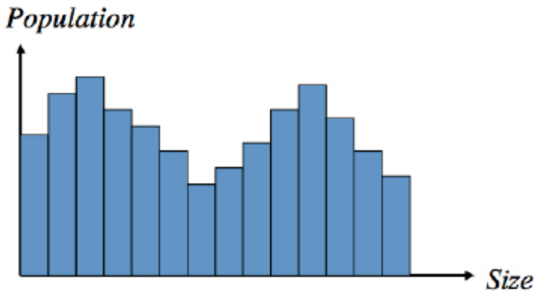
where $\frac{d\,r}{d\,t}$ is growth rate of a droplet with radius $r$:

$$\frac{d\,r}{d\,t} = \frac{A(T,p)\;S}{r}$$

$S = \frac{q_v}{q_{vs}} - 1$ is the supersaturation; $q_v$ is the ambient water vapor mixing ratio; $q_{vs}(p,T)$ is the saturated water vapor mixing ratio.






# BIN-RESOLVING WARM MICROPHYSICS:

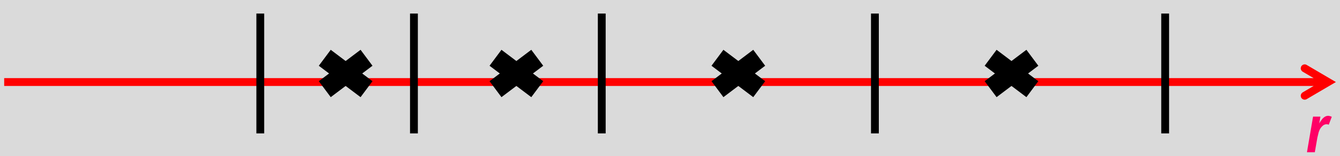
## ACTIVATION AND CONDENSATION

Continuity equation for activation and growth by condensation:

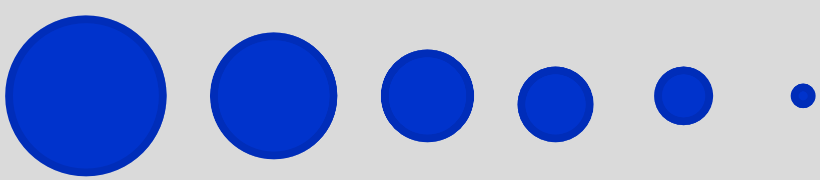
$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{d\,r}{d\,t}\;f(r,t)\right) = S_{nucl}$$

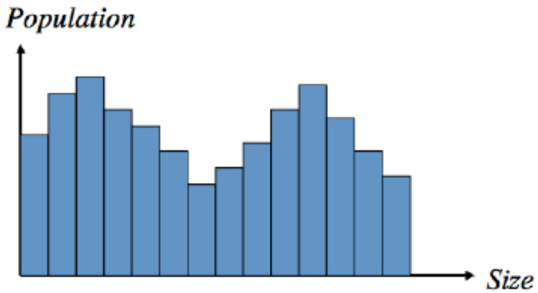
where $S_{nucl}$ is the source associated with activation of cloud droplets (CCN activation).

cloud droplets




aerosols (CCN)



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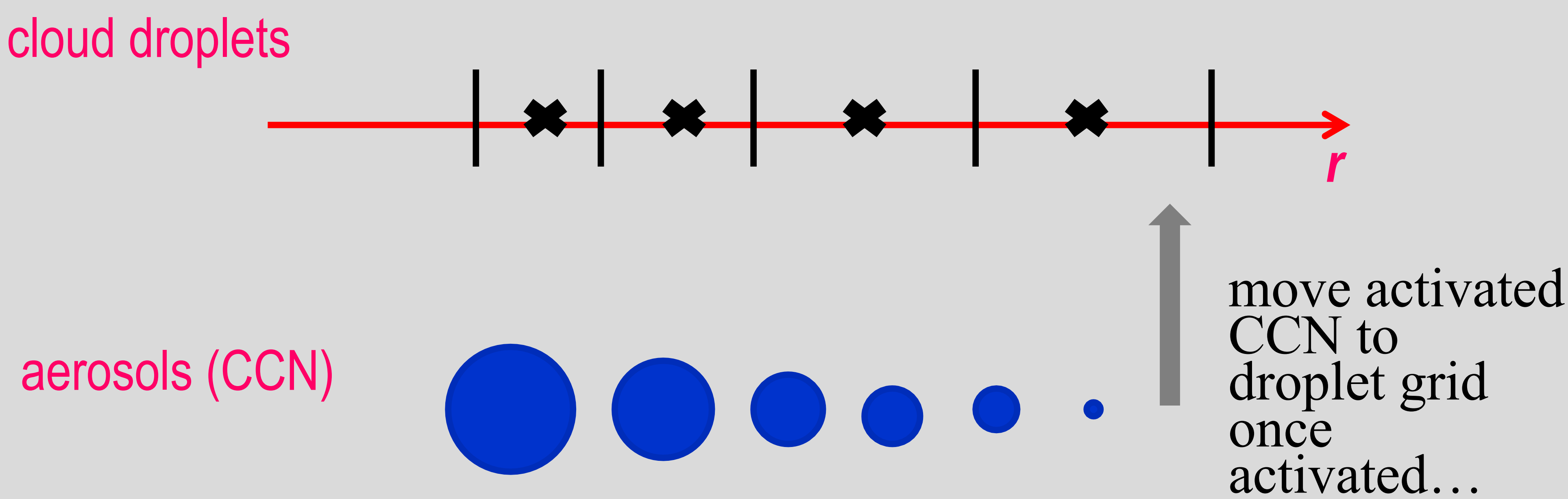


## *Twomey CCN activation:*

***N - total concentration of activated droplets***

***S – supersaturation ($S = q_v/q_{vs} - 1$)***

$$N = a\, S^b$$

***a, b – parameters characterizing CCN***

*$0 < b < 1$ (typically, $b=0.5$)*

*$a \sim 100\ cm^{-3}$ maritime/clean*

*$a \sim 1{,}000\ cm^{-3}$ continental/polluted*

***Activated CCN are inserted into the first bin (say, $r=1\ \mu m$)***

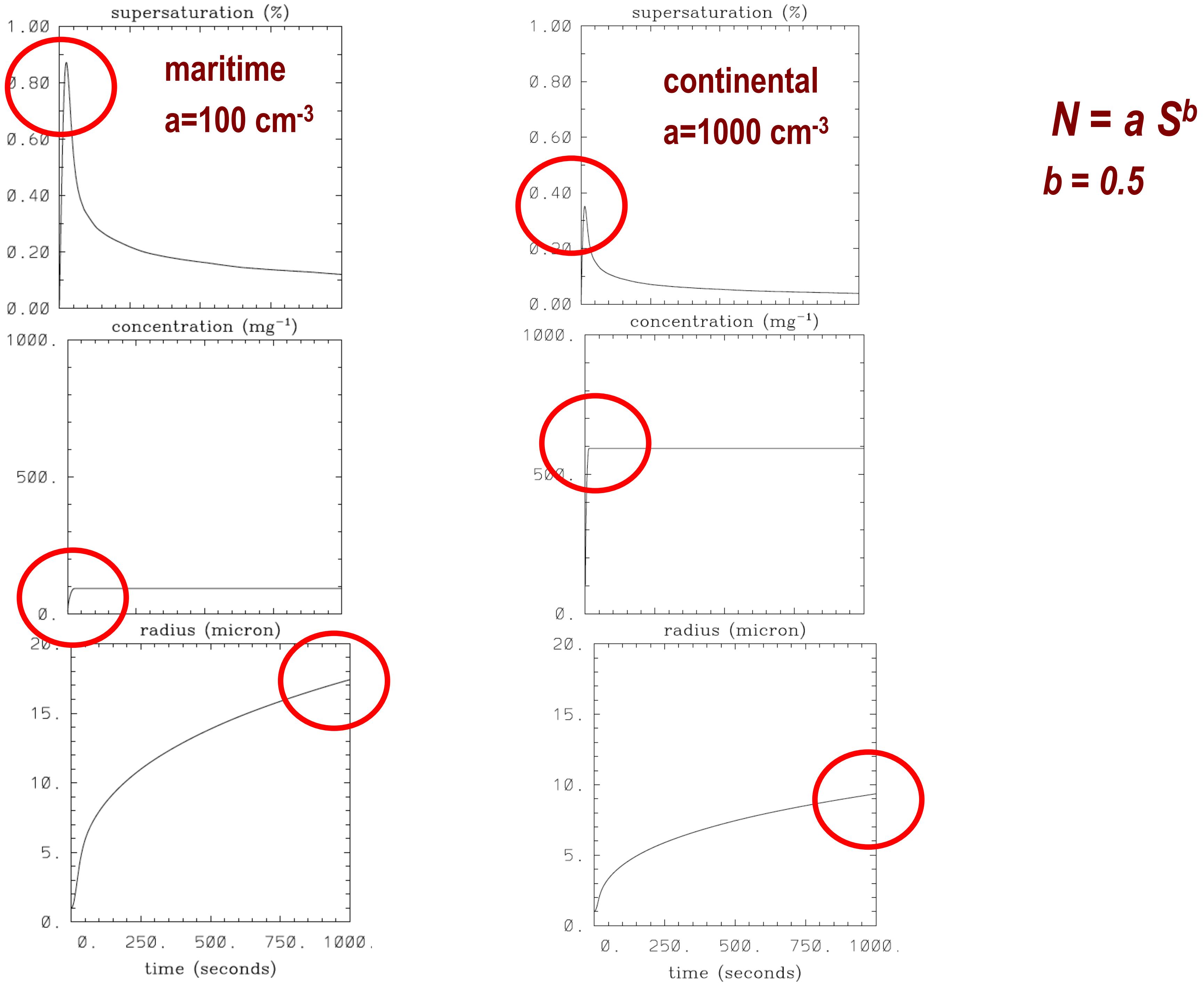
**Computational example:**

**Nucleation and growth of cloud droplets in a parcel of air rising with vertical velocity of 1 m/s;**

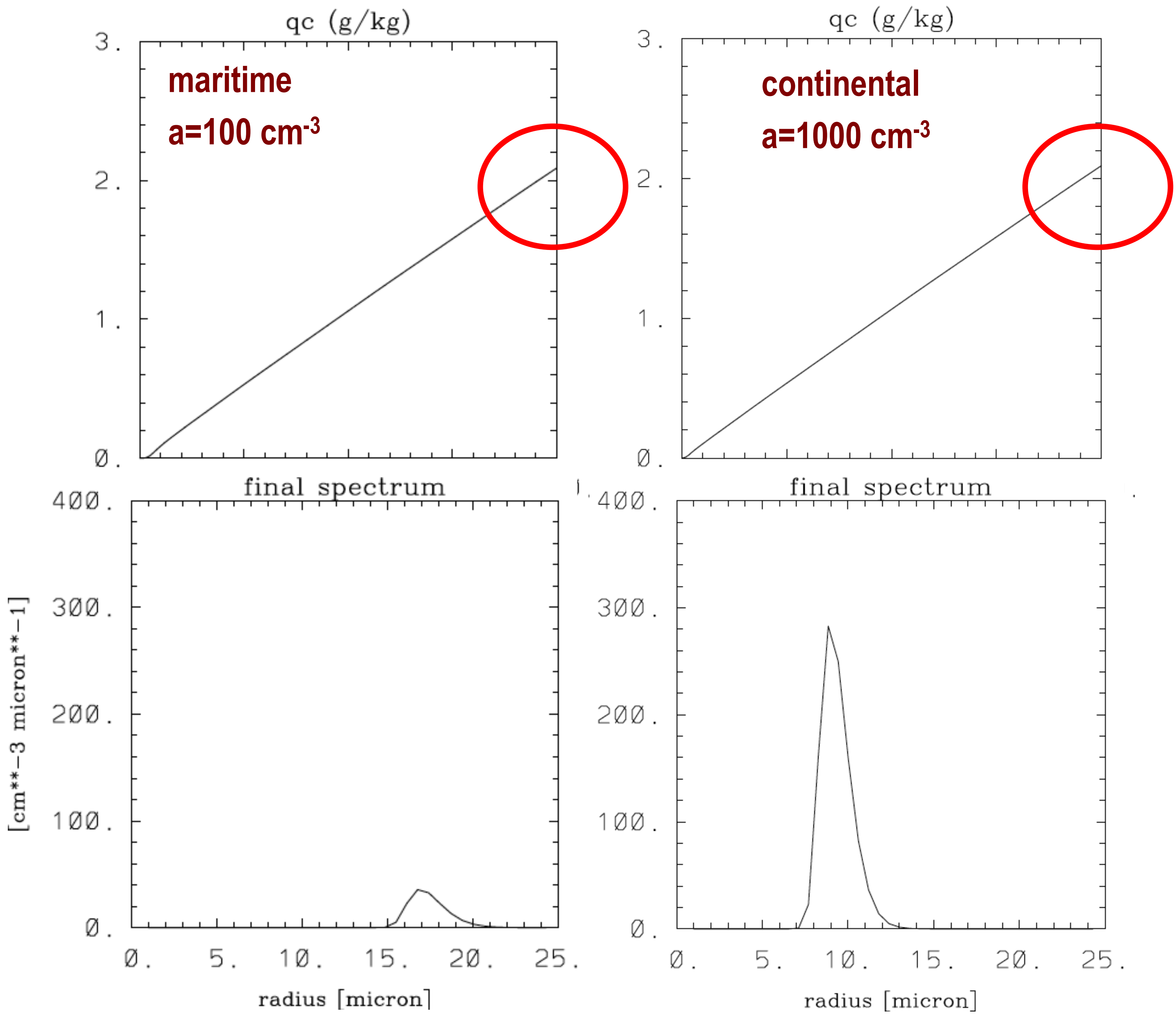
**60 bins used;**

**1D flux-form advection applied in the radius space;**

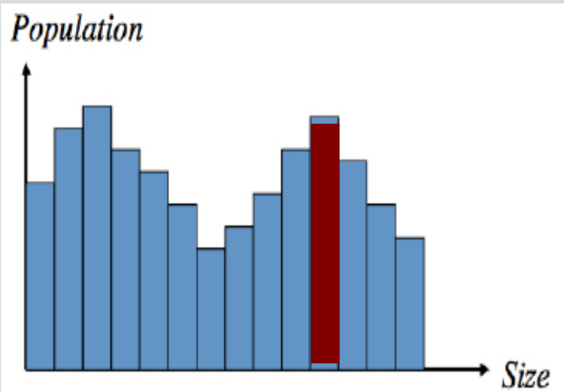
**Difference between continental/polluted and maritime/pristine aerosols**



maritime
a=100 cm-3
continental
a=1000 cm-3
N = a S^b
b = 0.5
supersaturation (%)
concentration (mg-1)
radius (micron)
time (seconds)



qc (g/kg)
maritime
a=100 cm-3
qc (g/kg)
continental
a=1000 cm-3
final spectrum
final spectrum
[cm**-3 micron**-1]
radius [micron]
radius [micron]




# GROWTH BY COLLISION/COALESCENCE

The Smoluchowski equation (aka *kinetic collection equation*, *stochastic coalescence equation*) for the spectral density function $f(m,t)$:

$$\frac{\partial f(m,t)}{\partial t} =$$

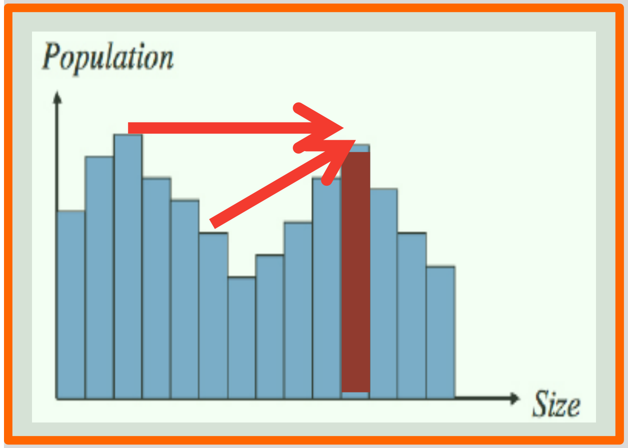
$$= \frac{1}{2}\int_0^m f(m-M,t)\, f(M,t)\, K(m-M,M)\, dM$$

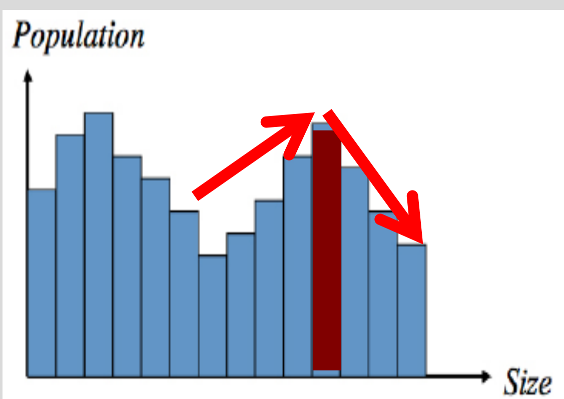
$$- f(m,t) \int_0^\infty f(M,t)\, K(m,M)\, dM$$

$m$, $M$ - droplet masses

$K(m,M)$ - *collection kernel*; frequency of collisions (per unit volume of air) between droplets with mass $m$ and $M$

# GROWTH BY COLLISION/COALESCENCE

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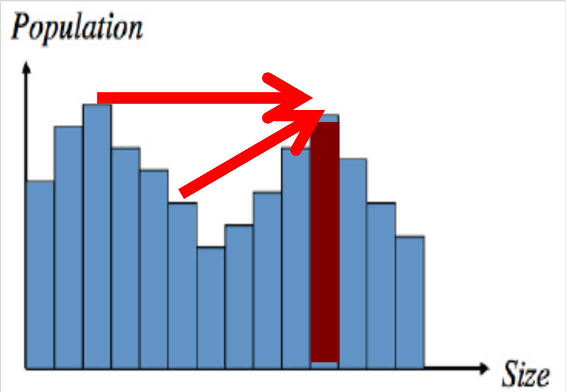


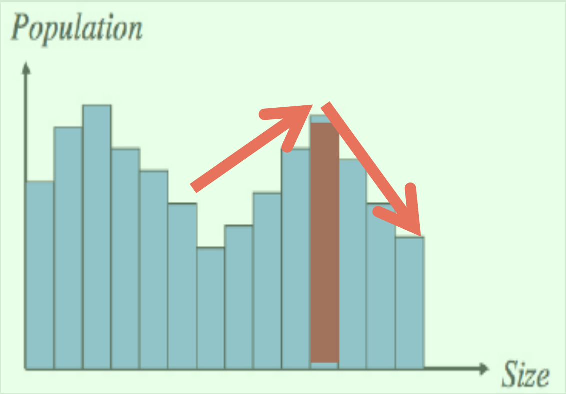


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$m$, $M$ - droplet masses

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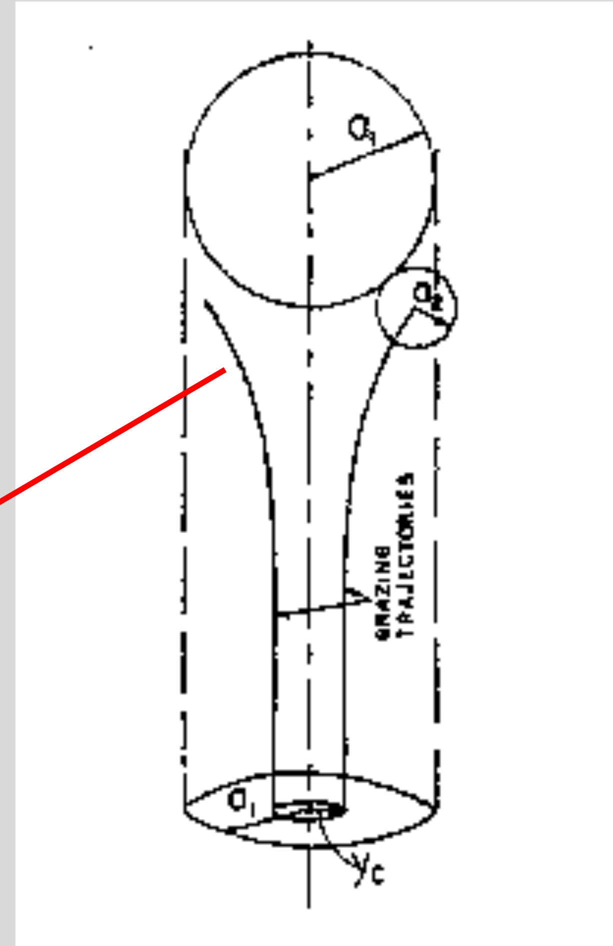
## Growth of water droplets by gravitational collision-coalescence:

$$K(m_{a1}, m_{a2}) = E_c \; \pi(a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$

**Collision efficiency:**

$$E_c = \frac{y_c^2}{(a_1 + a_2)^2}$$

Grazing trajectory




**Droplet inertia is the key; without it, there will be no collisions. This is why collision efficiency for droplets smaller than 10 µm is very small.**

$$K(m_{a1}, m_{a2}) = E_c \pi (a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$

TABLE 1. Radius ratio $r/R$.

| Collector drop radius ($\mu$m) | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 300 | 0.97 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 200 | 0.87 | 0.96 | 0.98 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 150 | 0.77 | 0.93 | 0.97 | 0.97 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 100 | 0.50 | 0.79 | 0.91 | 0.95 | 0.95 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 70 | 0.20 | 0.58 | 0.75 | 0.84 | 0.88 | 0.90 | 0.92 | 0.94 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.97 | 1.0 | 1.02 | 1.04 | 2.3 | 4.0 |
| 60 | 0.05 | 0.43 | 0.64 | 0.77 | 0.84 | 0.87 | 0.89 | 0.90 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.92 | 0.93 | 0.95 | 1.0 | 1.03 | 1.7 | 3.0 |
| 50 | 0.005 | 0.40 | 0.60 | 0.70 | 0.78 | 0.83 | 0.86 | 0.88 | 0.90 | 0.90 | 0.90 | 0.90 | 0.89 | 0.88 | 0.88 | 0.89 | 0.92 | 1.01 | 1.3 | 2.3 |
| 40 | 0.001 | 0.07 | 0.28 | 0.50 | 0.62 | 0.68 | 0.74 | 0.78 | 0.80 | 0.80 | 0.80 | 0.78 | 0.77 | 0.76 | 0.77 | 0.77 | 0.78 | 0.79 | 0.95 | 1.4 |
| 30 | 0.0001 | 0.002 | 0.02 | 0.04 | 0.085 | 0.17 | 0.27 | 0.40 | 0.50 | 0.55 | 0.58 | 0.59 | 0.58 | 0.54 | 0.51 | 0.49 | 0.47 | 0.45 | 0.47 | 0.52 |
| 20 | 0.0001 | 0.0001 | 0.005 | 0.016 | 0.022 | 0.03 | 0.043 | 0.052 | 0.064 | 0.072 | 0.079 | 0.082 | 0.080 | 0.076 | 0.067 | 0.057 | 0.048 | 0.040 | 0.033 | 0.027 |
| 10 | 0.0001 | 0.0001 | 0.0001 | 0.014 | 0.017 | 0.019 | 0.022 | 0.027 | 0.030 | 0.033 | 0.035 | 0.037 | 0.038 | 0.038 | 0.037 | 0.036 | 0.035 | 0.032 | 0.029 | 0.027 |

**Hall (*J. Atmos. Sci*. 1980)**
**(compilation of many theoretical studies and laboratory measurements)**

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| 200 | 0.87 | 0.96 | 0.98 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 150 | 0.77 | 0.93 | 0.97 | 0.97 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
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# Seminal 4-part series of papers by Berry and Reinhardt (JAS 1974)

**An Analysis of Cloud Drop Growth by Collection:
Part I. Double Distributions**

EDWIN X BERRY
*National Science Foundation, Washington, D. C. 20550*
RICHARD L. REINHARDT
*Sierra Nevada Corporation, Reno 89505*
(Manuscript received 29 January 1974, in revised form 9 April 1974)

**An Analysis of Cloud Drop Growth by Collection:
Part II. Single Initial Distributions[1]**

EDWIN X BERRY
*National Science Foundation, Washington, D. C. 20550*
RICHARD L. REINHARDT
*Sierra Nevada Corporation, Reno 89505*
(Manuscript received 1 May 1974)

The method, described in the appendix of Part I, and serving as Reinhardt's PhD dissertation, is considered highly accurate but very cumbersome. The main criticism is that it does not conserve water…

# Part 1

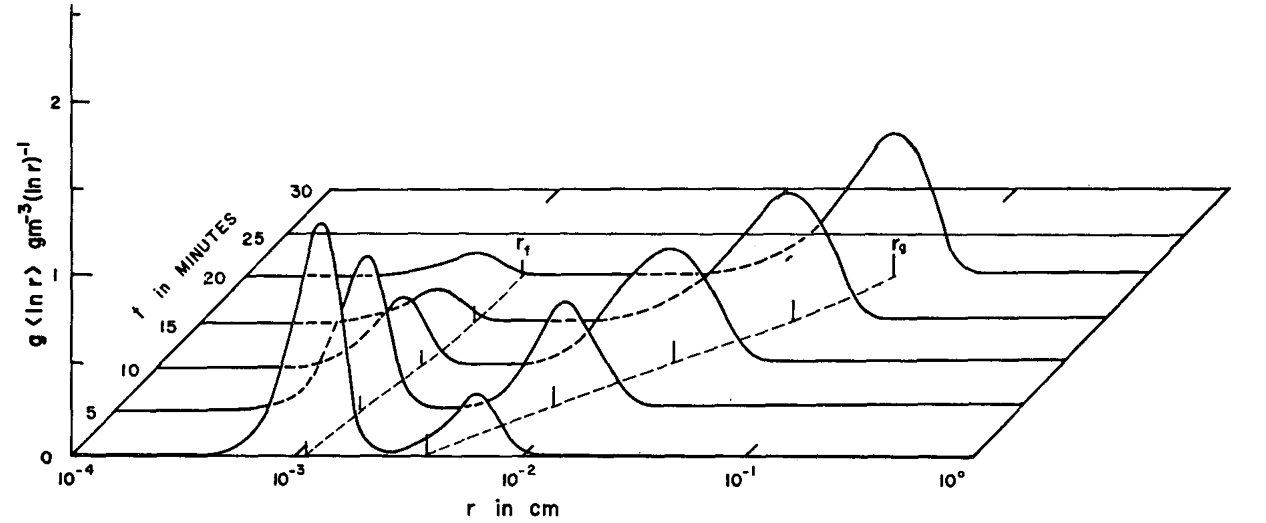


FIG. 2. Time evolution of the initial spectrum composed of 0.8 gm m$^{-3}$ with $r_f^0 = 10$ $\mu$m, and 0.2 gm m$^{-3}$ with $r_f^0 = 50$ $\mu$m, both with var $x = 1$.

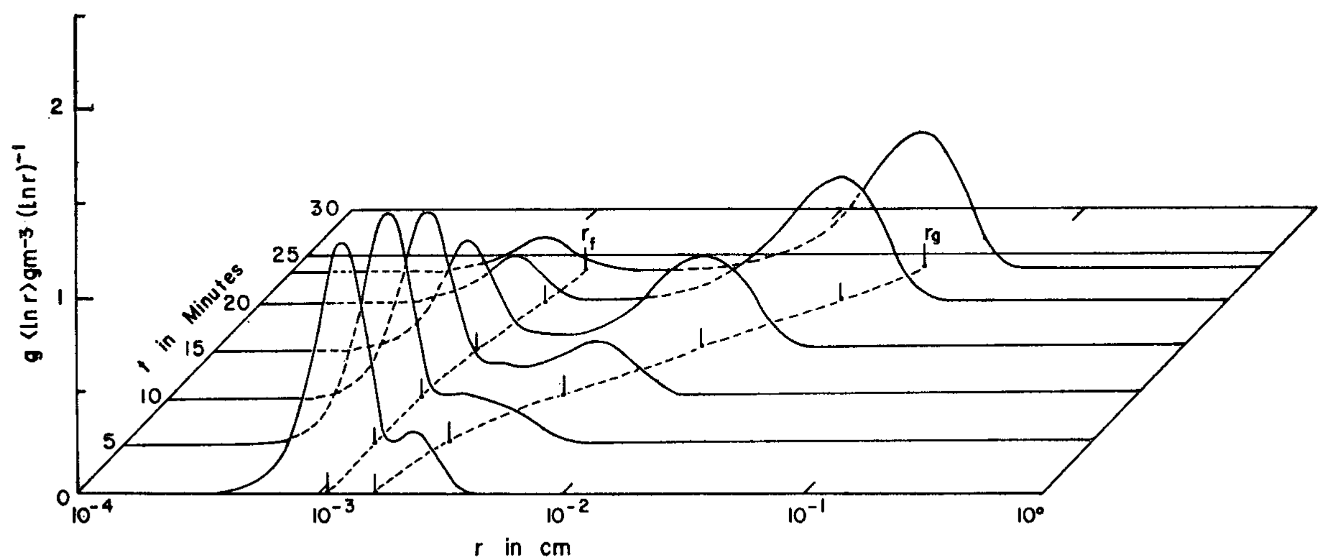


FIG. 3. As in Fig. 2 except for $r_f^0 = 20$ $\mu$m.

**Part 2**

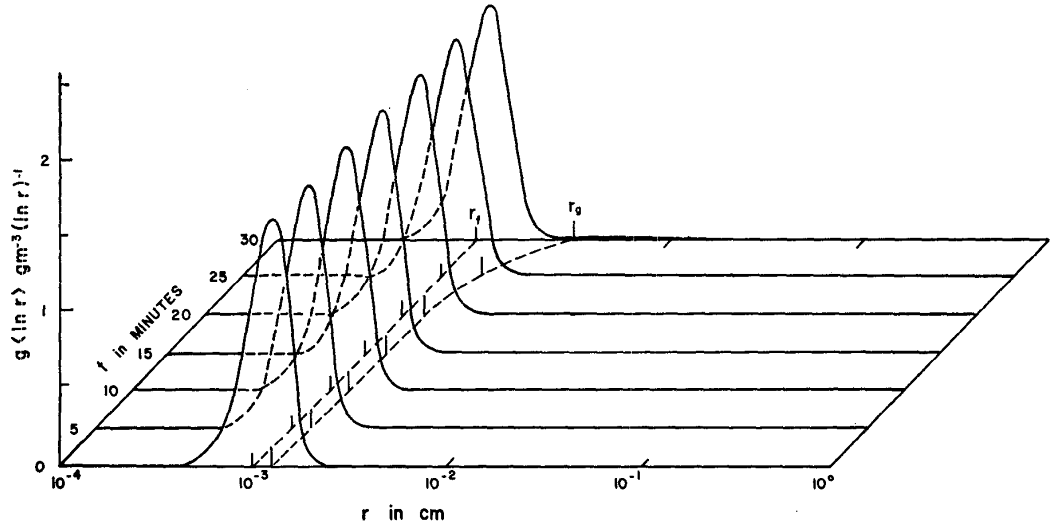


FIG. 1. Time evolution of the initial spectrum for $r_f^0=10$ μm, var $x=1$.

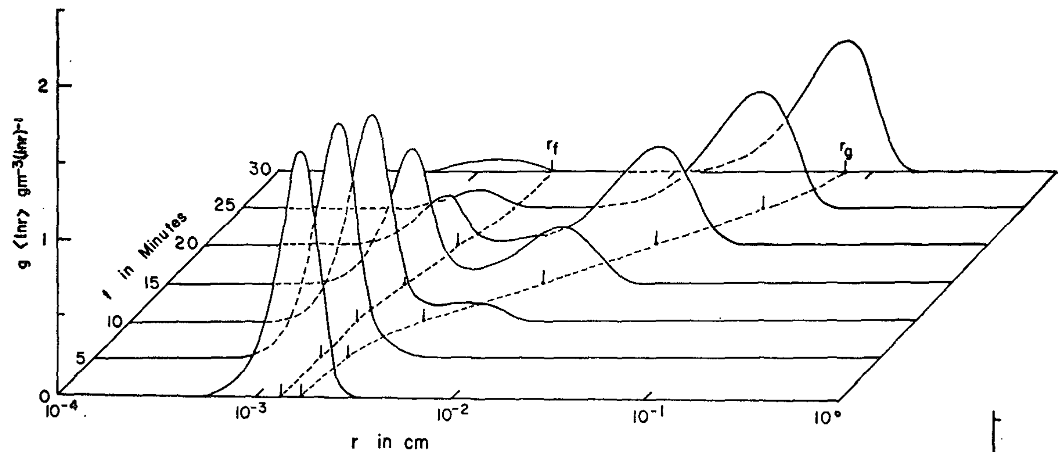


FIG. 4. Time evolution of the initial spectrum for $r_f^0=14$ μm, var $x=1$.

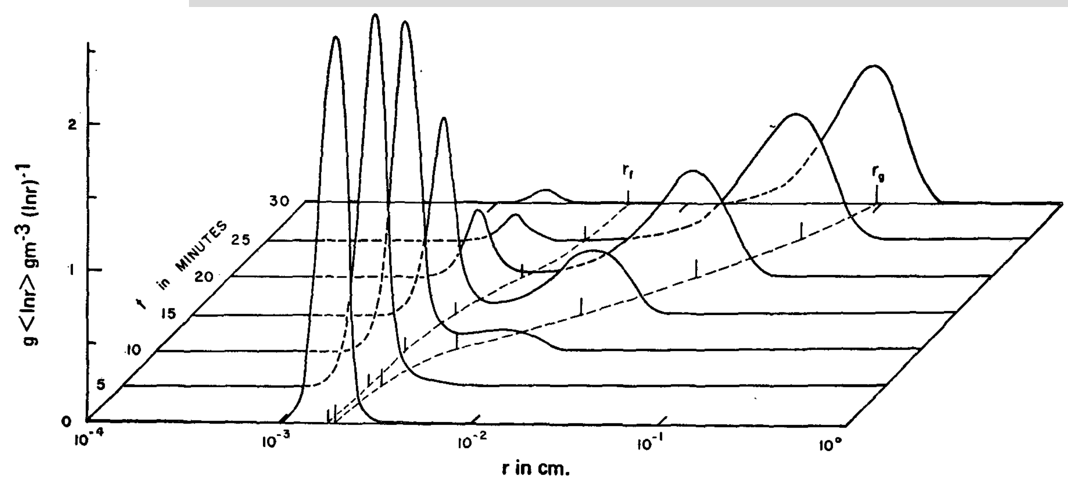


FIG. 5. Time evolution of the initial spectrum for $r_f^0=18$ μm, var $x=0.25$.

Simulation of the droplet size distribution is difficult.

For growths by both diffusion and collision/coalescence, numerical spreading of the distribution is of primary concern. Accurate methods are expensive because they require many bins.

For multidimensional cloud simulations, each bin variable needs to be advected in the physical space. This makes bin microphysics schemes extremely expensive because the numerical cost is approximately proportional to the number of bins used. Moreover, numerical spreading of the size distribution is also affected by the the advection in the physical space.

This why bulk schemes are the workhorse of the multidimensional (2D, 3D) cloud simulations…

# New trend: Lagrangian treatment of the condensed phase:

**The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model**

S. Shima,[a]* K. Kusano,[c] A. Kawano,[a] T. Sugiyama[a] and S. Kawahara[b]

**Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model**

M. Andrejczuk,[1] W. W. Grabowski,[2] J. Reisner,[3] and A. Gadian[1]

**Large-Eddy Simulations of Trade Wind Cumuli Using Particle-Based Microphysics with Monte Carlo Coalescence**

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**A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision**

T Riechelmann[1,3], Y Noh[2] and S Raasch[1]

**Eulerian dynamics, energy and water vapor transport:**

$$\frac{\partial(u\rho)}{\partial t}+\frac{\partial(uu\rho)}{\partial x}+\frac{\partial(wu\rho)}{\partial z}=-\frac{\partial p'}{\partial x}+\Phi_{m,x}+\frac{\partial(\kappa\rho\tau^{11})}{\partial x}+\frac{\partial(\kappa\rho\tau^{13})}{\partial z},$$

$$\frac{\partial(w\rho)}{\partial t}+\frac{\partial(uw\rho)}{\partial x}+\frac{\partial(ww\rho)}{\partial z}=-\frac{\partial p'}{\partial z}-\rho' g+\Phi_{m,z}+\frac{\partial(\kappa\rho\tau^{31})}{\partial x}+\frac{\partial(\kappa\rho\tau^{33})}{\partial z},$$

$$\frac{\partial(\theta\rho)}{\partial t}+\frac{\partial(u\theta\rho)}{\partial x}+\frac{\partial(w\theta\rho)}{\partial z}=\frac{\theta\rho L}{TC_p}f_{cond}+f_{surface-energy}+f_{rad}+\frac{\partial F_{\theta x}}{\partial x}+\frac{\partial F_{\theta z}}{\partial z},$$

$$\frac{\partial(q_v\rho)}{\partial t}+\frac{\partial(uq_v\rho)}{\partial x}+\frac{\partial(wq_v\rho)}{\partial z}=-f_{cond}+f_{surface-gas}+\frac{\partial F_{q_vx}}{\partial x}+\frac{\partial F_{q_vz}}{\partial z},$$

**Lagrangian physics of "super-particles"**

a single "super-particle" represents a number of the same airborne particles (aerosol, droplet, ice crystal, etc.) with given attributes

$$\frac{dx_i}{dt}=v_i$$

$$\frac{dv_i}{dt}=\frac{1}{\tau_p}(v_i^*-v_i)+g\delta_{i,2}$$

$$\frac{dr}{dt}=\frac{G}{r}(S^*-S_{eq})$$

**Coupling**

$$\Phi_{m,x}=\sum_{id} m_{id}\frac{M_{id}}{\Delta V}\frac{(u^*-u_{id})}{\tau_{p,id}}$$

$$\Phi_{m,z}=\sum_{id} m_{id}\frac{M_{id}}{\Delta V}\frac{(w^*-w_{id})}{\tau_{p,id}}$$

$$f_{cond}=\sum_{id}\frac{M_{id}}{\Delta V}\frac{dm_{id}}{dt}$$

$m_{id}$ – mass of the super-particle

$M_{id}$ – concentration of super-particles

$\Delta V$ – volume of the gridbox

**Andrejczuk et al. 2008, 2010**

# Why Lagrangian SD approach is appealing?

**- no numerical diffusion due to advection;**

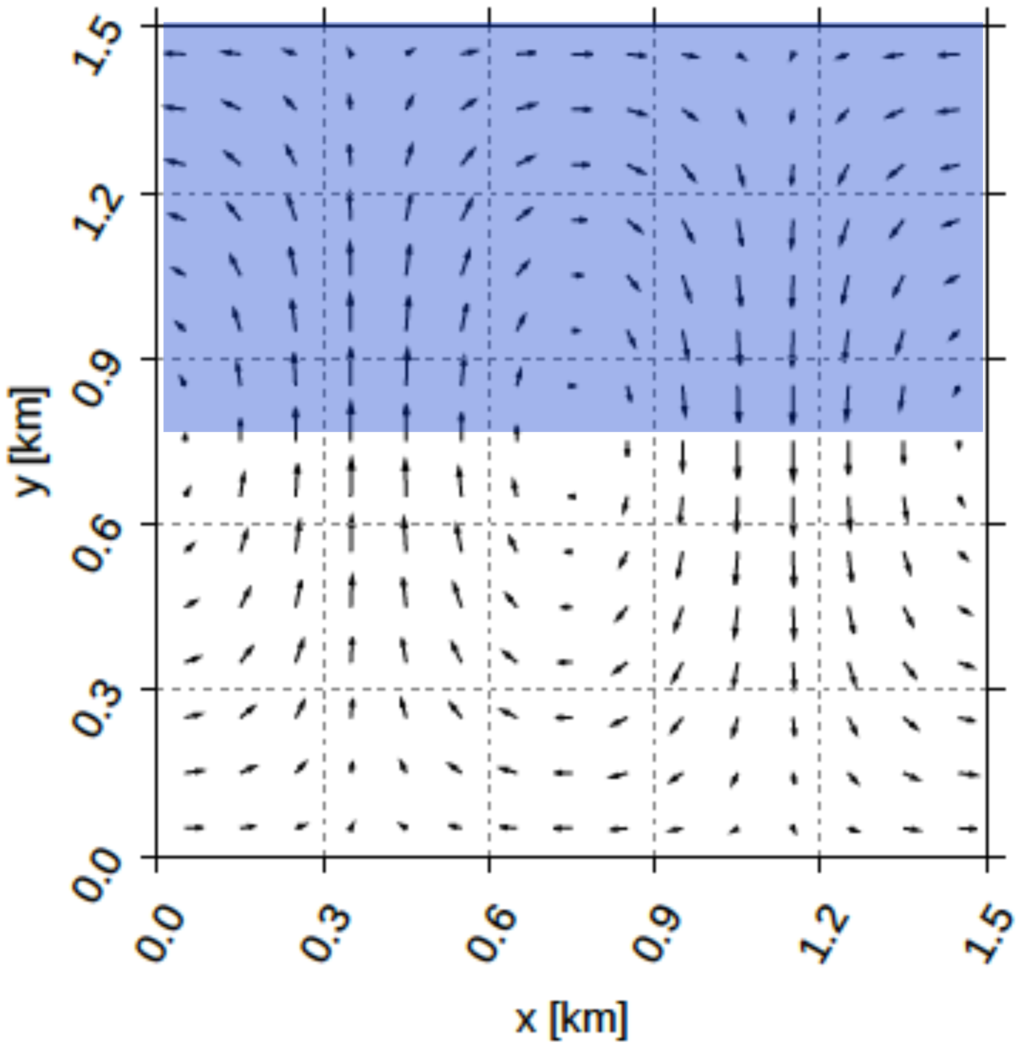
**- but sampling errors: one needs ~100 particles per gridbox for simple problems, many more with a longer list of attributes for appropriate sampling of the parameter space;**

**- straightforward for condensational growth of cloud droplets (initial sampling of the CCN distribution, growth/activation/evaporation of aerosol/droplet) – *ideal for entrainment/mixing!***

**- more complex for collisions (collision of two SDs creates a new SD: two methods in the literature to deal with this…);**

**- seems ideal to couple with sophisticated subgrid-scale models to represent effects of turbulence (e.g., randomly choose thermodynamic environment within a gridbox, use LEM approach, etc);**

**- easy representation of ice particle habits and diffusional varsus accretional growth.**


y [km]
x [km]
0.0
0.3
0.6
0.9
1.2
1.5

Geosci. Model Dev., 8, 1677–1707, 2015
www.geosci-model-dev.net/8/1677/2015/
doi:10.5194/gmd-8-1677-2015


Geoscientific Model Development
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# libcloudph++ 1.0: a single-moment bulk, double-moment bulk, and particle-based warm-rain microphysics library in C++

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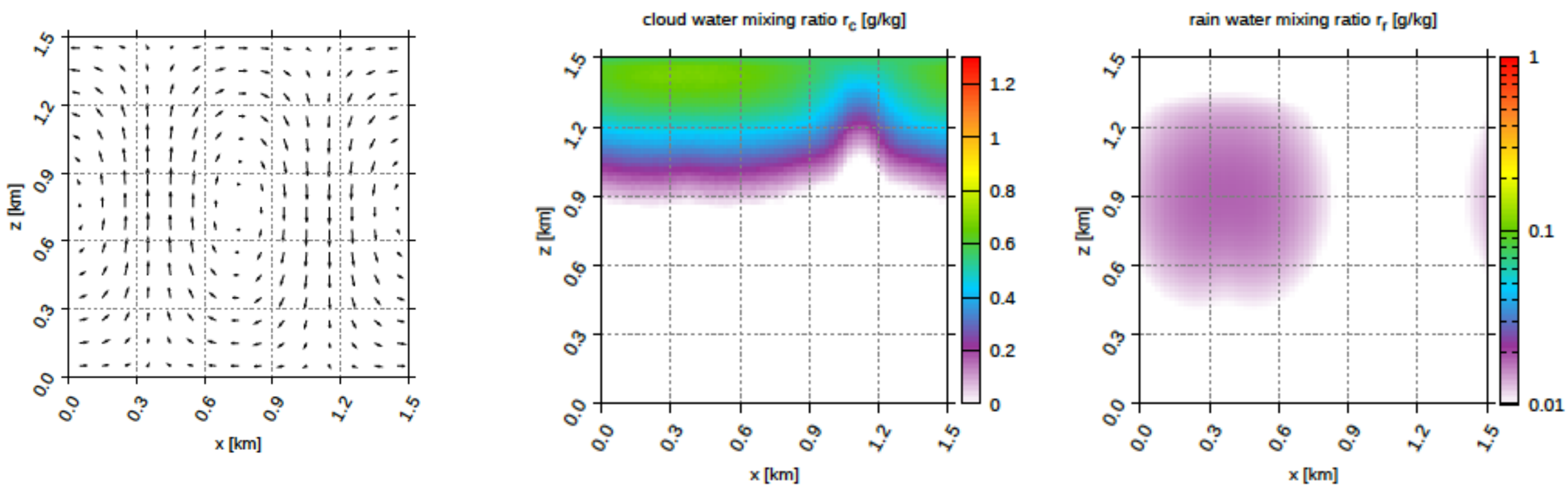
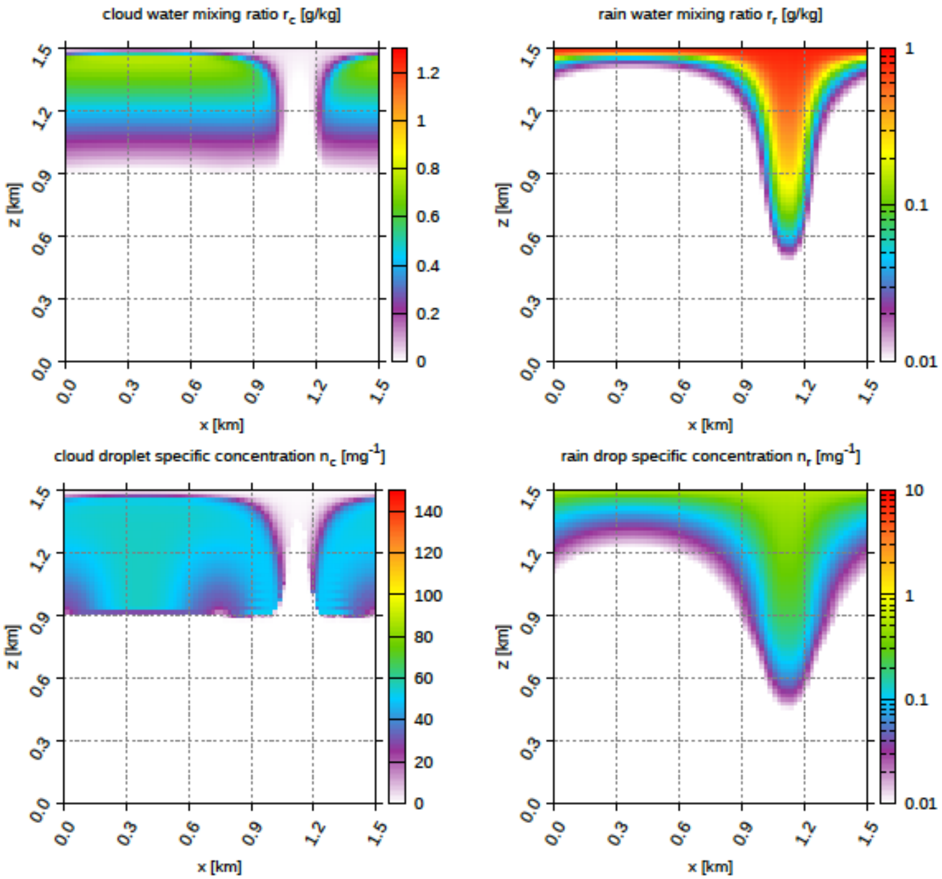
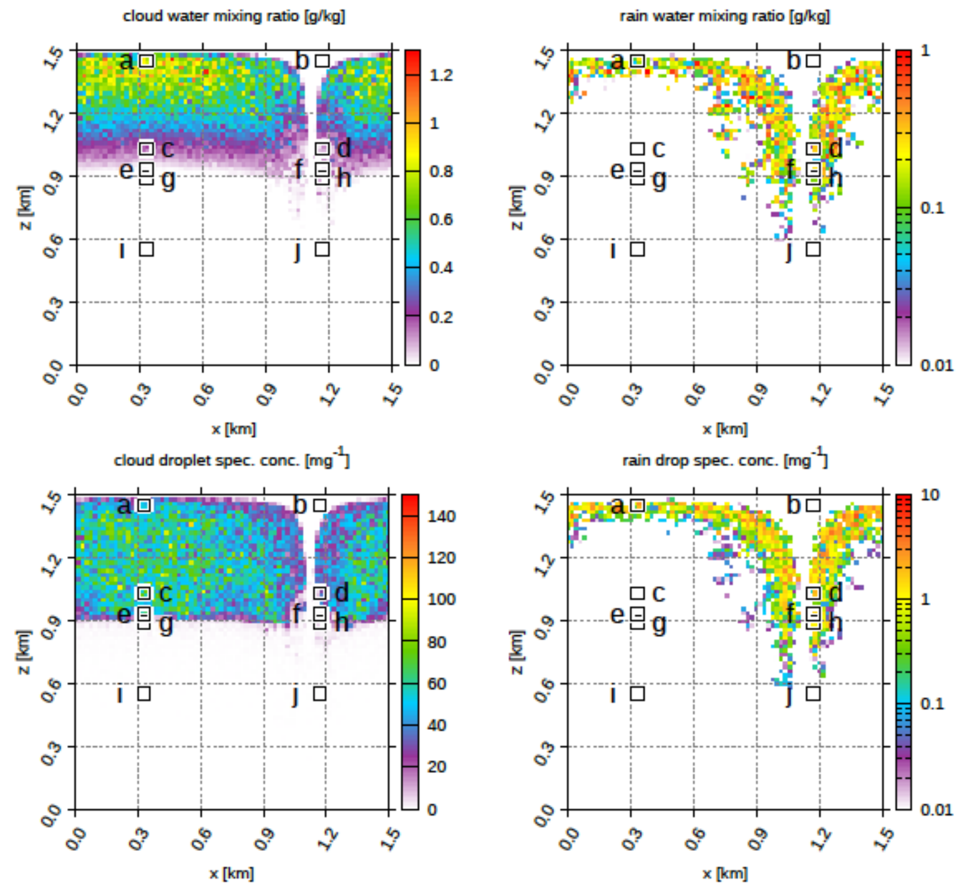
[1]Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland
[2]National Center for Atmospheric Research (NCAR), Boulder, CO, USA

## 1-moment Eulerian scheme






## 2-moment Eulerian scheme




## Lagrangian scheme (super-droplets)

**Arabas et al. GMD 2015**

**Summary:**

***Warm-rain microphysics:*** **cloud droplet activation, condensational growth, collisional growth.**

**Eulerian modeling warm-rain processes:**

**- bulk single-moment scheme: mixing ratios for cloud water and drizzle/rain water (activation irrelevant, no information about spectral characteristics, model resolution can be low);**

**- detailed (bin) microphysics: concentration (per unit mass) of cloud and drizzle/rain drop in each size (mass) category (~100 variables); supersaturation and droplet activation predicted, requires high spatial resolution (especially near cloud base); can be even more complicated if detailed information about aerosols is added;**

**- double-moment microphysics: mixing ratios and concentrations of cloud and drizzle/rain drops, supersaturation does not have to be predicted (but it can be; e.g., MG scheme), activation either predicted (MG; high resolution needed) or parameterized (e.g., as a function of the updraft speed; lower resolution possible).**

**Lagrangian modeling of warm-rain processes:**

**- relatively straightforward simulation of aerosol processing.**