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PROJECT REPORT ON MULTIDIMENSIONAL INITIALIZATION
FOR NWP MODELS

BY

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ABSTRACT

This report purports to be a brief review of the various techniques of preparing the initial data, from a given set of observed grid point data, to serve as input to the NWP model. It also contains the work done under this project, in the Institute. A critical examination of the comparative merits of each scheme and its suitability under Indian conditions, is also attempted.

1. INTRODUCTION

Numerical Weather Prediction consists in solving a set of partial differential equations, representing the atmospheric variables, as an initial value problem. The solution admits two basic types of waves, viz.(a)the slow, quasi-geostrophic, meteorologically significant waves, (b)the fast, inertia-gravity waves. The initial values of the time-dependent atmospheric variables, constitute one state of an atmospheric model and are obtained from observations of the real atmosphere. In the ideal physical situation it should be expected that the observations would specify, unambigously, one value for each and every state parameter at the selected initial time, $t = 0$. However, this ideal state never materializes in actual meteorological forecasting, owing to the following reasons :-

(a)Conventional pressure, temperature and wind observations are inadequately distributed around the planet and leave severe geographical gaps where no data are available.

(b)Conventional observations are point measurements which do not provide a correct sampling of the highly variable meteorological fields. Such measurements are not representative of true volume averages as required by numerical models.

(c)Conventional observations are subject to significant random instrumental errors.

In the absence of the ideal state, certain imbalances occur initially in the observed mass and wind fields, which manifests itself through the large horizontal divergence fields, changing rapidly with time. When this data is introduced into the model equations and integrated forward with time, fast inertia-gravity waves develop, which interfere with short-range forecasts upto 12 hrs. and longer range forecast of vertical velocity and precipitation. It is desirable, therefore, to eliminate entirely these waves from the model at initial time $t = 0$, or to reduce them as much as possible, To achieve this elimination or reduction, is the purpose of initialization.

2. PHYSICS OF INITIALIZATION

The physical processes by which the mass and the wind fields are brought into a state of mutual balance is called the

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'adjustment process'. In nature there is always a tendency for the changing mass and wind fields to adjust themselves towards a state of mutual balance and the energy of the imbalances are generally dispersed in the form of inertio-gravity waves. In a realistic mathematical model, it should be possible to bring out this adjustment process. Cahn(1945) investigated the oscillations resulting from a sudden addition of momentum to a rotating fluid body. He considered an initial current system of width 2a, flowing into the plane of the paper with a steady velocity u_0 (Fig.1). The v-component is initially taken to the zero and 'u' is defined as follows :-

$$u_0 = U\ (\text{Constant}) \quad \text{for } |y| \leq a$$
$$u_0 = 0 \quad \ldots\ldots \quad \text{for } |y| > a$$

The depth of the fluid system is D_0. In the northern hemisphere, the coriolis force will act to the right of the direction of the current, causing an accumulation of mass on the right bank. The pressure gradient force will act in the opposite direction and tend to balance the coriolis force. The governing equations are as follows :-

$$\frac{\partial u}{\partial t} = fv$$
$$\frac{\partial v}{\partial t} = -fu - g\frac{\partial h}{\partial y} \qquad \ldots\ldots \quad 2.1$$
$$\frac{\partial h}{\partial t} + D_0\frac{\partial v}{\partial y} = 0$$

Here $\frac{\partial u}{\partial x} = \frac{\partial h}{\partial x} = \frac{\partial v}{\partial x} = 0 \ ; \ h = D - D_0$

If one eliminates u and h by cross differentiation, one obtains :-

$$\frac{\partial^2 v}{\partial t^2} = -f^2 v + gD_0\frac{\partial^2 v}{\partial y^2} \qquad \ldots\ldots \quad 2.2$$

This is the well-known telegraphy equations whose solution is :-

$$v = -\frac{Ua}{\lambda} J_0\left\{\sqrt{f^2t^2 - \frac{y_1^2}{\lambda}}\right\} \qquad \ldots\ldots \quad 2.3$$

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where $\lambda = \frac{\sqrt{gD_0}}{f}$ is the Rossby radius of deformation and J_0 is the zero order Bessel function of 1st kind. The Rossby radius of deformation can be physically interpreted as the distance over which the gravitational tendency to render the free surface flat is balanced by the tendency of the coriolis acceleration to deform the surface. A plot of 'v' versus time ' t ' is as shown in Fig.2 at y = 0.

Now let us consider an incompressible, homogeneous, non-viscous, rotating fluid with a flat bottom and a free upper surface. The linearized form of the basic equations, which govern such a fluid is given as :-

$$\begin{aligned}\frac{\partial u}{\partial t} &= fv - g\frac{\partial h}{\partial x}\\ \frac{\partial v}{\partial t} &= -fu - g\frac{\partial h}{\partial y} \qquad \text{- - - - - - - - 2.4}\\ \frac{\partial h}{\partial t} &= -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\end{aligned}$$

'h' is the perturbed height of free surface : H is the mean height of the fluid surface. The coriolis parameter is assumed to be constant. Let us suppose that u,v,h can be represented as sums of 2-D Fourier component i.e.

$$u(x,y,t) = \sum_k \sum_l u_{kl}(t)\, e^{i(kx+ly)} \qquad \text{- - - - 2.5}$$

where u_{kl} represents the Fourier coefficient for kth and lth wavenumber components. In the same way 'v' and 'h' can also be represented. We also define a stream function 'ψ' and a velocity potential 'χ' which can also be represented in the form of Eq. (2.5). The rotational and divergent component of 'u' can be written as :

$$\begin{aligned}u_\psi &= -\frac{\partial \psi}{\partial y} = -il\psi\\ u_\chi &= \frac{\partial \chi}{\partial x} = ik\chi\end{aligned}$$

where $$\begin{aligned}u &= u_\psi + u_\chi = -il\psi + ik\chi \qquad \text{- - - - - - 2.6}\\ v &= v_\psi + v_\chi = ik\psi + il\chi\end{aligned}$$

Substituting Eq. 2.5 in Eq. 2.4, we get

$$\begin{aligned}\frac{\partial u}{\partial t} &= fv - ikgh\\ \frac{\partial v}{\partial t} &= -fu - ilgh \qquad \text{- - - - - - - - - - - 2.7}\\ \frac{\partial h}{\partial t} &= -iH(ku + lv)\end{aligned}$$

substituting for 'u' and 'v' from Eq.(2.6) and eliminating successively the terms containing 'ψ' and 'χ' respectively. from the L.H.S., we get two equations, viz.,

$$\frac{\partial \psi}{\partial t} = - f\chi \quad \ldots \quad 2.8$$

$$\frac{\partial \chi}{\partial t} = f\psi - gh \quad \ldots \quad 2.9$$

The equation for 'h' is given as

$$\frac{\partial h}{\partial t} = + H\left(k^2+l^2\right)\chi \quad \ldots \quad 2.10$$

This system is not capable of describing the adjustment process since it has a general solution which will oscillate indefinitely without change of amplitude. It is seen from Eqn.(2.10) that the height tendency is a function of the velocity potential. We, therefore, introduce a damping term, in the form $\nu\nabla^2\chi$ in the divergence equation (2.9), where 'ν' is a diffusion coefficient.

Let $\chi = \sum_k \sum_l \chi_{kl}\, e^{i(kx+ly)}$

$$\nabla^2\chi = -\left(k^2+l^2\right)\chi$$

The modified form of Eq.(2.9) becomes :

$$\frac{\partial \chi}{\partial t} = f\psi - gh - \nu\left(k^2+l^2\right)\chi \quad \ldots \quad 2.11$$

Our aim is now to get a convergence towards a balanced stationary state, represented by ψ_s, χ_s, h_s

Let $\psi_s = \frac{g}{f} h_s$

from Eqn. (2.8) we have $\chi = -\frac{1}{f}\frac{\partial \psi}{\partial t}$. Substituting for '$\chi$' in Eqn. (2.10) we have

$$- f\frac{\partial h}{\partial t} - H\left(k^2+l^2\right)\frac{\partial \psi}{\partial t} = 0$$

We define a parameter Ω such that $\frac{\partial \Omega}{\partial t} = - f\frac{\partial h}{\partial t} - H\left(k^2+l^2\right)\frac{\partial \psi}{\partial t}$

or $\Omega = - H\left(k^2+l^2\right)\psi - fh \quad \ldots \quad 2.12$

Here Ω is invariant with respect to time. If the initial mass and wind fields are not in geostrophic balance, we impose the condition that the stationary values of Ω is equal to its initial value i.e.

$$H(k^2+l^2)\psi_s + fh_s = H(k^2+l^2)\psi_i + fh_i$$

The stationary state is the balanced state in which ψ and h are related through the geostrophic relation.

$$\psi_s = \frac{g}{f}h_s \quad \text{or} \quad h_s = \frac{f}{g}\psi_s$$

Substituting for h_s in the above equation, we get .

$$\psi_s\left[H(k^2+l^2) + \frac{f^2}{g}\right] = H(k^2+l^2)\psi_i + fh_i$$

$$\therefore \psi_s = \frac{gH(k^2+l^2)}{[gH(k^2+l^2)+f^2]}\psi_i + \frac{fgh_i}{[gH(k^2+l^2)+f^2]}$$

$$\text{Let } \alpha = \frac{gH(k^2+l^2)}{gH(k^2+l^2)+f^2} \; ; \; 1-\alpha = \frac{f^2}{gH(k^2+l^2)+f^2}$$

$$\therefore \psi_s = \alpha\psi_i + (1-\alpha)\frac{g}{f}h_i$$

We now construct a hypothetical streamfunction $\psi_i' = \frac{g}{f}h_i$, which is in geostrophic balance with the initial geopotential height h_i ; i.e.

$$\psi_s = \alpha\psi_i + (1-\alpha)\psi_i' \qquad 2.13$$

We now consider the following cases :-

Case A $\alpha \rightarrow 1$

Eq. (2.13) now reduces to $\psi_s = \psi_i$. If $\alpha \rightarrow 1$ then $gH(k^2+l^2) >> f^2$ but ψ_i is the given initial wind field which is not in balance with the mass field, and since the ideal, geostrophically balanced stationary wind field is equal to the given initial wind field, it follows that the mass field will have to adjust to the initial wind field. In the tropics, since 'f' is very small, the condition $\alpha \rightarrow 1$ is fulfilled and the geopotential height field is adjusted to the initial wind field.

Case B $\alpha \rightarrow 0$

Eq. (2.13) now reduces to $\psi_s = \psi_i'$. It also implies that $gH(k^2+l^2) << f^2$, ψ_i' is a hypothetical streamfunction which is in geostraphic balance with the initial height field, 'h_i'. So we replace ψ_i' with the real height field as :

$$\psi_i' = \frac{g}{f} h_i = \psi_s$$

In this case we see that the balanced stationary wind field is equal to the given initial height (mass) field. So the initial wind field ψ_i has to adjust towards the given mass field. The aforesaid condition exists in the higher latitudes where 'f' is large.

It may also be noted that for waves with very large wave numbers (small wave lengths), case A is satisfied irrespective of the latitudes, and for small wavenumbers (large wavelengths) case B is satisfied.

The adjustment process in a baroclinic model is more complicated due to the existence of internal gravity waves. The adjustment processes associated with external inertia-gravity waves is faster than with internal modes. Some experiments were carried out with a 3-level baroclinic model in which a disturbance was artificially introduced in the mass field at a particular level, as well as extended to all levels. In the latter case, the recovery of the pattern from the unbalanced state to the original balanced one was rapid and sufficiently accurate. This was essentially due to the role of the external gravity wave in rapidly dissolving the imbalances, because of its fast phase speeds. On the other hand if the disturbances were limited to one layer, then the convergence to the original field was slow and inaccurate because now the adjustment process was done through the internal gravity modes.

3. METHODS OF INITIALIZATION

Leith (1980) developed the 'Slow manifold' concept which is very useful in graphically illustrating the different types of initialization procedures. It was seen that the solutions of the model equations contain two basic modes, viz. the Rossby and the gravity modes, resp.. The Rossby modes evolve slowly with time, while the gravity modes have high frequency oscillations. The 'slow manifold' is defined as the locus of all model states which are evolving slowly in time. In Fig.3, the amplitude 'Y' of the Rossby modes is the abscissa, while the amplitude of the gravity modes 'Z' is represented by the ordinate, Any model state on the abscissa can be said to be in geostrophic balance. The curve 'M' is the 'slow manifold' and represent a balanced state of the model. Its curved form is a reflection of the nonlinear nature of the balance. At Z, Y $\longrightarrow$ 0, the non-linear terms tend towards zero and the slow manifold becomes coincident with the Rossby manifold. As 'Y' becomes large, the slow and Rossby manifold diverge, suggesting that a high amplitude but slowly evolving model state would be far from geostrophic. The line D represents the data manifold. It is the locus of all model states which are obtained by keeping the spatial configuration of one of the dependent variables (geopotential, wind) fixed and varying the other variables.

Suppose we have a model state at point O, on the data manifold. It does not lie on the slow manifold, so that if 'O' is used as an initial state for the model, than gravity waves of magnitude proportional to the distance of 'O' from 'M' would be excited. As a specific case, let us suppose that 'D' represents the wind manifold, i.e. the spatial structure of the wind field is invariant along D. We assume that the wind observations are very accurate but the observations for geopotential is poor. If we performed unconstrained initialization, thus arriving at the point 'U', the original wind field would no longer be fitted. Since we had confidence in the wind observations, the initialized state 'U' would be unsatisfactory.

A better method would be to find the intersection of the data manifold D and the slow manifold M. This would imply that the original wind observations are fitted and yet no high frequencies would be excited in subsequent model integrations. This state is denoted by the point 'C' and indicates constrained initialization. In practice there are observations of both wind and height fields distributed irregularly in space and time and with varying accuracies. Some of these data will be inconsistent with the other data, so that it is impossible to fit all the data and still be on the slow manifold. The endeavour is to see that the data on which we have maximum confidence fits well, whereas the data whose accuracy is poor, does not fit.

The unconstrained initialization procedures can be subdivided into two classes, viz., (A) static initialization (B) Dynamic initialization.

(A) Static initialization : In this method, a wind law such as the geostrophic relation or the balance equation is used. The wind is derived from a stream function ψ such that :

$$\nabla^2 \psi = \vec{k} \cdot \nabla \times \vec{V}$$

where $\vec{V}$ denotes the vector horizontal wind

The height field (ϕ) is then derived from the balance equation, such as :

$$\nabla^2 \phi = \nabla \cdot (f \nabla \psi)$$

$$\text{or} \quad \nabla^2 \phi = \nabla \cdot (f \nabla \psi) + 2J\left(\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}\right)$$

The height field derived from the above equation is in balance with the rotational component of the wind field only, while the divergent component is totally ignored.

This method was examined by Krishnamurti (1969) and Kanamitsu (1975) and was found to suffer from the usual problem of adjustment in low latitudes. Krishnamurti found that this takes roughly the equivalent of 18 hours of integration, during which time the divergent part of the wind grows in magnitude and vertical motions increase by an order of magnitude. The other drawbacks in this method are as follows :

(a)A given wind field gives the same balanced height field, irrespective of the model states. Referring to Fig.3, we see that in this method the data point neither lies on the data manifold, nor on the slow manifold and is likely to generate high amplitude gravity waves if used as input to the model.

(b)The incorporation of physical processes(heat,friction etc.) in this diagnostic approach is virtually impossible. Even if this is done in an approximate way, it is difficult to maintain a balance state consistent with the model equations, i.e. maintain the data point on the slow manifold of Fig.3.

(c)The balanced height field is obtained from the solution of second order partial differential equation, whereas it occurs as a first order differential term in the model equations. This creates inconsistencies between the balanced field and the model solutions.

(B)Dynamic initialization (DI method) : In this method the model equations were themselves used to generate the balanced field. The height field, derived from the balance equation, and the observed wind field were used as initial data to integrate the model 'N' time steps forward. The model was then integrated 'N' time-steps backward from the initial time. The Euler time differencing scheme was used in the integration process, which has the property of damping high frequency waves. If u_N represents the value of u - component of wind at the Nth time step and u_0 its value at the initial time, then we have

$$u_N = u_0 \left[1 - (\omega\Delta t)^2 + (\omega\Delta t)^4 \right]^{N/2}$$

Here Δt represent the time-step used and ω' represents the angular frequency of a particular wave component in the u - field. It can be seen that the damping is directly proportional to the frequency of the wave. The amplitudes of the high-frequency waves are heavily damped, whereas the low frequency waves are left relatively unaffected. Each forward backward operation constitute one cycle. During the first ten cycles the height fields at times $t_0 + N\Delta t$ and $t_0 - N\Delta t$ resp., were averaged to give the modified values of the field at time, t_0 . The wind field was restored to its initial value at time t_0. During the next ten cycles the wind fields were averaged while the height fields were restored to its value at the end of the previous ten cycles. The above operations were repeated upto forty cycles. The wind and height fields at the end of the fortieth cycle, were taken as the balanced field and used as input to the model for forward time integration, Sinha and Kulkarni (1982) performed a number of numerical experiments with this method and found that the oscillations of the area ~~and~~ mean height tendency was considerably reduced in amplitude, as compared with those with the static initialization scheme. Fig. 4a and Fig. 4b, show, respectively, the height tendency plotted against time, in the case of time integration performed with initial data obtained with static and dynamic initialization methods. The lesser amplitude in the latter case suggests a reduced presence of gravity waves. The area of integration of the model extended from latitudes 5^0N to 35^0N and from longitudes 95^0E to 105^0E. In these experiments it was seen that the boundary conditions played an important role in the adjustment process. By putting $v = 0$ in the eastern and western boundaries, a fictitious anticyclone developed in the 24 hr. forecast field, due to the reflection of the gravity waves at the boundary, which inhibits the mutual adjustment process. It was also argued that the unbalanced part of the energy propagates as inertia gravity waves and should be allowed to propagate freely out of the region of integration.

Theoretically, one can include physical processes in this method of initialization, but such an inclusion led to practical difficulties because of the computational irreversibility of these processes. For example, if the frictional term is included, then during the forward integration it dissipates energy, but during the backward integration it produces amplification effect. Thus, small discripancies of the solutions between the forward and the backward processes can be amplified to a large difference.

The (DI) method without incorporating simulated effects, was successfully applied on the shallow water equations and the model was integrated upto 48 hours. The area-mean mass divergence, $\left(\frac{\partial \bar{\phi}}{\partial t}\right)$ was computed at hourly intervals and plotted as a function of time. It was seen that the fluctuations of this term was much less as compared to the fluctuations obtained by using only the static initialization method. The area of integration of the model extended from latitudes 5^{0}N to 35^{0}N and from longitudes 95^{0}E to 105^{0}E. In this experiment it was seen that the boundary conditions played an important role in the adjustment process. By putting $\vartheta = 0$ on the eastern and western boundaries, a fictitious anticyclone developed in the 24 hr. forecast field, due to the reflection of the gravity waves at the boundary which inhibits the mutual adjustment process. It was also argued that the unbalanced part of the energy propagates as intertia-gravity waves and should be allowed to propagate freely out of the region of integration.

The main drawbacks in this method are as follows :

(a) It has a slow rate of convergence towards the balanced state
(b) It was pointed out by Okland that space differences in a grid representation, greatly reduce the group velocity of the shorter waves, leading to a slower rate of convergence to a balanced state. Also damping of meteorological waves become inevitable, causing weak atmospheric systems to loose their identity.
(c) It does not distinguish between the type of waves, but only their frequencies. Thus gravity waves of large horizontal extent may sometimes remain almost unaffected in this initialization process.

4. NORMAL MODES INITIALIZATION

The set of equation 2.8-2.10 admits three linearly independent solutions for each wave vector $\vec{k}$. These independent solutions are called "normal modes" of the set of equations.

The modes are as follows :

1. Geostrophically balanced mode which turns out to be stationary in this simple model.

2. Two high frequency modes corresponding to propagating gravity waves along the forward and backward directions respectively. One can also view each normal mode as an eigenvector χ_i corresponding to each of the three eigenvalues :

$$\lambda_1 = 0 \quad ; \quad \lambda_2 = \sqrt{gHk^2 + f^2} \quad ; \quad \lambda_3 = -\lambda_2$$

In normal mode initialization, the initial data are projected into the normal modes, the coefficients of the unwanted modes are set to zero and the balanced initial fields are reconstituted from the remaining modes. If $\vec{Z}(t)$ represent the initial data, it can be expanded in the form

$$\vec{Z}(t) = \sum_j \vec{Z}_{k_j}(t) \exp(i k_j x)$$

where $\vec{Z}_{kj}(t)$ is the normal mode corresponding to wavenumber k_j Such an expansion of grid point data into the normal modes of the model allows filtering in a more selective and rational fashion. In the limit of infinitesimal grid interval, the expansion of initial data is given by the Hough functions of Laplace's tidal theory. In case of finite grid interval it is necessary to consider not only modes related to the Hough modes but also computational modes specific to the finite difference equations employed. The number of normal modes for zonal wave number is equal to the number of grid points. It is known that the fewer grid points contained in a wavelength, the less accurately the wave can be described. Thus, when the normal mode index is equal to or more than half the number of grid points, this mode cannot be represented properly. In multi-level models, the number of vertical modes is equal to the number of levels used in the models. In applying the normal mode initialization to a multi-level P.E. model, the model is first linearized about a basic state which is at rest, and the vertical dependence is separated from the horizontal, through the definition of appropriate structure functions. This precludes the inclusion of mountains, because the pressure gradient in the vicinity of the mountain would not be consistent with a basic state at rest.

Normal mode initialization are of two kinds, viz. :

(a) Linear normal mode initialization, in which the observational data are expanded in terms of the complete set of normal modes and then the expansion coefficients of unwanted computational and 'gravity' modes are set to zero. This method reduces the amplitude of the high frequency oscillation during the initial stages of time integration of the model, but they are regenerated later due to the non-linear interactions.

(b) Non-linear normal mode initialization, in which the time derivatives of the gravity mode coefficients are set equal to zero, while the gravity mode coefficients are modified in such a way that the linear contribution to the tendency of each coefficient compensate the contribution from the non-linear interactions between all the modes. This can be explained as follows :

Referring to the conceptual representation of Leith, we now project the equations of the model on to its normal modes. We obtain a set of ordinary differential equations which can be written symbolically as :

$$\dot{Z} = -i\lambda_z Z + N_z(Z, Y) \quad \cdots\cdots 4.1$$

$$\dot{Y} = -i\lambda_y Y + N_Y(Z, Y) \quad \cdots\cdots 4.2$$

where Z, Y are the column vectors of gravity mode and Rossby mode expansion coefficient, respectively; λ_z, λ_Y are diagonal matrices whose elements are the individual eigenfrequencies of the normal modes; N_z, N_Y are the projections of the non-linear and forcing terms of the model on the sets of normal modes Z and Y respectively. The terms $-i\lambda_z Z$ and $-i\lambda_y Y$ come from the linear terms of the model equations, which appear in this diagonalised form because the normal modes are eigenfunctions of the linearised equations. We follow the following steps, viz.

Step 1 : We apply linear initialization. The subscript indicates the iteration step

$$Z_0 = 0 \quad ; \quad Y_0 = Y$$

Step 2 : Put $\dot{Z} = 0$ in Eqn.(4.1), to get

$$Z_1 = N_z(Z, Y)/i\lambda_z$$

Step 3 : Repeat step 2 but use Z_1 instead of Z_0

$$Z_2 = N_z(Z, Y)/i\lambda_z$$

This step is repeated until convergence is obtained when $\dot{Z}$ on the L.H.S. is equal to 'Z' on the R.H.S. within tolerable limits. We shall denote this value Z_B Thus, finally we have

$$Z_B = N_z(Z_B, Y)/i\lambda_z \quad \ldots \; 4.3$$

For a f-plane model, Leith has identified Z_B as the low frequency ageostrophic flow.

The task of obtaining the normal modes is quite simple for a global or hemispherical model in the spectral forms, but creats problems for a limited area model, where some additional boundary mode coefficients have to be computed. These are derived from the boundary values. Otherwise periodic boundary conditions has to be used or the boundary is taken as a solid wall.

5. BOUNDED DERIVATIVE METHOD

Kreiss (1979,1980) developed a general theory for filtering motions of unwanted time scales from problems involving motions of multiple time scales, as in the hyperbolic system of partial differential equations of atmospheric motion. This is done by first scaling the parameters in the equations by their respective characteristic scales. The terms that contribute to large time derivatives are then identified and constrained to ensure that the time derivatives are of the order of the slow time-scales. In the case of atmospheric motions the slow waves have time derivatives which are of order (1) times their space derivatives while the fast waves have time derivatives of order ($1/R_0$) times their space derivatives, where R_0 , is the Rossby number. The time derivatives of the fast waves are constrained to be of order 1. The

The larger number of higher order time derivatives for which this condition is satisfied, the smoother is the time evolution of the solution. It was also shown by Kreiss that if this condition is satisfied at the initial time, then it continues to be satisfied for all times.

Browning, Kasahara and Kreiss(1980) applied this method to the shallow water equations, which included orography. The appropriate equations are as follows :

$$\begin{aligned} &u_t + uu_x + vu_y + gh_x - fv = 0 \\ &v_t + uv_x + vu_y + gh_y + fu = 0 \qquad \ldots \quad 5.1 \\ &h_t + (uh)_x + (vh)_y + h_0\,(u_x + v_y) - (uH)_x - (vH)_y = 0 \end{aligned}$$

where u, v are the horizontal components of velocity field, h_0 is the mean height of the homogeneous fluid above sea level, h is the deviation of the free surface height from h_0, $H(x,y)$ is the elevation of orography, g is the acceleration due to gravity and f is the coriolis parameter which is assumed to vary linearly with y.

After scaling the parameters with their characteristic scales, Eq. (5.1)becomes :

$$\begin{aligned} &u_t + uu_x + vu_y + \varepsilon^{-1}[\phi_x - fv] = 0 \\ &v_t + uv_x + vv_y + \varepsilon^{-1}[\phi_y + fu] = 0 \qquad \ldots \quad 5.2 \\ &\phi_t + (u\phi)_x + (v\phi)_y + \varepsilon^{-2}[1 - \varepsilon\Phi][u_x + v_y] \\ &\qquad - \varepsilon^{-1}[u\Phi_x + v\Phi_y] = 0 \end{aligned}$$

Here $\phi = \frac{h}{D}$ where 'D' is the representative magnitude of the deviation of the free-surface height from its mean and $\Phi = \frac{H}{H_0}$, H_0 being the mean elevation of orography. ε is the Rossby number. Putting $f = f_0 + \varepsilon\beta y$ where $\beta = \frac{\partial f}{\partial y}$ and seperating the terms containing some power of ε as its coefficient, we get

$$\begin{aligned} &u_t + uu_x + vu_y - \beta y v + a = 0 \\ &v_t + uv_x + vv_y + \beta y u + b = 0 \qquad \ldots \quad 5.3 \\ &\phi_t + u\phi_x + v\phi_y + \phi(u_x + v_y) + c = 0 \end{aligned}$$

we assume that a,b,c are smooth functions of x and y. The first order time derivatives u_t, v_t, h_t are $O(1)$, if and only if, the following conditions are satisfied.

$$\begin{aligned} \phi_x - f_0 v &= \varepsilon a \\ \phi_y + f_0 u &= \varepsilon b \qquad \ldots 5.4 \\ u_x + v_y &= \frac{\varepsilon}{1-\varepsilon\Phi}\left(u\Phi_x + v\Phi_y\right) + \frac{\varepsilon^2}{1-\varepsilon\Phi} c \end{aligned}$$

Put in a matrix form we get :

$$\begin{bmatrix} 0 & -f_0 & \frac{\partial}{\partial x} \\ f_0 & 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ \phi \end{bmatrix} = \varepsilon \begin{bmatrix} a \\ b \\ s^{-1}\left[u\Phi_x + v\Phi_y + \varepsilon c\right] \end{bmatrix}$$

where $s = (1-\varepsilon\Phi)$; $\delta = u_x + v_y$

The operator on the L.H.S. of Eqn.5.5 is a singular operator and so Eq.(5.5) has a solution if and only if.

$$-a_y + b_x = f_0 s^{-1}\left[u\Phi_x + v\Phi_y + \varepsilon c\right] \qquad 5.6$$

This is obtained by differentiating a and b with respect to y and x resp., and substituting for (u_x, v_y) in the equation for c. From Eq.5.3 we see that u_{tt}, v_{tt}, ϕ_{tt} ~~and~~ are of order unity, if and only if, a_t, b_t, and c_t are of order unity.

Considering the derivative c_t, we have

$$\varepsilon^2 c_t = s\delta_t - \varepsilon\left[\Phi_x u_t + \Phi_y v_t\right] \qquad \ldots 5.7$$

From the first two equations of Eq.5.3, we have

$$\begin{aligned} \delta_t = &-\nabla^2\phi + \beta y \zeta - \beta u + 2\left[u_x v_y - u_y v_x\right] \\ &- u\delta_x - v\delta_y + \delta^2 \end{aligned}$$

where $\zeta = v_x - u_y$

Substituting in Eq.5.7 we get :

$$\begin{aligned} -c_t = \varepsilon^{-2}&\left[\nabla^2\phi - \beta y\zeta + \beta u - 2\{u_x v_y - u_y v_x\} + u\delta_x \right. \\ &\left. + v\delta_y - \delta^2\right] - \varepsilon^{-3}\left[u_t\Phi_x + v_t\Phi_y\right] \end{aligned}$$

or

$$-\varepsilon^3 C_t = \varepsilon\left[\nabla^2\phi - \beta y \zeta + \beta u - 2\{u_x v_y - u_y v_x\} + u\delta_x + v\delta_y + \delta^2\right] - \left[u_t \Phi_x + v_t \Phi_y\right] \qquad \ldots 5.8$$

In an equatorial β -plane, $f_o = 0$ and D = 10. Therefore the divergence is one order of magnitude less i.e.

$$[\delta_t]_{Eq} \equiv \varepsilon\,[\delta_t]_{mid\text{-}latitude}$$

In this case the R.H.S. of Eq. 5.8 does not contain any terms of order ε . Now, neglecting terms of order ε^3 and less we have the final constraint for computing ϕ in terms of the wind component. It may be seen that the R.H.S. is a general elliptic equation whose solution at each grid point would yield the balanced mass and wind field. This solution when used as input to the model, is expected to maintain the solution in the slow time scale, i.e. the path of the model states would be along the slow manifold 'M' of Fig.3. Since there are two gravity modes to control, it is desirable to apply two constraints for proper initialization.

Kasahara(1982) applied this technique to a multi-level primitive equation model for the tropical region, in which friction and heating terms were also incorporated. The geopotential field was expressed as a sum of the hydrostatic and non-hydrostatic part. Defining the magnitude of the frictional terms as $|F| = \frac{C_D V_s^2}{d}$ where C_D is the drag coefficient whose value is 10^{-3}, 'd' is the thickness of the boundary layer, and V_s is the surface wind speed and diabatic heating term by $Q = \frac{C_p \Delta T}{\Delta t}$ where ΔT is change in temperature during a time interval Δt . He scaled these terms so that they were both of order unity. Applying the bounded derivative principle, he found that the vertical velocity field required to balance the given heating rate initially is given by

$$\omega = -\varepsilon\, k\, \frac{Q}{S\,p} \qquad 5.9$$

where $k = \frac{R}{C_p}$; $S = \frac{d\phi_s}{dp}\frac{d}{dp}(\ln \theta_s)$

where ϕ_s represents the dimensionless geopotential of the standard atmosphere, whose potential temperature distribution is θ_s. By solving a similar set of constraints as in the 2-D case, he obtained two elliptic equations, one for ϕ and the other for ω, which were solved simultaneously to yield the desired balanced initial state. The only information that is taken from the initial data are the rotational part of the velocity, Q and $\frac{\partial Q}{\partial t}$. Then the initial ω-field is computed from Eq.(5.9) and a similar equation for divergence, yields the divergence field.

6. COMPARISON OF THE B.D. AND N.M. METHODS

The basic requirements of any good initialization scheme are the following :

(a)It should have the ability to suppreses high frequency gravity waves.

(b)It should be able to provide consistent information to make up for deficiencies in the observation network, e.g. the divergent wind field or vertical motion field should be faithfully reproduced, consistent with synoptic theory. The vertical motion field should also evolve slowly and smoothly in time.

(c)The computation time should be small as compared to the time for integrating the model over the required time interval.

Judged from the viewpoint of these requirement, we now discuss the advantages and disadvantages of both the schemes.

The non-linear N.M. method was found to be very successful in suppressing high frequency waves, when applied to a model of the complexity of the ECMWF model. It was successful in generating upward motion ahead of the warm and cold fronts and downard motion behind the cold front, consistent with synoptic theory. It was also capable of generating cross-isobar flow in the model boundary layer and realistic mountain induced vertical motion.

However, when the unconstrained Macheuhauer scheme was applied to the ECMWF model, the scheme did not converge when convective heating and large scale condensation were included in the non-linear forcing. To avoid non-convergence, most of the model

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physics was not included, but this resulted in the suppression of the Hadley cell and the divergence in the tropics by a large factor. Puri and Bourke(1982) proposed a modified scheme of non-linear normal mode initialization by excluding these modes from the initialization process, which are significantly affected by convective processes. This scheme was able to maintain the strength of the Hadley circulation. Kitade ~~Krishnamurthy~~(1982) developed a non-linear NMI scheme, which is a modified version of Machenhauer's scheme and included complete physics of the model in the initialization process. He performed iteration with Eq.4.1 by putting 'Z' not equal to zero, but to a small value, which reduce by half at each iteration, i.e.

$$Z^{n} = \frac{1}{2}\left[\frac{N_z(Z,Y)}{i\lambda_z} + Z^{n-1}\right]$$

where 'n' denotes the iteration step. This method was found to preserve the divergence field better than in the adiabatic version. Another drawback in this method was that the separation in frequency between Rossby and gravity modes of the same equivalent depth, is small in the tropics, making identification difficult.

Semazzy and Navan(1986) made a comparative study of the two methods, using real time data as input to a barotropic model. They chose the Asiatic area bounded by latitudes 2°S and 62°N and longitudes 60°E and 120°E. This region contains an intense high pressure area centered around 30°N, 90°E, known as the "China high". This high was ascribed to be the result of computational problems arising out of the interpolation of height data between sigma and pressure coordinates. This high was found to be absent in the field initialized by BDI method, because it makes use of vorticity to construct the initial velocity field for starting the BDI iterations, and the vorticity value is kept unchanged thereafter. The same features were also noted in the NMI method. However, Daley(1978) suggested a constrained variational approach of the NMI method, by assigning different weights to the height field data, in the vicinity of the "high". A similar method has not been successfully used in the BDI. The two methods gave very similar results in the case of a barotropic model.

Bijlsma and Hafkenscheid (1986) made a comparison of the two methods using the ECMWF, baroclinic grid point model employed by Temperton and Williamson (1979). This is a limited area model extending from latitude 20^0N to 40^0N and longitude 20^0W to 20^0E. They also found that both the methods are virtually identical in their effect upon the initial fields, but the BDI method required less computation time.

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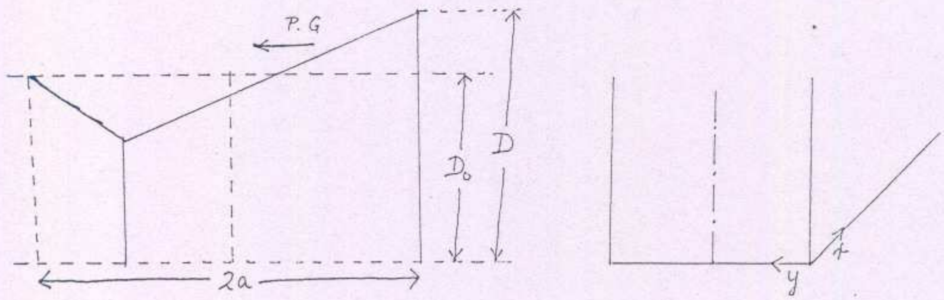

FIGURE 1

Plan of current system of the fluid to demonstrate inertia-gravity oscillations

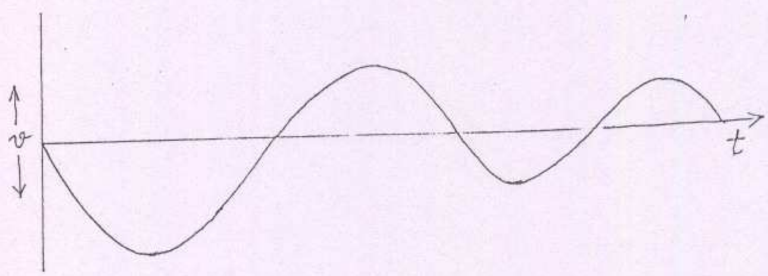

FIGURE 2

Schematic plot of the v-component of fluid motion vs. time

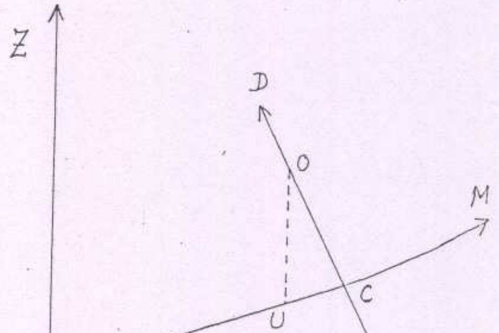

FIGURE 3

Leith's slow-manifold concept

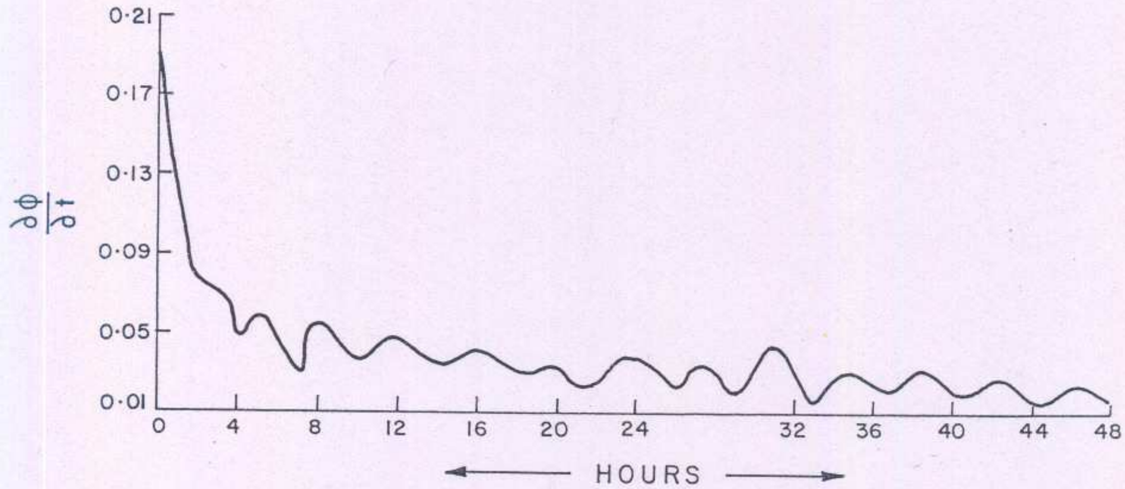
0·21
0·17
0·13
0·09
0·05
0·01
$\frac{\partial\phi}{\partial t}$
0
4
8
12
16
20
24
32
36
40
44
48
HOURS

FIG. 4(a)

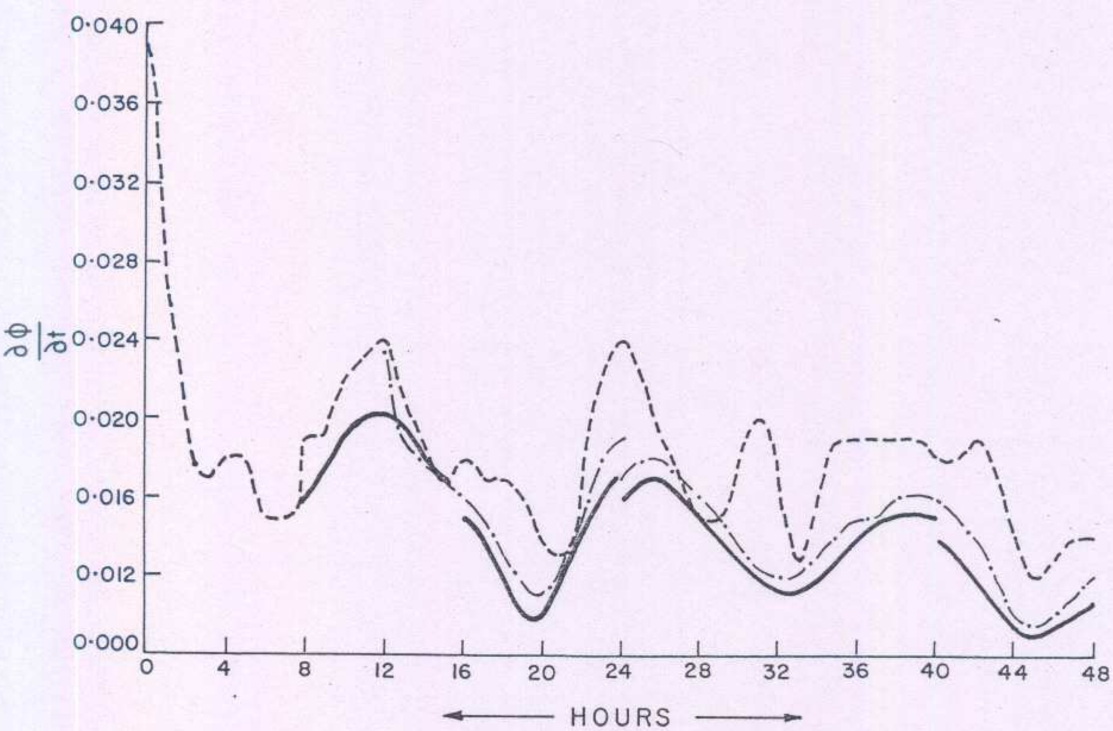
0·040
0·036
0·032
0·028
0·024
0·020
0·016
0·012
0·000
$\frac{\partial\phi}{\partial t}$
0
4
8
12
16
20
24
28
32
36
40
44
48
HOURS

FIG. 4(b)