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SOME ASPECTS OF SOLAR RADIATION

by

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Abstract.

Mathematical analysis involved in the computations of solar zenith angle and solar declination is presented in this study.These angles are used in almost every solar radiation related study.Certain inferences are drawn on the basis of these computations regarding the variability of solar zenith angle, solar declination, sun rise time, sun set time, duration of sun shine, latitude of polar night, hour angle, heliocentric distance and total solar radiation recieved at the top of the atmosphere. Variation of heliocentric distance is identified to be responsible for the asymmetry in the latitudinal-julian day distribution of total daily solar radiation at the top of the atmosphere.

Key Words : Heliocentric, Solar Declination, Zenith Angle, Eccentricity, Orbit, Radiation, Solar, Julian Day, Solastice, Equinox.

1. Introduction.

In almost all solar radiation related studies,one quite frequently comes across certain mathematical relations involving some solar angles related to the motion of the earth around the sun.It is found that in most of the works generally a reference is made to these formulas which are quite significant.An attempt is made to give a detailed mathematical analysis involved in the derivation of these relations and relevent implications are drawn from this analysis at the appropriate stages in this text.We feel that this sort of analysis is sometimes helpful in better understanding of certain basic features involved in solar radiative studies. In this study we have considered local time in all computations.

2. Solar Zenith Angle.

Solar zenith angle is normally computed from other angles that are known.In Fig.1 let P be the point of observation and OZ the zenith through this point.Assume that the sun is in the direction OS and let D be the point of earth's surface directly under the sun.Plane of OZ and OS will intersect the surface of the earth in a great circle.PD being the arc of this great circle is a geodesic.The angle ZOS,measured by the geodesic PD, is equal to sun's zenith distance θ_0.In the spherical triangle NPD, arc ND is equal to 90° minus the solar declination , δ, which is the angular distance of the sun north (positive) or south (negative) of the equator.Arc NP is equal to 90° minus the latitude , λ, of the point of observation.Angle ,h, is the hour angle i.e. angle through which the earth most (turn) rotate to bring the meridian of the point of observation ,P, directly under the meridian of the sun.The solar declination , δ,is a function of the day of the year only and is independent of the location of the point of observation.It , δ, varies from $+23^\circ\ 27'$ on 21^{st} June to $-23^\circ\ 27'$ on 22^{nd} December and is zero on equinoxes i.e. 21^{st} March and 23^{rd} September. Hour angle ,h, is a function of the time of the day and is zero at solar noon and increases (decreases) by 15° for every hour before (after) solar noon.Fig.2 is magnified version of the spherical geometry of Fig.1. In Fig.2 NA is tangent to the arc NP and NB is tangent to the arc ND.Fig.2 shows various angles of Fig.1. From Fig.2 we get

$$NA = NO \tan(\theta') \qquad (2.1.A)$$

$$NB = NO \tan(\theta) \qquad (2.1.B)$$

$$OA = NO \sec(\theta') \qquad (2.1.C)$$

$$OB = NO \sec(\theta) \qquad (2.1.D)$$

From triangle NAB we get :-

$$(AB)^2 = (AN)^2 + (NB)^2 - 2\, AN.NB \cos(h) \qquad (2.2)$$

From triangle OAB we get :-

$$(AB)^2 = (OA)^2 + (OB)^2 - 2\, OA.OB \cos(\theta_o) \tag{2.3}$$

From Eqs.(2.2),(2.3) we get :-

$$(AN)^2 + (NB)^2 - 2\, AN.NB \cos(h) = (OA)^2 + (OB)^2 - 2\, OA.OB \cos(\theta_o) \tag{2.4}$$

Subsituting Eq.(2.1) in Eq.(2.4) we get :-

$$(NO)^2 [\tan^2(\theta') + \tan^2(\theta) - 2\tan(\theta')\tan(\theta)\cos(h)] =$$

$$(NO)^2 [\sec^2(\theta') + \sec^2(\theta) - 2\sec(\theta')\sec(\theta)\cos(\theta_o)]$$

From this equation we get the following relation.

$$2 + 2\tan(\theta)\tan(\theta')\cos(h) = 2\sec(\theta)\sec(\theta')\cos(\theta_o)$$

This equation on simplification yields the following expression.

$$\cos(\theta_o) = \cos(\theta)\cos(\theta') + \sin(\theta')\sin(\theta)\cos(h) \tag{2.5}$$

From Fig.2 we find that

$$\theta = \pi/2 - \delta \tag{2.6.A}$$

$$\theta' = \pi/2 - \lambda \tag{2.6.B}$$

On using Eqs.(2.6.A).(2.6.B) in Eq.(2.5) we finally get the following relation.

$$\cos(\theta_o) = \sin(\lambda)\sin(\delta) + \cos(\lambda)\cos(\delta)\cos(h) \tag{2.7}$$

This equation gives solar zenith , θ_o, in terms of the angles δ, λ and h which are known.As $\cos(-h) = \cos(h)$,so solar zenith angle at a particular latitude on any day of the year is symmetric with respect to the noon. From Eq.(2.7) we find the solar zenith angle at any latitude at solar noon ,i.e. Cos(h) = 1 , as [λ - δ].Now, as at the time of sunrise or sunset at any latitude (except poles) Cos(θ_o) = 0 and h = H = half-day duration , so we get h = H = half-day duration from the following relation.

$$\mathrm{Cos}(H) = -\tan(\lambda)\tan(\delta) \qquad (2.8)$$

The half-day duration will be six hours if either tan(λ) = 0 i.e. equator on all days or tan(δ) = 0 i.e. equinoxes at all latitudes except the poles.The latitude of polar night is obtained by substituting H = 0 in the Eq.(2.8) and is given by the following relation

$$\mathrm{Lat} = 90^{\circ} - |\delta| \qquad (2.9)$$

3. Solar Declination.

The term **ANOMALY** is very frequently used in the computation of the solar declination. It is defined as "The angular distance describing the position of an orbiting body such as a planet".Anomaly is of three types viz. true anomaly , mean anomaly and eccentric anomaly.The apparent orbital motion of the sun about the earth is shown by the ellipse ADBFA in Fig.3 with earth located at the focus S.The circle AD'BGA represents the orbit of a fictitious sun which revolves with constant speed and a one year period.Let the sun and the fictitious sun be at perigee A at time t_o and at positions P and S' after some time t, respectively.Let "a" be the semi major axis , "b" the semi minor axis, e the eccentricity of the elliptical orbit and T the orbital period.Let the radius vector SP make an angle v with SA. Angle PSA , i.e. v, is called true anomaly.Distance SP i.e. the distance of the planet from the sun is called heliocentric distance. Let PR ,the perpendicular from P on CA, meet the circular orbit at Q.Angle QCA , i.e. E , is called eccentric anomaly.Let H be the point , in the plane of the ellipse, of intersection of the equator and the ecliptic i.e. where sun ascends north of the equator and it occurs at vernal equinox.Angle PSH i.e. θ , is the ecliptic longitude of the sun. Let ω be the value of θ when the sun is at perigee. Angles v, ω and θ are related by the following relation.

$$v = \theta - \omega \tag{3.1}$$

From a property of an ellipse we get :-

$$PR : QR = b : a \tag{3.2}$$

From Fig.3 we get

$$PR = r \, \text{Sin}(v) \tag{3.3}$$

$$\text{and} \quad QR = a \, \text{Sin}(E) \tag{3.4}$$

Subsitution of Eqs.(3.3),(3.4) in Eq.(3.2) gives

$$r \, \text{Sin}(v) : a \, \text{Sin}(E) = b : a \tag{3.5}$$

$$\text{i.e.} \quad r \, \text{Sin}(v) = b \, \text{Sin}(E)$$

From Fig.3 we further find that

$$SR = r \, \text{Cos}(v) = CR - CS = a \, \text{Cos}(E) - CS$$

$$= a \, \text{Cos}(E) - ae = a \, [\text{Cos}(E) - e]$$

$$\text{i.e.} \quad r \, \text{Cos}(v) = a \, [\text{Cos}(E) - e] \tag{3.6}$$

From Eqs.(3.5) and (3.6) we get

$$r^2 = b^2 \, \text{Sin}^2(E) + a^2 \, [\text{Cos}(E) - e]^2$$

On using the relation $b^2 = a^2(1 - e^2)$ in the above equation we get

$$r = a \, [1 - e \, \text{Cos}(E)] \tag{3.7}$$

From the trignometry we have the following relation

$$2r\ \mathrm{Sin}^2(v/2) = r[1 - \mathrm{Cos}(v)]$$

On using Eq.(3.6) and (3.7) the above relation becomes.

$$2r\ \mathrm{Sin}^2(v/2) = a(1 + e)[1 - \mathrm{Cos}(E)] \tag{3.8}$$

.Similarly we get

$$2r\ \mathrm{Cos}^2(v/2) = a(1 - e)[1 + \mathrm{Cos}(E)] \tag{3.9}$$

From Eqn.(3.8) and (3.9) we get

$$\tan(v/2) = (1 + e)^{1/2}\ (1 - e)^{-1/2}\ \tan(E/2) \tag{3.10}$$

We now introduce an angle , ϕ , lying between 0 and $\pi/2$ and defined as under.

$$\mathrm{Sin}(\phi) = e \tag{3.10.A}$$

On substituting Eq.(3.10.A) in Eq.(3.10) we get

$$\tan(v/2) = [1 + \tan(\phi/2)]\ [1 - \tan(\phi/2)]^{-1}\ \tan(E/2) \tag{3.11}$$

From the following identities

$$2\ \mathrm{Cos}(A) = e^{iA} + e^{-iA}$$

and $$2i\ \mathrm{Sin}(A) = e^{iA} - e^{-iA}$$

we get the following

$$\tan(v/2) = i(e^{iv} - 1)/(e^{iv} + 1) \tag{3.11.A}$$

$$\tan(E/2) = i(e^{iE} - 1)/(e^{iE} + 1) \tag{3.11.B}$$

Using the following transformation

$$x = \tan(\phi/2) \qquad (3.12)$$

and Eq.(3.11.A) and (3.11.B) ,Eq.(3.11) gives

$$e^{iv} = e^{iE} [1 - xe^{-iE}] [1 - xe^{iE}]^{-1} \qquad (3.13)$$

On using the following logarithmic series expension

$$\log(Z) = Z - Z^2/2 + Z^3/3 - \ldots$$

we get the following expression

$$v = E + 2 [x \operatorname{Sin}(E) + x^2/2 \operatorname{Sin}(2E) + x^3/3 \operatorname{Sin}(3E) + \ldots] \qquad (3.14)$$

Now from the transformation $x = \tan(\phi/2)$ we get

$$x = [1 - \operatorname{Cos}(\phi)] e^{-1} = [1 - (1 - e^2)^{0.5}] e^{-1} \qquad (3.14.A)$$

On using the following series expansion

$$(1 + x)^{1/2} = 1 + x/2 - x^2/8 + x^3/16 - x^4/128 + \ldots$$

we get the following expressions for x, x^2 and x^3.

$$x = e/2 + e^3/8 + e^5/16 + \ldots$$

$$x^2 = e^2/4 + e^4/18 + 5e^6/64 + e^8/614 + \ldots$$

$$x^3 = e^3/8 + 3e^5/32 + \ldots$$

Subsitution of these series expansions of different powers of x in Eq.(3.14) we get

$$v = E + (e + e^3/4 + e^5/16 + \ldots)\mathrm{Sin}(E)$$

$$+ (e^2/2 + e^4/8 + 5e^6/64 + \ldots)\mathrm{Sin}(2E)$$

$$+ (e^3/12 + e^5/16 + \ldots)\mathrm{Sin}(3E)$$

$$+ \ldots. \qquad (3.15)$$

This equation gives, true anomaly ,v, in terms of the eccentric anomaly ,E.For getting an expression of solar declination we need a relation connecting true anomaly and mean anomaly.This can be achieved by ,now, finding a relation between the eccentric anomaly and the mean anomaly.For getting such an expression we refer to the spherical geometry of Fig.3.

Kepler's second law of planetary motion states that "the aerial velocity is constant".

i.e. $$\text{Area SPA : Area of ellipse} = t - t_o : T \qquad (3.16)$$

and area of an ellipse is (πab)

$$\text{Area SPA} = \pi ab(t - t_o)/T \qquad (3.17)$$

On introducing mean angular motion $n = 2\pi/T$ we further get

$$\text{Area SPA} = nab(t - t_o)/2 \qquad (3.18)$$

We define mean anomaly "M" as

$$M = n(t - t_0) \qquad (3.19)$$

i.e. $$M = 2\pi(t - t_o)/T \qquad (3.20)$$

Substituting Eq.(3.20) in Eq.(3.17) we get

$$\text{Area SPA} = Mab/2 \qquad (3.21)$$

Now the area SPA is expressed in terms of the eccentric anomaly,E, in the following way.

$$\begin{aligned}
\text{Area SPA} &= \text{Area SPR} + \text{Area PRA} \\
&= 1/2\ SR.PR + \text{Area PRA} \\
&= 1/2\ [CR - CS]PR + \text{Area PRA} \\
&= 1/2\ [a\ Cos(E) - CS].PR + \text{Area PRA} \\
&= a/2\ [Cos(E) - e].PR + \text{Area PRA} \\
&= ab/2\ [Cos(E) - e]Sin(E) + \text{Area PRA} \qquad (3.22)
\end{aligned}$$

For computing area of the figure PRA we consider a small strip perpendicular to AB. Area PRA is thus considered as a sum total of areas of all such small strips.Let the strip VU meet the circle at W.

$$VU : WU = b : a \qquad (3.23)$$

i.e. $VU = (b/a)WU$

As sum of all such strips is equal to the area PRA , area thus becomes equal to (b/a) times the area of sector QRA. We thus get

$$\text{Area PRA} = (b/a)\ \text{Area of sector QRA} \qquad (3.23.A)$$

$$\text{Area of sector QRA} = \text{Area of sector CQA} - \text{area of triangle CQR} \qquad \ldots\ (3.23.B)$$

Since angle QCA is E, so the area of the sector CQA is $a^2E/2$. So

$$\begin{aligned}
\text{Area of triangle CQR} &= 1/2\ CR.QR \\
&= ab/2\ [E - Sin(E)Cos(E)] \qquad (3.23.C)
\end{aligned}$$

On using Eqs.(3.23.A),(3.23.B),(3.23.C), the Eq.(3.22) now reduces to the following form.

$$\text{Area SPA} = ab/2\ [E - e\ \mathrm{Sin}(E)] \tag{3.24}$$

From Eqs.(3.21) and (3.24) we get

$$M = E - e\ \mathrm{Sin}(E) \tag{3.25}$$

This equation is called "Kepler's Equation". It is a relation between eccentric anomaly ,E, and mean anomaly M.From Eq.(3.25) we also get

$$E = M + e\ \mathrm{Sin}(E) \tag{3.26}$$

Using Eq.(3.26) in Eq.(3.15) we get true anomaly ,v, in terms of the mean anomaly as under.

$$E = M + e\ \mathrm{Sin}(E) \qquad \text{from Eq.(3.26)}$$

As e is very small so to a first order aproximation

$$E = E_1 \tag{3.26.A}$$

Second order approximation of E i.e. a more accurate value is E_2 and

$$E_2 = M + e\ \mathrm{Sin}(E_1)$$

$$E_3 = M + e\ \mathrm{Sin}(E_2)$$

i.e. $$E_3 = M + e\ \mathrm{Sin}\ [M + e\ \mathrm{Sin}(E_1)]$$

$$= M + e\ \mathrm{Sin}(M) + e^2/2\ \mathrm{Sin}(2M)$$

$$E_4 = M + e\ \mathrm{Sin}(E_3)$$

$$= M + \mathrm{Sin}\ [M + e\ \mathrm{Sin}(M) + e^2/2\ \mathrm{Sin}(2M)]$$

$$E = E_4 = M + (e - e^3/8)\ \mathrm{Sin}(M) + e^2/2\ \mathrm{Sin}(2M) + 3e^3/8\ \mathrm{Sin}(3M) \qquad (3.27)$$

$$\text{i.e. } \mathrm{Sin}(E) = \mathrm{Sin}\ [M + (e - e^3/8)\mathrm{Sin}(M) + e^2/2\ \mathrm{Sin}(2M) + 2e^3/8\ \mathrm{Sin}(3M)] \qquad (3.27.A)$$

For simplicity in mathematical analysis we retain terms upto the order of e^2 only.Under this resriction Eq.(3.27.A) reduces to the following form.

$$\mathrm{Sin}(E) = \mathrm{Sin}\ [M + e\ \mathrm{Sin}(M) + e^2/2\ \mathrm{Sin}(2M)] \qquad (3.27.B)$$

Using the following series expansions of functions Sin(A) and Cos(A)

$$\mathrm{Sin}(A) = A - A^3/3! + A^5/5! - \ldots$$

$$\mathrm{Cos}(A) = 1 - A^2/2! + A^4/4! - \ldots$$

in Eq.(3.27.B) we further get

$$\mathrm{Sin}(E) = (1 - e^2/8)\ \mathrm{Sin}(M) + e/2\ \mathrm{Sin}(2M) + 3e^2/8\ \mathrm{Sin}(3M) \qquad (3.28)$$

$$\mathrm{Sin}(2E) = \mathrm{Sin}(2M) + e\ [\mathrm{Sin}(3M) - \mathrm{Sin}(M)] \qquad (3.29)$$

$$\mathrm{Sin}(3E) = \mathrm{Sin}(3M) \qquad (3.30)$$

Substituting Eqs.(3.28),(3.29),(3.30) in Eq.(3.16) we get

$$v = M + (2e - e^3/4)\,\mathrm{Sin}(M)$$
$$+ 5e^2/4\,\mathrm{Sin}(2M)$$
$$+ 13e^3/12\,\mathrm{Sin}(3M) + \ldots \qquad (3.31)$$

This formula is called "Equation of the Centre".Its importance lies in the fact that it relates true anomaly ,v, with the mean anomaly ,M, and the eccentricity ,e,. of the elliptical orbit.Following Katayama (1974) , letting ϵ denote the inclination of the earth's orbit, solar declination , δ, is given by the following relation.

$$\delta(t) = \mathrm{Sin}^{-1}[\mathrm{Sin}(\epsilon)\mathrm{Sin}(v + \omega)] \qquad (3.32)$$

where true anomaly v is computed using Eq.(3.31) and ω , the ecliptic longitude of the sun at perigee, is -1.3550737 radians (-77.64^{o}).Solar declination is positive for $0 \leq (v + \omega) \leq \pi$ and negative for $\pi \leq (v + \omega) \leq 2\pi$ .Solar declination is a function of the day of the year.The numerical value of the eccentricity ,e, of the earth's elliptical orbit is 0.01672.

The Solar zenith angle and solar declination thus computed are used in the computations relating to solar radiation. Following Sellers (1965) solar radiation incident at the top of the atmosphere ,q_s, is computed as

$$q_s = S\,[\overline{r}_E/r_E(t)]^2\,\mathrm{Cos}(\theta_o) \qquad (3.33)$$

where S is solar constant (langley minute^{-1}), $r_E(t)$ is the heliocentric distance at time "t" and $\overline{r}_E$ is the mean heliocentric distance.Substitution of Eq.(2.7) in Eq.(3.33) gives the following.

$$q_s = S\,[\overline{r}_E/r_E(t)]^2\,[\mathrm{Sin}(\lambda)\mathrm{Sin}(\delta) + \mathrm{Cos}(\lambda)\mathrm{Cos}(\delta)\mathrm{Cos}(h)] \quad (3.34)$$

The daily total solar radiation ,Q_s, incident on a horizontal surface at the top of the atmosphere is obtained by integerating Eq.(3.34) from sun rise to sun set as

$$Q_s = \int_{-H}^{H} q_s \, dt = \int_{-H}^{H} (q_s/\Omega) dh \qquad (3.35)$$

As $dh/dt = \Omega =$ the angular velocity of the earth $= 2\pi$ radians per day. Integeration of the above equation finally gives the following expression.

$$Q_s = (1440/\pi) \, S \, [\bar{r}_E/r_E(t)]^2 \, [H \, \mathrm{Sin}(\lambda)\mathrm{Sin}(\delta) + \mathrm{Cos}(\lambda)\mathrm{Cos}(\delta)\mathrm{Sin}(H)] \qquad (3.36)$$

where solar constant is in langley minute^{-1} and Q_s is in units of langley day^{-1}. Following Katayama (1974) we get the ratio $[\bar{r}_E/r_E(t)]$ as a series of mean anomaly ,M, as

$$[\bar{r}_E/r_E(t)] = (1 + e^2) - (e - 3e^3/8)\,\mathrm{Cos}(M) - (e^2/2 - e^4/3)\,\mathrm{Cos}(2M) - (3e^3/8 - 135e^5/64)\,\mathrm{Cos}(3M) - \ldots \qquad (3.37)$$

Eq.(3.37) is used while computing Q_s from Eq.(3.36).

4. Results.

Daily latitudinal variations of solar zenith angle are shown in Figs.4(a),(b),(c) at 12 hours, 9 hours and 6 hours ,respectively. Fig.4(a) shows that solar zenith angle is symmetrical with respect to summer and winter solastices i.e. 21^{st} June and 22^{nd} December. Similar symmetry of zenith angle is found from its distribution given in Fig.4(b) at 9 hours. However, from Fig.4(c) it is found that solar zenith angle is 90° at equator on all days of the year. It also shows that zenith angle has (i) latitudinal symmetry with respect to equator on all days of the year and (ii) daily symmetry about 21^{st}

June at all latitudes.

The daily latitudinal variations of the times of sun rise are shown in Fig.5(a).In this figure area covered by the index "U" denotes the points of the latitude - julian day cross section for which the concept of daily sun rise is undefined.Fig.5.(a) shows sun rise time as 6 hours at (i) equator on all days of the year and (ii) both equinoxes ,i.e. vernal and autumnal, at all latitudes.It also shows that time of sun rise is symmetrical about (i) equator on all days of the year and (ii) at all latitudes on both summer and winter solastices i.e. 21^{st} June and 22^{nd} December.It also shows that the time of sun rise increases (decreases) from north pole to south pole during vernal to autumnal (autumnal to vernal) equinox. Fig.5(b) shows daily latitudinal variations of the time of sun set.It gives sun set time as 18 hours at (i) equator on all days of the year and (ii) at all latitudes on both equinoxes. It also shows that the time of sun set is symmetrical about (i) equator throughout the year and (ii) at all latitudes on both summer and winter solastices (21^{st} June and 22^{nd} December). Area covered by index "U" is same as in Fig.5(a). It also shows that the time of sun set decreases (increases) from north pole to south pole during vernal to autumnal (autumnal to vernal) equinox.The daily latitudinal variation of the duration of sun shine is shown in Fig.5(c).It shows full day of 12 hour duration at (i) equator on all days of the year and (ii) both equinoxes at all latitudes.Duration of sun shine (full day) exhibits same latitudinal and daily symmetry as times of sun rise and sun set.It also shows that the duration of sun shine decreases (increases) from north pole to south pole during during vernal to autumnal (autumnal to vernal) equinox.

Profile of the diurnal variation of the hour angle (in degrees) is shown in Fig.6.It is a linear variation at the rate of 15^{o}/hour with 0^{o} at solar noon.Profile of the daily variation of solar declination is shown in Fig.7.It is not a linear variation.It shows symmetry with respect to 21^{st} June.Its value is 0^{o} on both equinoxes.It varies from $+23^{o}\ 27'$ on 21^{st} June to $-23^{o}\ 27'$ on 22^{nd} December.

Profile of the daily variation of the latitude of polar night is shown in Fig.8.It exhibits symmetry with respect to 21^{st} June.It shows polar nights are in polar latitudes on days around equinoxes.Latitude of polar night is minimum on summer and winter solastices i.e. 21^{st} June & 22^{nd} December. Profile of the daily variation of $[\overline{r}_E/r_E(t)]^2$ is shown in Fig.9.It is seen from this figure that this factor never exceeds unity by more than 3.5 percent.It varies from 1.0344 on 3^{rd} January to 0.9671 on 4^{th} july.This figure shows that this parameter is symmetric about 4^{th} july.

Daily variation of total solar radiation received at the top of the atmosphere is shown in Fig.10.This distribution is seen to be symmetric with respect to both summer and winter solastices. A maxima is found in each (northern and southern) hemisphere. Value of the southern hemispheric maxima is more than the value of northern hemispheric maxima. Both hemispheric maxima are found to be located equidistant from the equator. Locations of the two hemispheric maxima are marked as "X" in Fig.10. Areas of Fig.10 marked as "N" mean that these areas are covered by night. The distribution of Q_s is slightly asymmetric and this asymmetry may be attributed to term $[\overline{r}_E/r_E(t)]^2$ (say 'A') of Eq.3.36. From Fig.9 it is found that 'A' increases from its minimum value (4^{th} July) to its maximum value (3^{rd} January) by 6.4% of the minimum value and variation of Q_s from its northern hemispheric maxima to southern hemispheric maxima is 6.1% of the former. In these computations term 'A' is computed every day whereas Q_s is computed at the interval of seven days. Both these quantities viz. 'A' and Q_s attain their maximum values during January. Furthermore sun is located in the southern hemisphere during the month of January as a result of which the southern hemisphere gets more solar insolation.The closeness of these limits of variations suggest that 'A' may be responsible for the asymmetry in Q_s.The date of perigee varies annually fron 2^{nd} January to 5^{th} January. The mean date of perigee ,Katayama (1974), for the years 1950-1972 is 3.36 and accordingly the value of t_o in Eq.(3.20) is taken as 2.36 days. Results obtained in this analysis are found to be consistent with List (1963).

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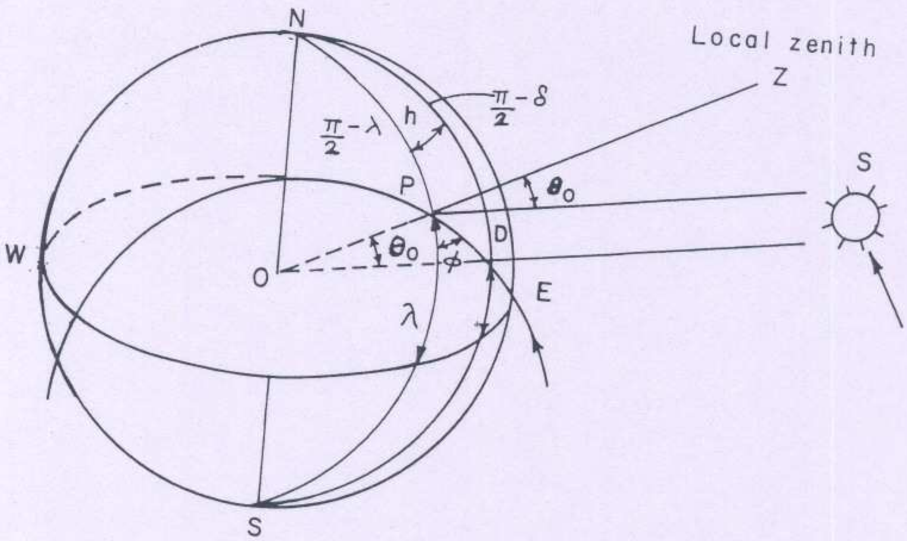

Fig.1 Spherical geometry for computing solar zenith angle.

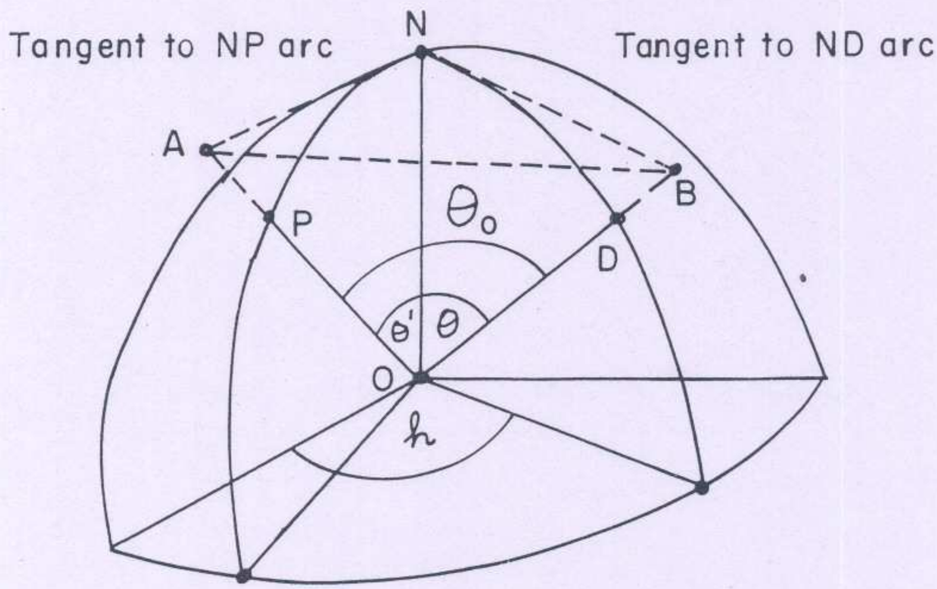

Fig.2 A part of Fig.1.

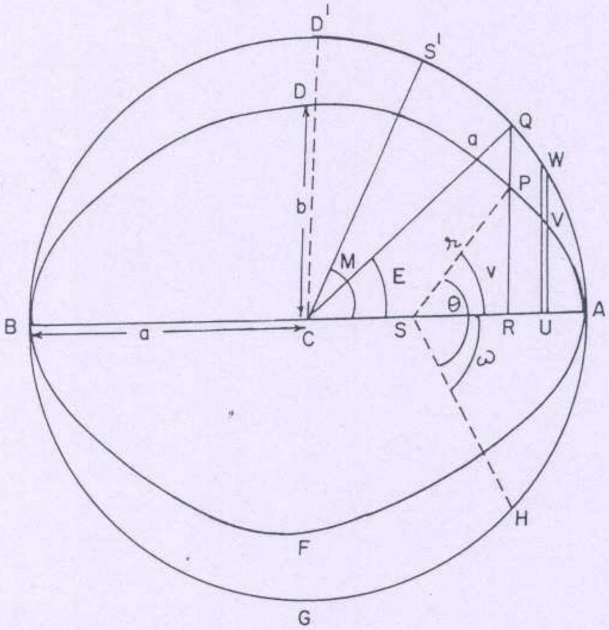

Fig.3 Spherical geometry for computing solar declination.

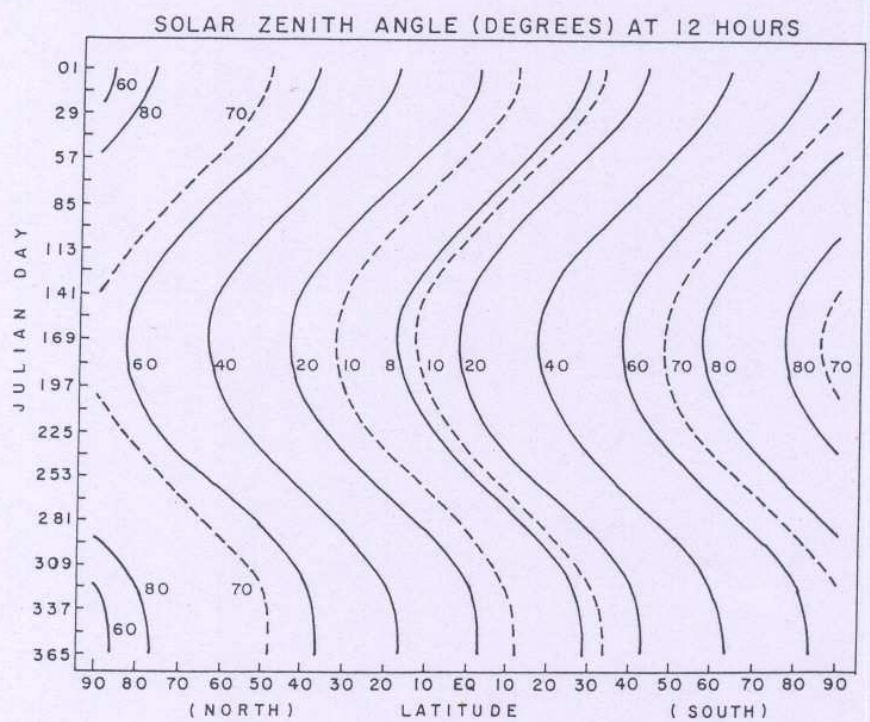

Fig.4(a) Latitude - julian day cross section of the variation of solar zenith angle (degrees) at 12 hours.

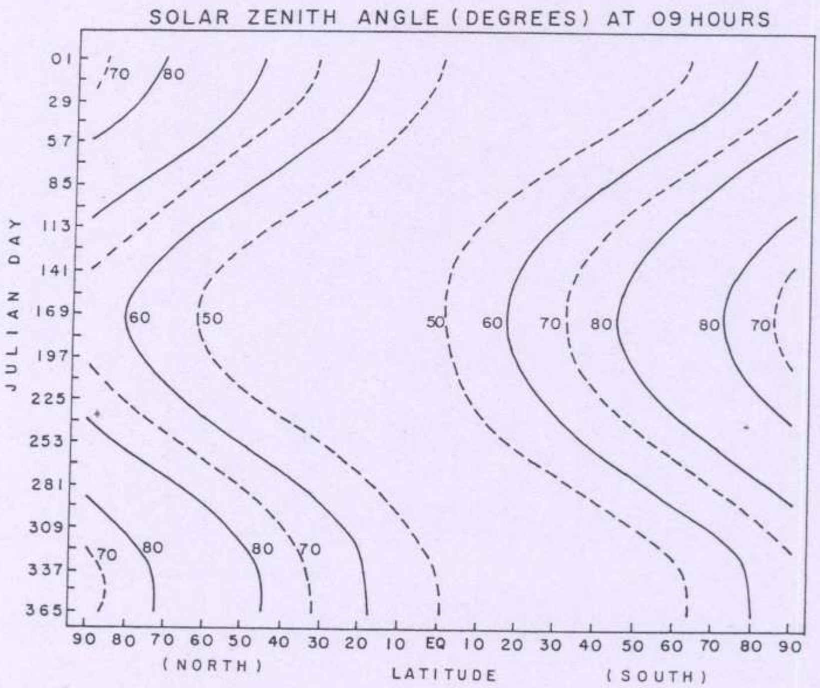

Fig.4(b) Same as Fig.4(a) but at 9 hours.

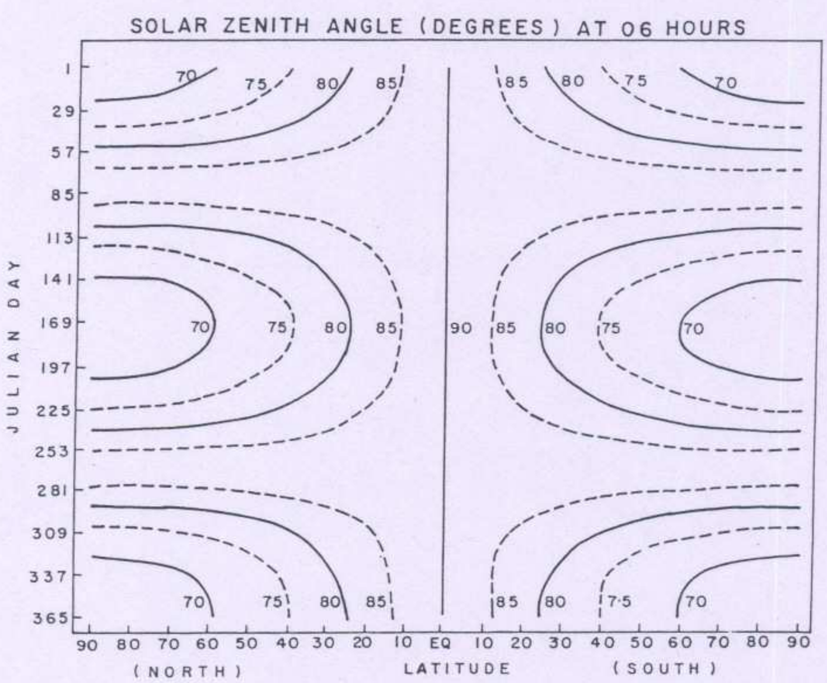

Fig.4(c) Same as Fig.4(a) but at 6 hours.

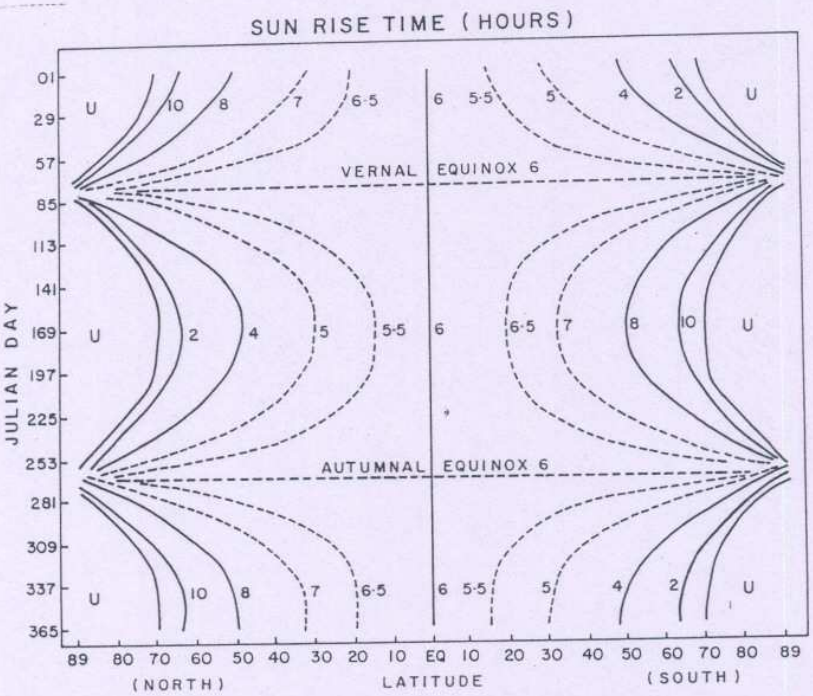

Fig.5(a) Latitude - julian day cross section of the variation of the time (hours) of sun rise.

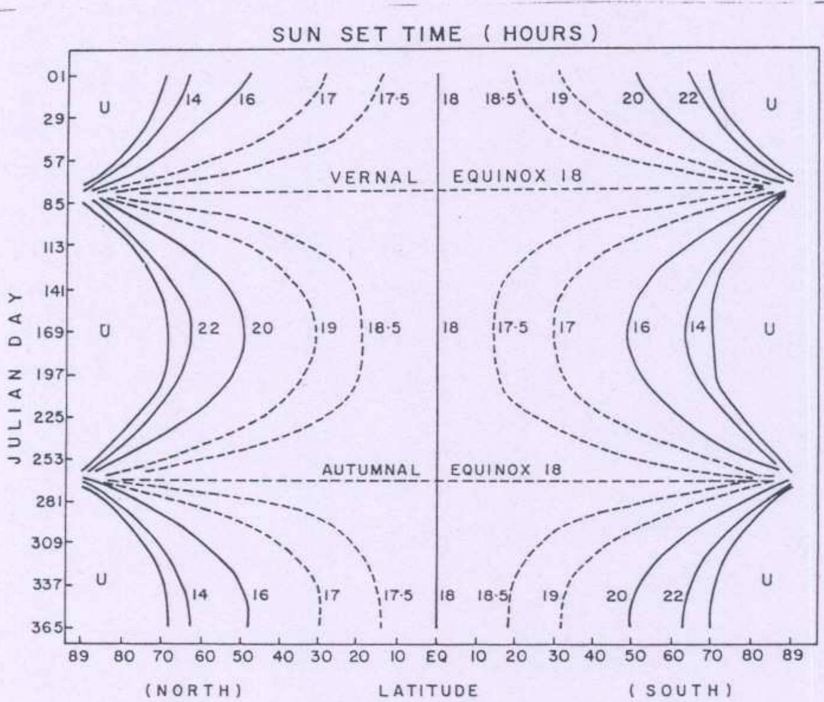

Fig.5(b) Same as Fig.5(a) but for sun set.

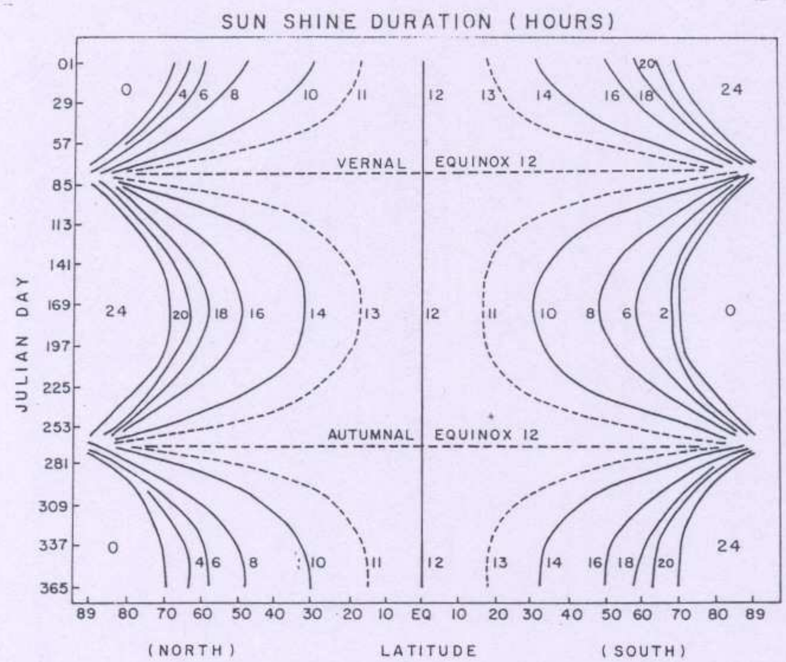

Fig.5(c) Same as Fig.5(a) but for the duration of sun shine.

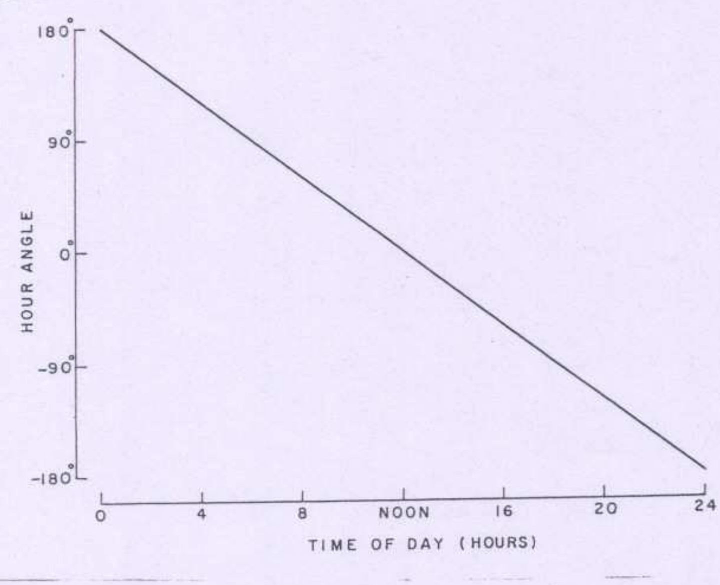

Fig.6 Diurnal variation of hour angle (degrees).

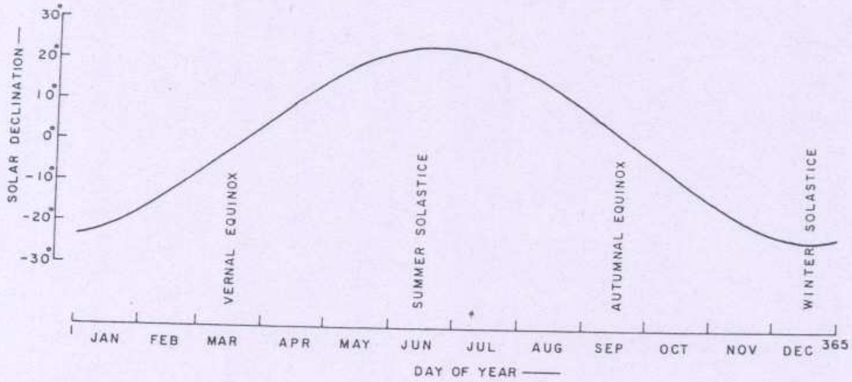

Fig.7 Daily variation of solar declination (degrees).

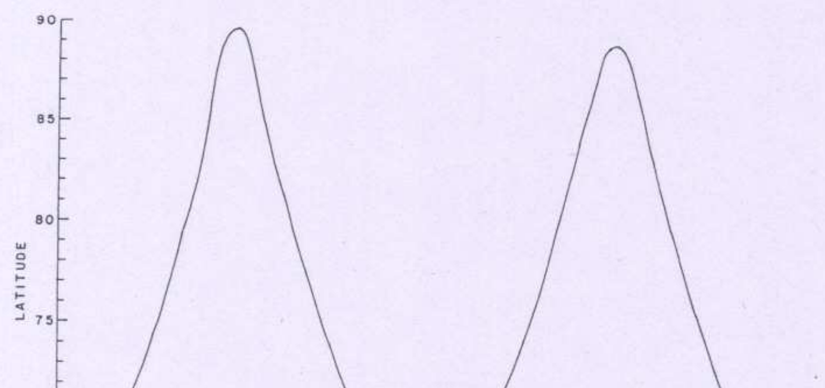

Fig.8 Daily variation of the latitude of polar night in degrees.

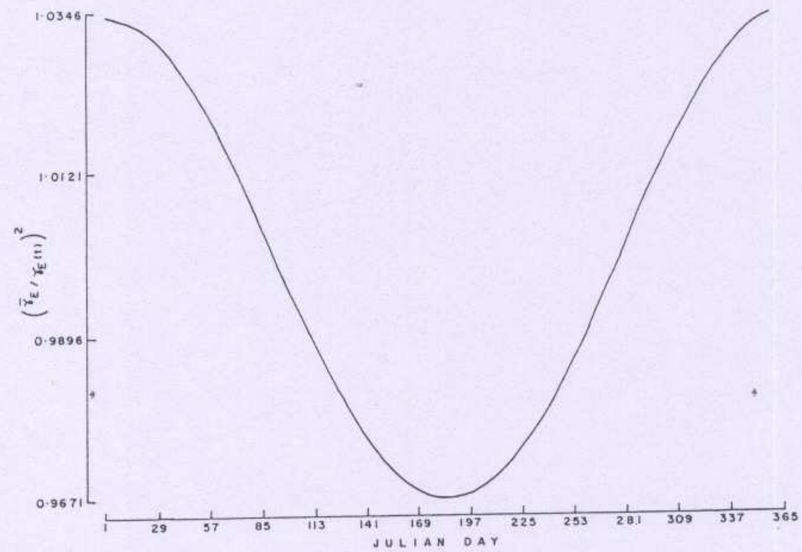

Fig.9 Daily variation of $(r_E/r_E(t))^2$.

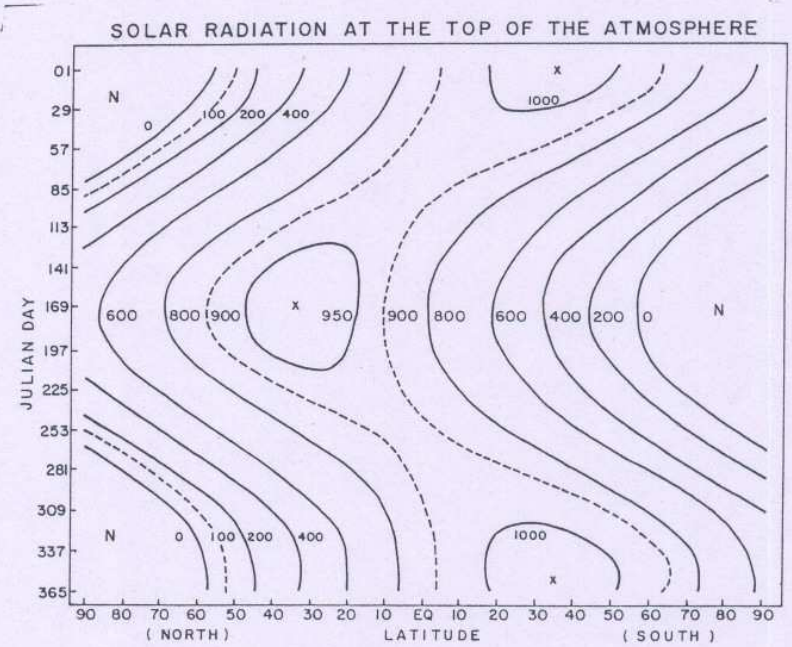

Fig.10 Latitude - julian day cross section of the variation of total solar radiation at the top of the atmosphere in langleys per day. Locations of hemispheric maxima are marked as "X".