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DETERMINISTIC CHAOS AND NUMERICAL WEATHER PREDICTION

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ABSTRACT A brief review of deterministic chaos and possible applications to Meteorology and Atmospheric Physics is given in this paper. References for further reading are given at the end of the paper.

Introduction

Deterministic chaos is the name given to the irregular behaviour of simple deterministic systems and has emerged as an area of intensive research in dynamics since the early 1980s. A dynamical system is simply a system that changes its behaviour over time, two disparate examples being the atmospheric weather patterns and the stock market price fluctuations. The dynamical system consists of rules describing the way some quantity or a set of quantities undergoes a change. The model is based on differential equations and their representations in space. This geometrical and topological foundation for dynamics and the study of moving things is attributed to the French mathematician Jules Henri Poincare (1908). The beginnings of chaos theory originated with Poincare´s study of a familiar (as yet unresolved) problem in dynamics, the three body problem. A dynamical system is called deterministic if given a set of initial conditions precisely, then the state of the system can be determined precisely by the evolution equations at any later time. Such equations are also termed linear equations. Students of the physical sciences are taught mostly linear laws of dynamics which envisage total predictability of the dynamical system starting from a set of known initial conditions. When confronted with a nonlinear problem the equations are immediately linearized by using an approximation as a special case. But in the real world many problems are inherently nonlinear with irregular chaotic behaviour and cannot be linearized meaningfully without losing the physics of the problem. Chaos is a Greek word that has come to mean confusion, disorder, a complete lack of structure and organisation. Chaos theory or chaos science is a branch of both mathematical and physical sciences.

Traditionally (unit 1970), the complexity in the behaviour of dynamical systems was attributed to indeterminism in the form of external noise or complexity in the structure of the deterministic system. Recently, mathematical studies indicate that simple deterministic systems can exist which are intrinsically capable of manifesting extremely complicated behaviour. Further, there is evidence for the existence of patterns of order deeply embedded in the field of chaos. Chaos is not random since it is shown to obey strict mathematical rules that derive from equations that can be written down and studied. These ongoing discoveries would have profound implications for every branch of science as well as for economics and other areas of human interest. This is because random chance governs almost all events in the universe. Chaos, conventionally associated with random chance is now shown to follow deterministic laws and therefore stochastic processes may also have an underlying order and therefore predictable.

2. Origin of Chaos in Dynamical Systems

The origin of complicated (irregular, unpredictable) behaviour in a deterministic system must atleast be related to the imprecision in specifying the initial conditions. This, however is not sufficient for unpredictable behaviour as there are many systems for which the small initial errors remain small. As a result, apart from this imprecision other ingredients are necessary for such a system to allow complicated behaviour. To begin with, a mechanism is required to blow up initial errors exponentially. This is called 'sensitive dependence on initial conditions'. This mechanism is yet to be identified. The important unsolved problem in modern chaos dynamics is how to characterize mathematically (statistically) the implicit order in the field of chaos. Even though the temporal and spatial aspects of the evolution of such systems are ultimately closely related, nevertheless comparatively little is known about the latter. The spatial evolution of the field of chaos may be important in the forecasting of the intensification of weather systems. The final goal in the study of chaos dynamics is to formulate structurally stable mathematical models for dynamical systems, i.e., models which do not change their quantitative behaviour under small perturbations and which enable prediction of time averages which are independent of initial conditions.

3. Universal route to chaos

Simple deterministic model systems (as well as real ones) can have irregular (chaotic, turbulent) behaviour as well as regular (laminar) types of motion. A problem that has recently attracted a great deal of attention concerns the way in which this transition comes about. In fact the history of the subject extends atleast as far back as 1940s when the Soviet physicist Landau put forward together with his scenarios for the onset of turbulence, the view that the knowledge of the route to turbulence would be of value in understanding turbulence itself. Until very recently the hope was that such routes would either be unique or that their number would be finite and small.

One of the more exciting recent developments in chaos theory has been the idea of universality. One of the leaders of this development has been physicist Mitchell Feigenbaum, formerly of Los Alamos and now at Cornell University. Essentially universality states that nature appears to have chosen a limited number of patterns of behaviour that lead to chaos and that these patterns have not only qualitative aspects in common but quantitative ones as well-that there is order in chaos. One of these patterns in the transition to chaos is called period doubling. It is the process by which the periodic behaviour of a particular system alters and finally becomes erratic as a particular parameter is changed. The behaviour undergoes a very specific alteration. In such systems the behaviour normally reproduces itself after a given period or time interval, when the parameter is changed - temperature, say in heating a fluid, or food supply in a population of grazing animals - the period doubles. It now takes two of the original time intervals for the behaviour to repeat. According to Feigenbaum, this process of successive period doubling recurs continually until at a certain value of the parameter it has doubled ad infinitum, so that the behaviour is no longer periodic. Period doubling is then a characteristic route for a system to follow as it changes from a simple periodic to a complex aperiodic motion. In fact period doubling is seen in the noise in electronic circuits, the size of a population of whales, the transition to turbulence in a pot of water being heated and in the dripping faucet. In 1975 Feigenbaum began exploring the properties of some simple nonlinear equations using a programmable pocket calculator and the process called iteration. He found that the ratio of the change in parameter values needed to switch from one period doubling to the next is a constant and he called this constant 'convergence rate'. He identified the two universal constants a=-2.5029 and d = 4.6692 now named Feigenbaum's constants. a and d are independent of the details of the nonlinear equations for the period doubling sequence. Delbourgo, a physicist of the University of Tasmania extended the above studies in 1986 and identified the universal relation $3d=2a^2$ for a wide domain of period triplings, quadruplings etc. A great deal of work has been done in order to understand the origin of this 'universality' which has been observed in many real systems. It is however, not been possible to identify a single universal route to chaos since experiments (both numerical and real) show that even within one system many different routes to chaos can exist for different ranges of the parameters. As a result, the understanding of the route to chaos does not necessarily lead to the understanding of chaotic regime itself.

4. Atmospheric flows

Describing turbulent fluid flows e.g., atmospheric flows and predicting their quantitative behaviour realistically has not been possible even after a century of research in turbulence. The reasons for the difficulty lie partly in the mathematics of turbulence theory. The equations of fluid flows are derived directly from Newton's laws of motion. They are called Navier-Stokes equations. The Navier-Stokes equations neatly express in mathematical language the fundamental laws that govern fluid flow. These equations however do not have exact analytic solutions and in the averaged forms normally applied are insufficient to determine a solution. In order to simulate the atmospheric convective activity which is basically a problem of transport of heat by a gas, researchers must account for the conservation of mass and momentum (Navier-Stokes equations), the conservation of energy (first law of thermodynamics) and the relationship between density, temperature and pressure (Boyle´s law). For computer models or numerical simulations these equations must be converted to a form in which they can be applied only at a relatively small set of grid points. Variables defined at each grid point make up a numerical model that is coded into computer language and put into the machine. The number of variables involved in modelling turbulent flow is so staggering that they cannot be adequately handled with advanced computers. Even at the swift rate of development of computers they will not be able to resolve fully developed turbulence.

5. The geometry of turbulent trajectories in fluid flows

In trying to represent the extraordinary complexities that arise in the behaviour of a turbulent fluid, physicists invoke an abstraction called phase space. Phase space is a multidimensional space in which differential equations form patterns generated by a point that continuously traces a trajectory. The location of the point at any time contains all the information needed to describe its state. Poincaré (1908) had originally used this geometrical approach to analyze the motions of three body systems (earth, sun and moon). One point in phase space represents a single measurement of the state of the fluid as it evolves in time. When the collection of points that represent all the measurements is connected, a trajectory is produced that lies on the surface of an object (another abstraction) sitting somewhere in phase space. This object is usually a strange attractor-strange because of its convoluted shape, attractor because it refers to the final destination of the trajectories.

The chaotic (strange) attractor was actually known to theorists 60 years ago, but it only emerged into the consciousness of science between 1961 and 1971 because of the availability of computer graphical display facilities for doing mathematics. The results can be seen graphically and interpreted immediately. This enabled solution of a much richer variety of problems than possible in the nineteenth century using analytical techniques. The first strange attractor (although he did not use the term) in a real world physical system was found by Edward Lorenz at the Massachusetts Institute of Technology in 1963. Lorenz had been a mathematician before World War II, a student of the great George Birkhoff (1884-1944). Birkhoff continued what Poincare had begun, the geometrical study of dynamical systems. By 1932 Birkhoff had developed the theoretical basis of what would now be called chaos. Lorenz started with Navier-Stokes equations of the entire planet´s atmosphere in order to understand weather patterns. He arrived at a simple looking system of three ordinary differential equations which however did not have analytical solutions. He then calculated using a computer, the time evolution of the air flow trajectories. He found that the trajectory went round and round in a geometrical figure that was vaguely two dimensional. On that surface it behaved very erratically. Lorenz´s model for atmospheric turbulence therefore predicts inherent uncertainity of long range weather forecasts. Lorenz´s model of the onset of chaos is described in a classical paper 'Deterministic Nonperiodic Flow' published in 1963 in the Journal of Atmospheric Sciences of the American Meteorological Society. The shape of the Lorenz's strange attractor, a sort of undulating torus with two holes is now called Lorenz's butterfly or Lorenz's mask depending on which of the two objects the viewer thinks it most resembles. The phrase, strange attractor, was first published in a paper in 1971 by David Ruelle, a mathematical physicist at the Institute of Advanced Scientific Studies in France and Floris Takens, a mathematician at the State University of Groningen in the Netherlands. Until very recently all the work done on turbulence had been highly mathematical and theoretical, since quantitative technique for characterising a dynamical system in phase space were not known. Now, however, experimental physicists have begun to get involved after quantitative technique became available in 1983 enabling scientists, using more accurate data from experiments, to construct attractors as the dynamical systems evolved in time.

6. Self similar fractal geometry of strange attractors

The strange attractor design for dynamical systems in chaos is found to have fractal geometrical shape. The word fractal indicates broken or fractured geometrical structure and was first coined by mathematician Benoit Mandelbrot of Harvard University and IBM's Thomas J. Watson Research Centre who vigorously promulgated the use of fractals since the late 1950s. The fractal dimension of an object, usually a noninteger or fractional value is a measure of the extent to which it fills space on which it is embedded. The fractal dimension of a cloud which is not an exact sphere is therefore less than 3. The ever changing shape of the cloud is a typical strange attractor pattern of the three dimensional trajectories of turbulent air flows inside the cloud. The fractal geometrical shape of the strange attractor is also self similar i.e., possesses identical internal structure throughout its space time domain. Self similarity implies growth by replication of a basic design e.g., the intricate beautical patterns of 'Rangoli' which are considered auspicous in Indian tradition are formed by a continous harmonious repetition of a single basic design. Self similarity implies scale invariance since the internal structure remains the same for all spatial scales. Mandelbrot's studies showed that nature abounds in self similar fractal structures (coastlines, trees, lightining, mountains etc.)

Deterministic chaos in atmospheric flows

Atmospheric flows consist of a complete spectrum of fluctuations ranging from the turbulence scale of a few centimeters to the planetary scales of thousands of kilometres. Traditional meteorological studies were confined to the immediately perceived turbulence scale and the synoptic scale (100-1000 km) because of the lack of a sufficiently dense network of observatories. The advent of radar and then satellites in the fifties and sixties gave access to meteorologists to view the complete spectrum of weather phenomena comprising the turbulence, convective, meso-, synoptic and planetary scales. Advances in remote sensing and insitu measurement technique, have enabled to record the following new observational results. Fluctuations occur over a wide range of space and time scales, in particular, the energy spectrum of wind in the horizontal is of the scaling (power law) form f^{-B} where f is the frequency and B is a exponent of value -5/3 for horizontal spectrum. Such scale invariant atmospheric fluctuations are also consistent with the observed universal fractal geometry to the global cloud cover pattern. Shaun Lovejoy a physicist now at the University of McGill, Canada established in 1982 conclusively the applicability of fractals in meteorology by showing that cloud and rain areas project on the Earth along shapes whose boundaries are fractal curves with fractal dimension 1.35 for sizes ranging from 0.16 km to 1000 km. This finding expressed mathematically the intuitive view of cloud and rain shapes as composed of billows upon billows since fractal geometry implies structure with a hierarchy of identical shapes, Furthermore, in the last 10 years, many experiments, particularly in the velocity field show that atmospheric circulations are scale invariant from the turbulence scale to the planetary scale. Deterministic chaos therefore underlies meteorological and climatological fluctuations in atmospheric flows and probably determines weather patterns from meteorological to climatological scales.

8. Deterministic chaos, meteorology and atmospheric physics

Atmospheric weather systems are coherent structures consisting of discrete cloud cells, forming patterns of rows/streets, mesoscale (up to 100 km) cloud clusters (MCC) and spiral bands which maintain their identity for the duration of their appreciable life time in the turbulent shear flows (wind speed varying with height) of the earth's atmosphere. The existence of coherent structures (seemingly systematic motion) in turbulent flows has been well established during the last 20 years of research in turbulence. It is still, however, debated whether these structures are the consequence of some kind of instabilities or whether they are manifestation of some instrinsic universal properties of any turbulent flow. Experimental data and theoretical studies indicate that the steady state atmospheric boundary layer may consist of a hierarchy of intrinsically helical fluctuations (vortices). The coherent helical geometry of weather system is, indicated by the following observations. (1) All basic atmospheric flow structures appear to be distinctly helical. These include such outstanding examples of organised geophysical motion as medium scale tornado generating squall lines, hurricanes etc. (2) Geophysical flows give an implicit indication of upscale transfer of a certain amount of energy inserted at much smaller scale (3) The helical nature of the most violent geophysical phenomena - a supercell storm - is shown beyond any doubt. Further, observations show that energy injected at some scale is not dissipated but on the contrary is transmitted to larger and larger scales i.e., inverse energy cascade occurs. At present the concept of inverse cascade is well accepted and probably plays an important role in geophysical flows. These observational results indicate that the geometrical pattern of deterministic chaos in atmospheric flows consists of a hierarchical system of vortices i.e., swirls within swirls existing as a unified whole.

Deterministic chaos and weather prediction

Numerical weather prediction models, in particular long range prediction models do not give realistic forecasts because of the following inherent limitations. (1) The nonlinear governing equations for atmospheric flows do not have exact analytic solutions and being sensitive to initial conditions give chaotic solutions characteristic of deterministic chaos. (2) The governing equations do not incorporate the dynamical interactions and co-existence of the complete spectrum of turbulent fluctuations which form an integral part of the large coherent weather systems (3) Limitations of available computer capacity necessitates severe truncation of the governing equations, thereby generating errors of approximations. (4) The computer precision related roundoff errors magnify the earlier mentioned uncertainties exponentially with time and the model predictions become unrealistic. The accurate modelling of weather phenomena therefore requires alternative concepts and computational techniques. The newly emerging field of deterministic chaos may hopefully provide suitable mathematical and statistical techniques for more accurate long range weather prediction.

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