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ESTIMATION OF NONLINEAR KINETIC ENERGY EXCHANGES INTO INDIVIDUAL TRIAD INTERACTIONS IN THE FREQUENCY DOMAIN BY USE OF THE CROSS-SPECTRAL TECHNIQUE

by

D. R. CHAKRABORTY

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Estimation of Nonlinear Kinetic Energy Exchanges into Individual Triad Interactions in the Frequency Domain by use of the Cross-Spectral Technique

D.R. Chakraborty

Indian Institute of Tropical Meteorology, Pune-411008.

Abstract :

In order to determine nonlinear energy exchanges into individual triad interactions in the frequency domain, spectral formulas are derived by use of the corss-spectral technique. Nonlinear product terms are calculated using the conventional interaction Fourier coefficients. The proposed method of computation can be applied to the problem of maintenance and predictability of low frequency modes. Nonlinear aliasing errors associated with this approach can be either neglected or completely eliminated by Fourier interpolation.

Key Words : Frequency domain, cross-spectral, predictability, nonlinear, aliasing error.

1. Introduction .

Obsrvations have shown the presence of low frequency motions on the time scale of roughly 30 to 50 days. There are several regional and global aspects of these oscillations that have been emphasized in recent literature. Among these one observational aspect relevant to this problem is the energy exchange in the frequency domain. The maintenance of low frequency modes has to be addressed via detailed computations of energetics in the frequency domain using daily analysed data sets over global tropics covering many years. These studies are somewhat analogous to the estimates of energetics in the zonal wavenumber domain. In the frequency domain, the kinetic to kinetic energy exchanges can occur among long term time mean flows and other frequencies, or among triads of frequencies.

Following Krishnamurti (1978) synoptic interpretation of kinetic energy transfer due to wave-wave interaction is possible if observations are carefully composited around episodes of large energy exchanges. In order to show this one of the triads of the long wave to medium wave interaction $\langle K_L . K_M \rangle$ has been selected. An inspection of the daily record of triad interactions show that the triad 1,3,4 is one of the major contributions to the wave-wave exchange $\langle K_L \cdot K_M \rangle$ which is positive, and thus provides a net transfer of kinetic energy from long to medium waves. The results here are integrals over the latitude belt 10°S to 20°N. Those dates for which 1,3,4 shows pronounced negative values have been selected from the graph. These are dates when a large loss of kinetic energy was encountered by wavenumber one when it interacted with wavenumbers 3 and 4. It was felt that a composite streamline geometry of the maps during these episodes of intense energy transfer would illustrate the mechanism of wave-wave energy exchange. The map of wavenumber 1 is illustrated in Fig.1. It shows the well-known southwest to northeast tilt of long waves. Two regions of strong velocity, essentially zonal, located around 40°E, 10°N and 140°W, 10°N are important features in the present context. Fig.2 portrays the streamline for zonal wavenumbers 3 plus 4 composited for the same dates. A comparison of the two diagrams shows that the tilt of shorter waves (shown by dashed line) removes westerly momentum from the westerly jet over this region of wavenumber 1. In the region of strong easterliels (between 10°W and 70°W at 10°N) for wavenumber 1, the tilt of shorter waves (southeast to northwest) results in a removal of easterly momentum away from the easterly wind maximum. Thus, the wave-wave exchange has a local interpretation. Similar interpretation of kinetic to kinetic energy exchanges among triad of frequencies will have significant bearing on the maintenance of low frequency waves.

Sheng and Hayashi (1990a, 1990b) studied the energetics in the frequency domain using two versions of the FGGE III b data set, processed at GFDL and ECMWF. They also applied the analysis of spectral energetics in the frequency domain to several observed datasets and those simulated by a GFDL general circulation model. Their results showed that in the tropics the kinetic energy is transferred from transients of longer time scales to those of shorter time scales. But over the northern hemisphere the direction of transfer of energy is from high frequency to low frequency wave. Yasunari (1980, 1981) emphasized the relationship between the low frequency oscillations and the Northern Hemisphere Summer monsoon. The results of energetics calculations, performed by Sheng (1986) show that the low frequency modes on the time scale of 30 to 50 days receive a substantial amount of kinetic energy from the high frequencies.

The computation of energy exchanges in the frequency domain carried out by Krishnamurti et al (1990) for the control and the anomaly experiments in their study of predictability of low frequency modes showed the energy exchange from the higher to the lower frequencies is very small.

All the earlier studies did not determine nonlinear energy exchanges into individual triad interaction in the frequency domain. It is of considerable interest to have synoptic interpretation of wave-wave exchanges of kinetic energy in the frequency domain and to identify the dominant triad interactions responsible for maintenance of low frequency mode over global tropical belt. For this purpose spectral formulation of this problem derived by use of the cross-spectral technique is presented in this study.

2.Formulation.

a.Wavenumber cross spectra.

It is assumed that space-time series data u(λ ,t) and v(λ ,t) are cyclic and discrete in longitude. These series are represented by a Space-Fourier series with discrete wavenumbers (n) as

$$u(\lambda,t) = \sum_{n=0}^{N} \left[C_n^u(t) \cos n\lambda + S_n^u(t) \sin n\lambda \right] \qquad (1)$$

In particular, $C^u = u_o$ (zonal mean) an $S^u = 0$.

The sample wavenumber cospectra Pn(u,v) is defined as

$$Pn\,(u,v) = \tfrac{1}{2}\,(C_n^u\ C_n^v + S_n^u\ S_n^v) \qquad (2)$$

b.Kinetic energy spectra.

The equations of motion and continuity in spherical pressure coordinate system in flux form as :

$$\frac{\partial u}{\partial t} = -\left[\frac{\partial}{\partial x} u u + \frac{\partial}{\partial y} v u + \frac{\partial}{\partial p} \omega u - \frac{\tan\phi}{a} u v\right] + 2\Omega \sin\phi . v - g\frac{\partial Z}{\partial x} + F_u$$

$$\frac{\partial v}{\partial t} = -\left[\frac{\partial}{\partial x} u v + \frac{\partial}{\partial y} v v + \frac{\partial}{\partial p} \omega v + \frac{\tan\phi}{a} u u\right] - 2\Omega \sin\phi . u + \frac{g}{a}\frac{\partial Z}{\partial \phi} + F_v$$

$$0 = -\left[\frac{RT}{p\theta}\right]\theta - g\frac{\partial Z}{\partial p} \qquad \cdots \quad \cdots \quad \cdots (3)$$

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} \qquad \cdots \quad \cdots \quad \cdots (4)$$

where $\frac{\partial}{\partial x}(\ ) = \frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}(\ )$

Kinetic energy per unit mass K(n) for frequency n is defined by

$$K(n) = \tfrac{1}{2}\,[Pn(u,u)+Pn(v,v)] \qquad (5)$$

where Pn is the frequency cospectrum.

Taking a cospectrum operation between u and both sides of Eq.(i) yields

$$Pn\left(u, \frac{\partial u}{\partial t}\right) = 0 = -\left[P_n\left(u, \frac{\partial}{\partial x} u u\right) + P_n\left(u, \frac{\partial}{\partial y} v u\right) + P_n\left(u, \frac{\partial}{\partial p}\omega u\right) - \frac{\tan\phi}{a} P_n(u, u v)\right] + 2\Omega \sin\phi . P_n(u, v) - P_n\left(u, g\frac{\partial Z}{\partial x}\right) + P_n(u, F_u) \qquad \cdots \quad \cdots (6)$$

The term Pn $\left(u, \frac{\partial u}{\partial t}\right)$ vanishes. For the meridional momentum equation we have

$$P_n\left(v, \frac{\partial v}{\partial t}\right) = 0 = -\left[P_n\left(v, \frac{\partial}{\partial x} u v\right) + P_n\left(v, \frac{\partial}{\partial y} v v\right) + P_n\left(v, \frac{\partial}{\partial p}\omega v\right) + \frac{\tan\phi}{a} P_n(v, u u)\right] - 2\Omega \sin\phi . P_n(v, u) - P_n\left(v, \frac{g}{a}\frac{\partial Z}{\partial \phi}\right) + P_n(v, F_v) \qquad \cdots \quad \cdots (7)$$

Following Hayashi (1980), the spectral Kinetic energy equation can be obtained by adding (6) and (7).

$$= -[P_n(u, \frac{\delta}{\delta x} uu) + P_n(u, \frac{\partial}{\delta y} vu) + P_n(u, \frac{\partial}{\delta p} \omega u)$$
$$+ P_n(v, \frac{\delta}{\delta x} uv) + P_n(v, \frac{\partial}{\delta y} vv) + P_n(v, \frac{\partial}{\delta p} \omega v)$$
$$+ \frac{\tan\phi}{a} [P_n(u, uv) - P_n(v, uu)]$$
$$- P_n(u, g \frac{\delta z}{\delta x}) - P_n(v, \frac{g}{a} \frac{\delta z}{\partial \phi}) + P_n(u, F_u) + P_n(v, F_v)$$

By definition of cross spectra, we can derive the following relationship

$$- P_n(u, g \frac{\delta z}{\delta x}) - P_n(v, \frac{g}{a} \frac{\delta z}{\delta \phi})]$$
$$= -g [\frac{\partial}{\delta x} P_n(u, z) + \frac{\partial}{\delta y} P_n(v, z) - P_n(\frac{\delta u}{\delta x}, z) - P_n(\frac{\delta v}{\delta y}, z)]$$
$$= -g [\frac{\partial}{\delta x} P_n(u, z) + \frac{\delta}{\delta y} P_n(v, z) + P_n(\frac{\partial \omega}{\delta p}, z)]$$
$$= -g [\frac{\partial}{\delta x} P_n(u, z) + \frac{\partial}{\delta y} P_n(v, z) + \frac{\delta}{\partial p} P_n(\omega, z) - P_n(\omega, \frac{\delta z}{\delta p}) \quad \cdots (8)$$
$$= -g [\frac{\delta}{\delta x} P_n(u, z) + \frac{\partial}{\delta y} P_n(v, z) + \frac{\partial}{\delta p} P_n(\omega, z) - P_n(\omega, \alpha)$$

In deriving (8) we have used the continuity equation (4) and the hydrostatic equation (3). The final form of the kinetic energy equation in the frequency domain is written as

$$0 = \langle K.K(n) \rangle - P_n(\omega, \alpha)$$
$$- g [\frac{\partial}{\delta x} P_n(u, z) + \frac{\partial}{\delta y} P_n(v, z) + \frac{\partial}{\delta p} P_n(\omega, z) - D(n)$$

Following Hayashi (1980) and Sheng (1986) the nonlinear kinetic energy transfer spectrum $\langle K.K(n) \rangle$ is written as

$$\langle K.K(n) \rangle = -[P_n(u, \frac{\partial}{\delta x} uu) + P_n(u, \frac{\partial}{\delta y} vu) + P_n(u, \frac{\delta \omega u}{\delta p})$$
$$+ P_n(v, \frac{\partial}{\delta x} uv) + P_n(v, \frac{\partial}{\delta y} vv) + P_n(v, \frac{\delta}{\delta p} \omega v)$$
$$+ \frac{\tan\phi}{a} [P_n(u, uv) - P_n(v, uu)] \quad \cdots \cdots (9)$$

and $D(n) = - P_n(u, Fu) - P_n(v, Fv)$

$A(n).K(N) = -Pn(\omega,\alpha)$ which denotes the baroclinic conversion from APE to KE at frequency n. According to Fjortoft (1953) a transfer of kinetic energy from or into a frequency n occurs when the frequencies of oscillations a,b and c are respectively n, (n+m) and m as related by $n = (n \pm m) \mp m$.

It is important to point out at this stage that Eq.(9) corresponds to othe time change of amplitude of periodic oscillation with frequency n. Therefore, according to Hayashi (1982), it indeed represents the generation, maintenance and dissipation of spectral energy.

Following Hayashi (1980), the nonlinear energy transfer spectra $\langle K.K(n)\rangle$ can be further partitioned into two parts as

$$\langle K.K(n)\rangle = \langle L(n)\rangle + \langle K(0).K(n)\rangle$$

Here $\langle L(n)\rangle$ is the transfer of kinetic energy into frequencies excluding 0 (time mean), while $\langle K(0).K(n)\rangle$ is the transfer of energy into frequency n by interaction between the mean flow and frequency n.

By definition, $\langle L(n)\rangle$ is given by

$$\langle L(n)\rangle = -[Pn\left(u, \frac{\delta}{\delta x}u'u'\right) + P_n\left(u, \frac{\delta}{\delta y}v'u'\right) + P_n\left(u, \frac{\delta}{\delta p}\omega'u'\right) + P_n\left(v, \frac{\delta}{\delta x}u'v'\right) + P_n\left(v, \frac{\delta}{\delta y}v'v'\right) + P_n\left(v, \frac{\delta}{\delta p}\omega'v'\right) + \frac{\tan\phi}{a}\left[P_n(u, u'v') - P_n(v, u'u')\right] \quad \ldots\ldots(10)$$

The interaction between the time-mean and the time transient motions is given by

$$\langle K(0).K(n)\rangle = \langle K.K(n)\rangle - \langle L(n)\rangle$$

Where the prime denotes deviation from time mean for frequency spectra.

C. Explicit expressions to have triad interaction in the frequency domain.

The theory of harmonic analysis shows that if X_1, $X_2, \ldots, X_s, \ldots, X_r$ are equi-spaced values of any observed parameter $X = f(t)$ at the time epochs $t_1, t_2, \ldots, t_s, \ldots, t_r$, the data series can be exactly represented by a finite series of n harmonics $f(t_s) = \sum_n (XOC_n \cos nt_s + XOS_n \sin nt_s)$

Where $t_s = \frac{2\pi}{r} s$ $\qquad$ $(s = 1, 2, 3, \ldots\ldots, r)$

and $n = \frac{1}{2}(r-1)$ or $r/2$ depending upon r is odd or even. The expressions for the Fourier coefficients are

$$XOC_k = \frac{2}{r} \sum_{\sigma=1}^{r} X_\sigma \cos k t_\sigma \qquad (k = 1, 2, 3, \ldots, n)$$

$$XOS_k = \frac{2}{r} \sum_{\sigma=1}^{r} X_\sigma \sin k t_\sigma$$

Similarlly, any time transient field $X' = f'(t)$ can be expressed as :

$$f'(t_s) = \sum_n (XTC_n \cos nt_s + XTS_n \sin nt_s) \qquad (s = 1, 2, 3, \ldots, r)$$

Pn [U, U'V']

$$= \frac{1}{T} \int_{-T/2}^{T/2} \sum_n (UOC_n \ COSnt + UOS_n \ SINnt) . \sum_r (UTC_r \ COSrt + UTS_r \ SINrt) . \sum_s (VTC_s COSst + VTS_s SINst) dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} \sum_n \sum_r (UOC_n . UTC_r . COSnt.COSrt + UOC_n . UTS_r . COSnt.SINrt$$

$$+ UOS_n . UTC_r . SINnt.COSrt + UOS_n . UTS_r . SINnt.SINrt)$$

$$X \sum_s (VTC_s \ COS_s t + VTS_s \ SINSt) dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} \sum_n \sum_r \sum_s (UOC_n . UTC_r . VTC_s \ COSnt.COSrt \ COSSt$$

$$+ UOC_n . UTS_r . VTC_s . COSnt . SINrt.COSst$$

$$+ UOS_n . UTC_r . VTC_s . SINnt.COSrt. COSst$$

$$+ UOS_n . UTS_r . VTC_s . SINnt.SINrt.COSst$$

$$+ UOC_n . UTC_r . VTS_s . COSnt . COSrt . SINst$$
$$+ UOC_n . UTSr . VTS_s . COSnt . SINrt . SINst$$
$$+ UOS_n . UTC_n . VTS_s . SINnt . COSrt . SINst$$
$$+ UOS_n . UTS_r . VTS_s . SINnt . SINrt . SINst) \, dt$$

$$= \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n . UTC_r . VTC_s + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n} \end{bmatrix} UOS_n . UTS_r . VTC_s$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n . UTS_r . VTS_s + \frac{1}{2}\begin{bmatrix} \sum_{r+s=n} \\ \sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOS_n . UTC_r . VT_s$$

Similarly,

$$P_n \left[U, \frac{\delta}{\delta x} U'U'\right] = P_n \left[U, U' . \frac{\delta u'}{\delta x}\right] + P_n \left[U, \frac{\delta u'}{\delta x} . U'\right]$$
$$= 2P_n \left[U, U' \frac{\delta u'}{\delta x}\right]$$

$$P_n \left[U, U' \frac{\delta u'}{\delta x}\right]$$
$$= \frac{1}{T}\int_{-T/2}^{T/2} \sum_n (UOC_n \; COSnt + UOS_n \; SINnt)$$

$$X \sum_r (UTC_r \; COSrt + UTS_r \; SINrt)$$

$$X \sum_s \left(\frac{\delta uTC_s}{\delta x} . COSst + \frac{\delta uTS_s}{\delta x} . SINst\right) \quad dt$$

$$= \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n . UTC_r . \frac{\delta uTC_s}{\delta x} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n} \end{bmatrix} UOS_n . UTS_r . \frac{\delta uTC_s}{\delta x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n \cdot UTS_r \cdot \frac{\partial UTS_s}{\partial x} + \begin{bmatrix} \sum_{r+s=n} \\ -\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOS_n \cdot UTC_r \cdot \frac{\partial UTS_s}{\partial x}$$

$$P_n\ [U, \frac{\partial}{\partial y} v'u'] = P_n\ [U, U' \cdot \frac{\partial v'}{\partial y}] + P_n\ [U, \frac{\partial u'}{\partial y} \cdot V']$$

$$P_n\ [U, U' \frac{\partial v'}{\partial y}]$$

$$=\frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n \cdot UTC_r \cdot \frac{\partial VTC_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n} \end{bmatrix} UOS_n \cdot UTS_r \cdot \frac{\partial VTC_s}{\partial y}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n \cdot UTS_r \cdot \frac{\partial VTS_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ -\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOS_n \cdot UTC_r \cdot \frac{\partial VTS_s}{\partial y}$$

$$P_n\ [U, \frac{\partial u'}{\partial y} \cdot V']$$

$$=\frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n \cdot \frac{\partial UTS_r}{\partial y} \cdot VTC_s$$

$$+ \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n} \end{bmatrix} UOS_n \cdot \frac{\partial UTS_r}{\partial y} \cdot VTC_s$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n} \end{bmatrix} UOC_n \cdot \frac{\partial UTS_r}{\partial y} \cdot VTS_s$$

$$+\frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ -\sum\limits_{r-s=n}\\ +\sum\limits_{r-s=-n}\end{array}\right] uos_n \cdot \frac{\partial uTC_r}{\partial y} \cdot vTS_s$$

$$P_n \ [U, \frac{\partial \omega' U'}{\partial p}] = P_n \ [U, U' \frac{\partial \omega'}{\partial p}] + P_n \ [U, \omega' \cdot \frac{\partial u'}{\partial p}]$$

$$P_n \ [U, U' \frac{\partial \omega'}{\partial p}]$$

$$= \frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ +\sum\limits_{r-s=n}\\ +\sum\limits_{r-s=-n}\end{array}\right] UOC_n \cdot UTC_r \cdot \frac{\partial}{\partial p} WTC_s$$

$$+\frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ +\sum\limits_{r-s=n}\\ -\sum\limits_{r-s=-n}\end{array}\right] uos_n \cdot \frac{\partial WTC_s}{\partial p} \cdot uTS_r$$

$$+\frac{1}{2}\left[\begin{array}{l}-\sum\limits_{r+s=n}\\ +\sum\limits_{r-s=n}\\ +\sum\limits_{r-s=-n}\end{array}\right] UOC_n \cdot UTS_r \cdot \frac{\partial}{\partial p} WTS_s$$

$$+\frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ -\sum\limits_{r-s=n}\\ +\sum\limits_{r-s=-n}\end{array}\right] uos_n \cdot \frac{\partial WTS_s}{\partial p} \cdot uTC_r$$

$$P_n \ [U, \omega' \frac{\partial u'}{\partial p}]$$

$$= \frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ +\sum\limits_{r-s=n}\\ +\sum\limits_{r-s=-n}\end{array}\right] UOC_n \cdot WTC_c \cdot \frac{\partial uTC_r}{\partial p}$$

$$+\frac{1}{2}\left[\begin{array}{l}+\sum\limits_{r+s=n}\\ +\sum\limits_{r-s=n}\\ -\sum\limits_{r-s=-n}\end{array}\right] uos_n \cdot WTC_s \cdot \frac{\partial}{\partial p} uTS_r$$

$$+\frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n \cdot uTC_r \cdot \frac{\partial vTS_s}{\partial x}$$

$$P_n\left[V, \frac{\partial}{\partial y} v'v'\right] = 2P_n\left[V, V'\frac{\partial v'}{\partial y}\right]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot VTC_r \cdot \frac{\partial vTC_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} vOS_n \cdot vTS_r \cdot \frac{\partial vTC_s}{\partial y}$$

$$+\begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot VTS_r \cdot \frac{\partial vTS_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n \cdot vTC_r \cdot \frac{\partial vTS_s}{\partial y}$$

$$P_n\left[V, \frac{\partial u'v'}{\partial x}\right] = P_n\left[V, U'\frac{\partial v'}{\partial x}\right] + P_n\left[V, V'\frac{\partial u'}{\partial x}\right]$$

$$P_n\left[V, U'\frac{\partial v'}{\partial x}\right]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot UTC_r \cdot \frac{\partial vTC_s}{\partial x} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} vOS_n \cdot uTS_r \cdot \frac{\partial vTC_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot UTS_r \cdot \frac{\partial vTS_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . uTC_r . \frac{\partial vTS_s}{\partial x}$$

$$P_n\left[V, \frac{\partial}{\partial y} v'v'\right] = 2P_n\left[V, V'\frac{\partial v'}{\partial y}\right]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n . VTC_r . \frac{\partial vTC_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . vTS_r . \frac{\partial vTC_s}{\partial y}$$

$$+ \begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n . VTS_r . \frac{\partial vTS_s}{\partial y} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . vTC_r . \frac{\partial vTS_s}{\partial y}$$

$$P_n\left[V, \frac{\partial}{\partial x} u'v'\right] = P_n\left[V, U'\frac{\partial v'}{\partial x}\right] + P_n\left[V, V'\frac{\partial u'}{\partial x}\right]$$

$$P_n\left[V, U'\frac{\partial v'}{\partial x}\right]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n . UTC_r . \frac{\partial vTC_s}{\partial x} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . uTS_r . \frac{\partial vTC_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n . UTS_r . \frac{\partial vTS_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} UOC_n \cdot WTS_s \cdot \frac{\partial uTS_r}{\partial p} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ -\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} uOS_n \cdot WTS_s \cdot \frac{\partial uTC_r}{\partial p}$$

$$P_n \; [V, V' \frac{\partial u'}{\partial x}]$$

$$P_n \; [V, \frac{\partial}{\partial x} u'v'] = P_n \; [V, U' \frac{\partial v'}{\partial x}] + P_n \; [V, V' \frac{\partial u'}{\partial x}]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} VOC_n \cdot VTC_s \cdot \frac{\partial uTC_r}{\partial x} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n}\end{bmatrix} vOS_n \cdot vTC_s \cdot \frac{\partial uTS_r}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} VOC_n \cdot VTS_s \cdot \frac{\partial uTS_r}{\partial x} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ -\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} vOS_n \cdot vTS_s \cdot \frac{\partial uTC_r}{\partial x}$$

$$P_n \; [V, U' \frac{\partial v'}{\partial x}]$$

$$= \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} VOC_n \cdot UTC_r \cdot \frac{\partial vTC_s}{\partial x} + \frac{1}{2}\begin{bmatrix} +\sum_{r+s=n} \\ +\sum_{r-s=n} \\ -\sum_{r-s=-n}\end{bmatrix} vOS_n \cdot uTS_r \cdot \frac{\partial vTC_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum_{r+s=n} \\ +\sum_{r-s=n} \\ +\sum_{r-s=-n}\end{bmatrix} VOC_n \cdot UTS_r \cdot \frac{\partial vTS_s}{\partial x}$$

$$+\frac{1}{2}\begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot WTS_s \cdot \frac{\partial vTS_r}{\partial p} + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n \cdot WTS_s \cdot \frac{\partial vTC_r}{\partial p}$$

$$P_n \quad [V, U'U']$$

$$=\frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot UTC_r \cdot UTC_s + \frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . uTS_r . uTC_s$$

$$+\frac{1}{2}\begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} VOC_n \cdot UTS_r \cdot UTS_s + \frac{1}{2}\begin{bmatrix} \sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} vOS_n . uTC_r . uTS_s$$

$$L(n) = -[P_n\left(U, \frac{\partial}{\partial x} U'U'\right) + P_n\left(U, \frac{\partial}{\partial y} u'v'\right)$$

$$+P_n\left(u, \frac{\partial}{\partial p} \omega' u'\right) + P_n\left(V, \frac{\partial}{\partial x} U'V'\right)$$

$$+P_n\left(V, \frac{\partial}{\partial y} v'v'\right) + P_n\left(V, \frac{\partial}{\partial p} \omega' V'\right)$$

$$+\frac{\tan\phi}{a}[P_n(U, U'V') - P_n(V, U'V')]$$

$$= -[\frac{1}{2}\begin{bmatrix} +\sum\limits_{r+s=n} \end{bmatrix} 2UOC_n UTC_r \cdot \frac{\partial uTC_s}{\partial x}$$

$$\left[\begin{array}{l} +\sum\limits_{r-s=n} \\ \\ +\sum\limits_{r-s=-n} \end{array}\right] \begin{array}{l} +UOC_n\ UTC_r \cdot \frac{\partial}{\partial y} VTC_s \\ +UOC_n \cdot \frac{\partial UTC_r}{\partial y} VTC_s\ + \\ UOC_n \cdot UTC_r \cdot \frac{\partial}{\partial p} WTC_s\ + \\ UOC_n \cdot \frac{\partial}{\partial p} UTC_r \cdot WTC_s \end{array}$$

$$+ VOC_n . UTC_r \cdot \frac{\partial}{\partial x} VTC_s^n + VOC_n \cdot \frac{\partial UTC_r}{\partial x} . VTC_s$$

$$+2VOC_n \cdot VTC_r \cdot \frac{\partial}{\partial y} VTC_s$$

$$+VOC_n \cdot VTC_r \cdot \frac{\partial WTC_s}{\partial p} + VOC_n \cdot \frac{\partial VTC_r}{\partial p} . WTC_s$$

$$+ \frac{\tan\phi}{a} (UOC_n . UTC_r \cdot VTC_s - VOC_n \cdot UTC_r \cdot UTC_s)]$$

$$+\tfrac{1}{2} \left[\begin{array}{l} +\sum\limits_{r+s=n} \\ \\ +\sum\limits_{r-s=n} \\ \\ -\sum\limits_{r-s=-n} \end{array}\right] \begin{array}{l} 2UOS_n \cdot UTS_r \cdot \frac{\partial UTC_s}{\partial x} \\ +UOS_n \cdot UTS_r \cdot \frac{\partial VTC_s}{\partial y} \\ + UOS_n \cdot \frac{\partial UTS_r}{\partial y} . VTC_s \\ + UOS_n \cdot UTS_r \cdot \frac{\partial}{\partial p} WTC_s \\ + UOS_n \cdot \frac{\partial UTS_r}{\partial p} . WTC_s \end{array}$$

$$+ VOS_n \cdot UTS_r \cdot \frac{\partial VTC_s}{\partial x} + VOS_n \cdot \frac{\partial}{\partial x} UTS_r . VTC_s$$

$$+ 2VOS_n \cdot VTS_r \cdot \frac{\partial}{\partial y} VTC_s$$

$$+VOS_n \cdot VTS_r \cdot \frac{\partial WTC_s}{\partial p} + VOS_n \cdot \frac{\partial VTS_r}{\partial p} . WTC_s$$

$$+ \frac{\tan\phi}{a} (UOS_n \cdot UTS_r \cdot VTC_s - VOS_n \cdot UTS_r \cdot UTC_s)$$

$$+\tfrac{1}{2} \left[\begin{array}{l} -\sum\limits_{r+s=n} \\ \\ +\sum\limits_{r-s=n} \\ \\ +\sum\limits_{r-s=-n} \end{array}\right] \begin{array}{l} 2UOC_n \cdot UTS_r \cdot \frac{\partial UTS_s}{\partial x} \\ +UOC_n . UTS_r \cdot \frac{\partial}{\partial y} VTS_s \\ +UOC_n \cdot \frac{\partial UTS_r}{\partial y} . VTS_s \\ + UOC_n . UTS_r \cdot \frac{\partial}{\partial p} WTS_s \end{array}$$

$$+ UOC_n \cdot \frac{\delta UTS_r}{\delta p} \cdot WTS_s$$

$$+VOC_n \cdot UTS_r \cdot \frac{\delta VTS_s}{\delta x} + VOC_n \cdot \frac{\delta UTS_r}{\delta x} \cdot VTS_s$$

$$+2VOC_n \cdot VTS_r \cdot \frac{\delta VTS_s}{\delta y}$$

$$+VOC_n \cdot VTS_r \cdot \frac{\delta WTS_s}{\delta p} + VOC_n \cdot \frac{\delta VTS_r}{\delta p} \cdot WTS_s$$

$$+ \frac{\tan\phi}{a} (UOC_n \cdot UTS_r \cdot VTS_s - VOC_n \cdot UTS_r \cdot UTS_s)$$

$$+\frac{1}{2} \left[\begin{array}{l} + \sum_{r+s=n} \\ - \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] \begin{array}{l} 2UOS_n \cdot UTC_r \cdot \frac{\delta UTS_s}{\delta x} \\ +UOS_n \cdot UTC_r \cdot \frac{\delta VTS_s}{\delta y} \\ +UOS_n \cdot \frac{\delta UTC_r}{\delta y} \cdot VTS_s \\ +UOS_n \cdot UTC_r \cdot \frac{\delta WTS_s}{\delta p} \\ +UOS_n \cdot \frac{\delta UTC_r}{\delta p} \cdot WTS_s \end{array}$$

$$+VOS_n \cdot UTC_r \cdot \frac{\delta VTS_s}{\delta x} + VOS_n \cdot \frac{\delta UTC_r}{\delta x} \cdot VTS_s$$

$$+2VOS_n \cdot VTC_r \cdot \frac{\delta VTS_s}{\delta y}$$

$$+VOS_n \cdot VTC_r \cdot \frac{\delta WTS_s}{\delta p} + VOS_n \cdot \frac{\delta VTC_r}{\delta p} \cdot WTS_s$$

$$+ \frac{\tan\phi}{a} (UOS_n \cdot UTC_r \cdot VTS_s - VOS_n \cdot UTC_r \cdot UTS_s)\Big]$$

$$\langle L(n) \rangle$$

$$= -\Big[\frac{1}{2} \left[\begin{array}{l} + \sum_{r+s=n} \\ + \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] \begin{array}{l} UOC_n \cdot UTC_r \cdot \left(2 \frac{\delta UTC_s}{\delta x} + \frac{\delta VTC_s}{\delta y} + \frac{\tan\phi}{a} \cdot VTC_s\right) \\ +UOC_n \cdot \left(\frac{\delta UTC_r}{\delta y} \cdot VTC_s + \frac{\delta UTC_r}{\delta p} \cdot WTC_s\right) \\ +VOC_n \cdot UTC_r \cdot \left(\frac{\delta VTC_s}{\delta x} - \frac{\tan\phi}{a} \cdot UTC_s\right) \end{array}$$

$$+VOC_n \cdot VTC_r \left(2 \cdot \frac{\delta VTC_s}{\delta y} + \frac{\delta WTC_s}{\delta p} \right)$$

$$+VOC_n \left(\frac{\delta}{\delta x} uTC_r \;.VTC_s + \frac{\delta}{\delta p} vTC_r \;.WTC_s \right)$$

$$+\tfrac{1}{2} \begin{bmatrix} +\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} \begin{array}{l} UOS_n . UTS_r \left(2 \frac{\delta}{\delta x} uTC_s + \frac{\delta}{\delta y} vTC_s + \frac{\delta}{\delta p} WTC_s + \frac{\tan\phi}{a} VTC_s \right) \\ +UOS_n \left(\frac{\delta}{\delta y} uTS_r \;.VTC_s + \frac{\delta}{\delta p} uTS_r \;.WTC_s \right) \\ +VOS_n . UTS_r \left(\frac{\delta}{\delta x} vTC_s - \frac{\tan\phi}{a} . UTC_s \right) \end{array}$$

$$+VOS_n . VTS_r \left(2 . \frac{\delta}{\delta y} vTC_s + \frac{\delta}{\delta p} WTC_s \right)$$

$$+VOS_n . \left(\frac{\delta}{\delta x} uTS_r \;.VTC_s + \frac{\delta}{\delta p} vTS_r \;.WTC_s \right) \ldots\ldots\ldots \text{(C)}$$

$$+\tfrac{1}{2} \begin{bmatrix} -\sum\limits_{r+s=n} \\ +\sum\limits_{r-s=n} \\ -\sum\limits_{r-s=-n} \end{bmatrix} . \begin{array}{l} UOC_n . UTS_r . \left(2 \frac{\delta}{\delta x} uTS_s + \frac{\delta}{\delta y} vTS_s + \frac{\delta}{\delta p} WTS_s + \frac{\tan\phi}{a} VTS_s \right) + uOC_n \left(\frac{\delta}{\delta y} uTS_r . vTS_s + \frac{\delta}{\delta p} uTS_r . WTS_s \right) \\ +VOC_n . UTS_r . \left(\frac{\delta}{\delta x} vTS_s - \frac{\tan\phi}{a} UTS_s \right) \end{array}$$

$$+VOC_n . VTS_r . \left(2 \frac{\delta}{\delta y} vTS_s + \frac{\delta}{\delta p} WTS_s \right)$$

$$+VOC_n . \left(\frac{\delta}{\delta x} uTS_r \;.VTS_s + \frac{\delta}{\delta p} vTS_r . WTS_s \right)$$

$$+\tfrac{1}{2} \begin{bmatrix} +\sum\limits_{r+s=n} \\ -\sum\limits_{r-s=n} \\ +\sum\limits_{r-s=-n} \end{bmatrix} \begin{array}{l} UOS_n . UTC_r \left(2 \frac{\delta}{\delta x} uTS_s + \frac{\delta}{\delta y} vTS_s + \frac{\delta}{\delta p} WTS_s + \frac{\tan\phi}{a} .VTS_s \right) \\ + UOS_n . \left(\frac{\delta}{\delta y} uTC_r \;.VTS_s + \frac{\delta}{\delta p} uTC_r . WTS_s \right) \\ + VOS . UTC_r . \left(\frac{\delta}{\delta x} vTS_s - \frac{\tan\phi}{a} . UTS_s \right) \\ + VOS_n . VTC_r . \left(2 \frac{\delta}{\delta y} vTS_s + \frac{\delta}{\delta p} WTS_s \right) \end{array}$$

$$+ VOS_n . \left(\frac{\delta}{\delta x} uTC_r \;.VTS_s + \frac{\delta}{\delta p} vTC_r \;.WTS_s \right)]$$

3. Concluding Remarks :

In the present study spectral formulas are derived by use of the cross-spectral technique to compute nonlinear energy exchanges into individual triad interactions in the frequency domain. Nonlinear product terms are formulated using the interaction Fourier coefficients. A glance at (C) reveals that L(n) is composed of many contributions due to disturbances with frequencies r and s, where r and s satisfy either of the condition $r+s = n$ or $|r-s| = n$. Thus L (n) can be expressed as

$$L(n) = \sum LN\ (n,r,s)$$

LN (n,r,s) denotes a contribution to the net L(n) due to the specified combination of frequencies r and s. It is proposed to investigate the energetics in the frequency domain by using the technique in the following areas :

(1) Synoptic interpretation of wave-wave exchanges of kinetic energy in the frequency domain and identification of the dominant triad interactions responsible for maintenance of low frequency mode over global tropics.

(2) Nonlinear triad interactions with emphasis on the contrast between the extra-tropics and the tropics.

(3) Interseasonal variability of nonlinear triad interactions over tropics.

The role ofthe time-mean flow on low frequency transients for kinetic energy transfer which is an important aspect of the atmospheric energy cycle in the frequency domain cannot be studied from the present formulations.

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References.

Hayashi,Y., 1980 : Estimation of nonlinear energy transfer spectra by the cross-spectral method, J.Atmos.Sci., 37, pp 299.

Krishnamurti,T.N., 1978 : Large scale features of the tropical atmosphere. Published in Meteorology over the tropical oceans, Roy.Meteor.Soc., Bracknell Encland, pp.31.

Krishnamurti,T.N., P.K. Jayakumar,J.Sheng, N.Surgi and A.Kumar, 1985 : Divergent circulations on the 30 to 50 day time scale, J.Atmos.Sci., 42, pp 364.

Krishnamurti,T.N., M.Subramanian, D.K.Oosterhof and G.Daughenbaugh,1990 : Predictability of Low Frequency Modes, Meteorol.Atmos.Phys., 44, pp 63.

Sheng,J., 1986 : On the energetics of low frequency motions.Ph.D. Dissertation. (Available from the Department of Meteorology, Florida State University, Tallahassee, FL-32306, U.S.A.).

Sheng,J., and Y. Hayashi, 1990 a : Estimation of atmospheric energetics in the frequency domain during the FGGE year, J.Atmos.Sci., 47, pp 1255.

Sheng,J. and Y. Hayashi,1990 b : Observed and simulated energy cycles in the frequency domain, J.Atmos.Sci., 47, pp 1243.

Yasunari,T., 1980 : A quasi-stationary appearance of 30 to 40 day period in the cloudiness fluctuations during the summer monsoon over India, J.Meteor.Soc.Japan, 58, pp 225.

Yasunari,T., 1981 : Structure of an Indian summer monsoon system with around 40-day, J.Meteor.Soc.Japan, 59, pp 336.

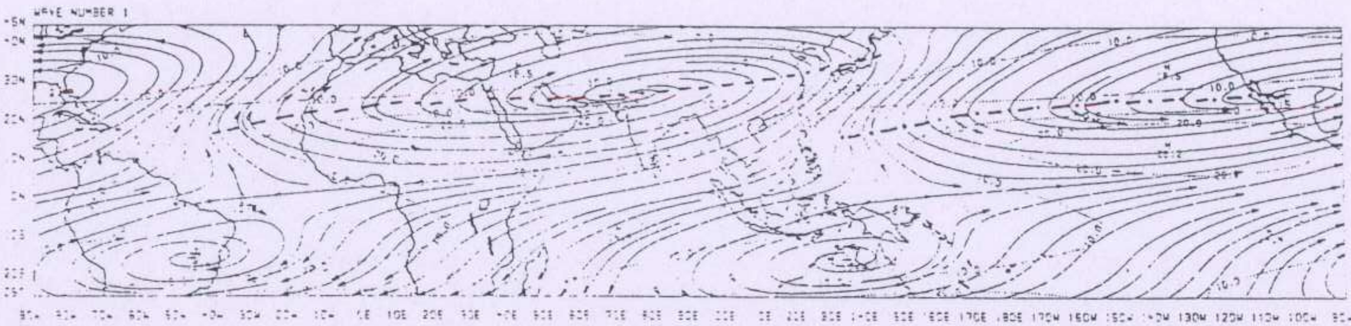

Figure 1. Streamlines (solid lines) and isotachs (dashed lines, knots) for zonal wavenumber-one flows composited for those dates during which ⟨1,3,4⟩ was minimum.

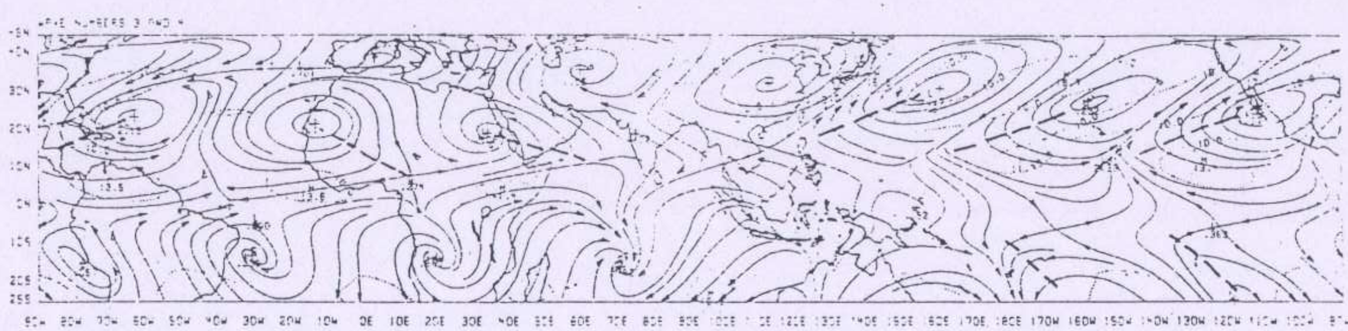

Figure 2. As Fig. 1, for zonal wavenumbers 3 and 4.