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FRACTAL NATURE OF MONTBLEX TIME SERIES DATA

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ABSTRACT

Fluctuations in meteorological parameters occur on all time scales ranging from the instrumentation time resolution limit to the total duration for which data is available. Recent studies have identified the self-similar *fractal* structure of such fluctuations. Self-similarity implies long-range correlations. It is well known that variance of time series data increases with increase in time resolution. Traditional statistical methods do not provide a general approach to describing variances of such *fractal* time series. It is therefore important to quantify this resolution dependent variance for parameterization of boundary layer proceses for predictability studies of different time scales in numerical weather prediction (NWP) models. In this paper it is shown that this resolution dependent variance is described by the fractal dimension **D**. The relative dispersion **RD** (**RD**=standard deviation/mean) of the temporal distribution for a given time resolution **t** is given by

$$\mathbf{RD}(t) = \mathbf{RD}(t_0)\left[\frac{t}{t_0}\right]^{1-D}$$

where t_0 is the available data resolution. The fractal dimension **D** is related to the conventional correlation coefficient **r** between adjacent values in the time series as $\mathbf{r} = 2^{3-2D} - 1$. A fractal dimension less than, equal to or greater than *1.5* indicates respectively, a positive, zero, and negative correlation between adjacent values. A fractal dimension *1.5* corresponds to zero correlation, i.e., random fluctuations and $\mathbf{RD}(t)=\mathbf{RD}(t_0)/\sqrt{(t/t_0)}$ in agreement with conventional statistics.

In this paper, the fractal structure of temporal fluctuations of temperature time series is illustrated and the increase in variance with increase in time resolution is quantified in terms of the fractal dimension **D**.

Key words : Temporal fractals, fractal time series, fractal dimension, correlation coefficient.

1. Introduction

Fluctuations in atmospheric flows as recorded in pressure, temperature and other meteorological parmeters occur on all space (time) scales from the turbulence scale of a few millimetres (seconds) to hundreds of kilometres (years). It is important to quantify the space-time pattern of fluctuations for predictability studies. *Lovejoy and Schertzer* (1986) have documented the self-similar fractal geometry of the space-time fluctuations in atmospheric flows as manifested in the fractal geometry of cloud shapes and the inverse power law form for power spectra of temporal fluctuations of meteorological parmeters. The fractal shape of cloud over the Indian region has been identified (Jain, 1989; Jayanthi et al., 1990). Real world dynamical systems, i.e., systems which evolve with time exhibit self-similar fluctuations in space and time and are recently identified as signatures of **self-organized criticality** (Bak et al., 1988). Self-organized criticality in the temporal (years) evolution of rainfall pattern over the Indian region has been identified (Mary Selvam et al., 1992; Mary Selvam, 1993). In this paper it is shown that the *fractal* nature of meteorological time series enables quantification of the relation between variances at different time resolutions, i.e., the variability at shorter time resolutions may be inferred from more convenient measurements at larger time-averaged measurements. Standard statistical procedures (Snedecor and Cochran, 1980; King et al.,1990) do not provide for quantifying the observed increase in variance associated with higher resolution (space-time) measurements of the *fractal* (self-similar) time series with long-range spatiotemporal correlations. The *fractal* description of the relationship between variances of the time-averaged variable at different time-resolutions will help identify suitable time-interval for predictability studies.

2. Data and Analysis Techniques

One-minute resolution MONTBLEX temperature data (Rudrakumar et al., 1991a; Rudrakumar et al.,1991b; Rudrakumar and Prabhu, 1991) at 1,2,4,8,15 and 30 m levels at the three stations, Delhi, Varanasi and Jodhpur were used for the study. The locations/heights of the stations, the period of study, the number of minutes of continuous data and the mean temperature at available levels are given in Table 1.The data were obtained from the Computer and Data Division of the Indian Institute of Tropical Meteorology,Pune - 411 008,India.

The self-similar (*fractal*) fluctuations in time for a representative sample time series is shown in Figure 1. The rise and fall of diurnal solar heating related temperature fluctuation seen in the long-period record at the upper right-hand corner of Figure 1 is repeated (self-similar) on a smaller (time-magnitude) scale throughout the record duration as seen in the magnified short period record in the center of Figure 1.

In statistics, the relative dispersion **RD** analysis compares the variance of a variable as the measurement resolution increases. The coefficient of variation, namely **RD**, is an index of variability or heterogeneity within the domain.

RD equals the standard deviation divided by the mean. In general, the observed degree of heterogeneity increases as resolution of the method increases. When these increases are proportional, the relation can be *fractal* or at least describable by a *fractal* relation, if it holds true over a sizable range of observation unit sizes. Given that a *fractal* relation exists between the observed **RD** of temperature and **t**, the averaging time interval in minutes, the relation can be expressed by the equation (King et al., 1990).

$$\mathbf{RD(t) = RD(t_0)}\left[\frac{\mathbf{t}}{\mathbf{t_0}}\right]^{1-D} \tag{1}$$

where $\mathbf{t_0}$ is the highest available time resolution (1 minute in the present study). The reason for calling this a *fractal* relationship rather than just a power law relationship is that the possible slopes of the relationship are bounded by limits, and the *fractal dimension* **D**, gives insight into the nature of the data. These equations, describing the relation between a measured quantity and the temporal resolution of measurement, are also found in mathematical constructs and natural phenomena having *fractal* geometry (Mandelbrot, 1983; Peitgen and Richter, 1986; Peitgen and Saupe, 1988).

Traditional statistical methods deal with perfectly random variations where any one set of observations at a particular size of observed unit serves to characterise it completely such that

$$\mathbf{RD(t)} = \frac{\mathbf{RD(t_0)}}{\sqrt{(\mathbf{t/t_0})}} \tag{2}$$

A general approach to describing variances, in particular for the ubiquitous self-similar (correlated) time series is not provided in conventional statistics. There is no law that says that log **RD** versus log **n** for the number **n** of aggregates of non-random variables should show self-similarity. However, the grouping of neighbours must give a monotonically decreasing **RD**, since averaging smoothes out extreme fluctuations.

The *fractal dimension* **D** gives a measure of the temporal correlation **r** between defined time intervals. *Van Beek et al.* (1989) have derived the general expression

$$\mathbf{r = 2^{3-2D} - 1}$$

for precise description of heterogeneity of regional and myocardial blood flows over a wide range of domain sizes. Details of derivation of the above equation are given in the following.

Experimental time series data show that a sample interval with high values for the time series variable (temperature in the present study) tends to have a neighbour that also has a relatively high temperature, and low temperature intervals tend to have low temperature neighbours.Persistence or correlation between neighbouring events in time occur on all time scales from seconds to years. The fluctuations in time are therefore self-similar or *fractals* in time and can be described by the fractal law in Eq. (1), namely,

$$\mathbf{RD(t) = RD(t_0)\left[\frac{t}{t_0}\right]^{1-D}}$$

when two adjacent time intervals are taken together $(\mathbf{Y_1+Y_2})$, the expectation (**E**) for the combined temperature is

$$\mathbf{E\ (\ Y_1 + Y_2\ ) = 2\mu}$$

where μ is the average temperature for unit time interval. *Mendenhall and Scheaffer* (1973) show the expected variance of $\mathbf{Y_1 + Y_2}$ to be

$$\mathbf{Var(Y_1 + Y_2) = Var\ (Y_1) + Var\ (Y_2) + 2Cov\ (Y_1\ ,Y_2)}$$

where **Cov** is the covariance. Because it is assumed that

$$\mathbf{Var\ (Y_1) = Var\ (Y_2) = Var\ (Y)}$$

we find

$$\mathbf{RD(Y_1 + Y_2) = \frac{2^{-0.5}\sqrt{[Var(Y) + Cov(Y_1, Y_2)]}}{\mu}} \qquad (3)$$

But it has been empirically found (Eq. 1) that

$$\mathbf{RD(Y_1+Y_2) = RD\ (Y)\ .\ 2^{1-D}} \qquad (4)$$

where, by definition

$$\mathbf{RD(Y) = \frac{\sqrt{Var(Y)}}{\mu}} \qquad (5)$$

Therefore from Eqs. (3) to (5)

$$\mathbf{\frac{\sqrt{Var(Y)}}{\mu} 2^{1-D} = \frac{2^{\ 0.5}\sqrt{[Var(Y) + Cov(Y_1 + Y_2)]}}{\mu}}$$

or

$$\mathbf{1 + \frac{Cov(Y_1, Y_2)}{Var(Y)} = 2^{3-2D}} \qquad (6)$$

Since the traditional correlation coefficient **r** is given as

$$r = \frac{\mathrm{Cov}(Y_1 +, Y_2)}{\sqrt{[\mathrm{Var}(Y_1)\mathrm{Var}(Y_2)]}} = \frac{\mathrm{Cov}(Y_1, Y_2)}{\mathrm{Var}(Y)}$$

We have from Eq. 6

$$r = 2^{3-2D} - 1 \qquad (7)$$

The above equation is an important derivation since it summarises the properties of the data set. If there is no temporal correlation, **r**=*0*, so that when the temperature fluctuations are completely random the *fractal dimension* **D**=*1.5* and Eq. (1) reduces to Eq. (2) consistent with traditional statistics. This gives a maximal slope in the plots of **RD** versus the time interval for averaging.. At the opposite extreme, with perfect correlation, **r**=*1*, temperature fluctuations are uniform and the *fractal dimension* is *1.0*. Regions of averaging time intervals for which **D** is a constant have averaging-time independent correlation coefficient between neighbours.

Eq. (1) and (7) can be applied to all temporal or spatial *fractals*. If a *fractal* relationship is a reasonably good approximation, even over a decade or so, then it will prove useful in considerations of temporal functions and might be useful in initiating research for the underlying basis for correlation.

In the present study, the relative dispersion **RD** was computed for groups of consecutive **n** values, **n** ranging from *1* at the highest resolution (*1 minute*) to **n**=*T/10* where *T* is the total duration in minutes of temperature record. For each **n**, say *10*, the **RD** was calculated as the mean of *10* sets, starting successively from first, second, third, tenth data value so that all possible combinations of consecutive *10* values are averaged. The standard deviation of the mean **RD** was also computed. The mean **RD**, its standard deviation for averaging time intervals ranging from *1* to *600* minutes are shown in Figures 2-4 respectively for the three stations Delhi, Varanasi and Jodhpur for all available levels.

The exponent **1-D** was computed from Eq. (1) and was plotted in Figure 5 for all levels at the three stations.

3. Discussion and Conclusion

The important results of the present study are as follows.

(1) **RD** remains constant for averaging time intervals up to *100* minutes and thereafter decreases progressively with increase in averaging-time interval at the three stations at all available levels (Figures 2-4).

(2) **RD** decreases in general with increase in height of observation concomitant with decrease in mean temperature. **RD** increases with height when there is an inversion, i.e., increase in temperature with height (Table 1 and Figs. 2-4).

(3) The variation of the exponent **1-D** in Eq. 1 and therefore the *fractal dimension* **D** with averaging time interval (Fig. 5) is identical for all levels at each station.

(4) The *fractal dimension* **D** is equal to *1*, for averaging time intervals up to *100*-minutes indicating averaging-time interval independent one-to-one correlation of fluctuations between neighbours. **D** gradually increases for averaging-time intervals greater than *100* minutes and becomes equal to *1.5* at averaging-time intervals equal to about *400* minutes. Therefore, the positive correlation of fluctuations between neighbours gradually decreases with increasing averaging-time interval and becomes zero at averaging-time intervals of about *400* minutes.

The above study gives a quantitative measure for averaging-time interval related variability of *fractal* time series. The results of analysis show that temperature time series have *multifractal* structure, i.e., the *fractal dimension* **D** is different for different ranges of averaging-time intervals. *Fractal* analysis of temperature time series at all levels for Delhi, Varanasi and Jodhpur show that *100*-minute averaging- time is representative of fluctuations of higher resolution up to *1* minute. Easy-to-maintain lower resolution data acquisition systems may therefore suffice for providing data input for modelling boundary layer flows.

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Table 1 : Details of MONTBLEX - 90 Tower Data used in the study

Station	Location		Period	Mean temperature °C at levels (m) /Duration in minutes (within brackets)					
Altitude(m)	Lat	Long	Date / Time (IST)	1 m	2 m	4 m	8 m	15 m	30 m
Delhi (216)	28° 35'	77° 12'	19 Sept/0000 to 22 Sept/0945	27.33 (4911)	27.04 (4910)	**26.62** (4908)	**26.82** (4909)	**27.40** (4910)	26.69 (4904)
Varanasi (76)	25° 18'	83° 1'	10 June/0000 to 12 June/1138	33.47 (3579)	33.47 (3579)	33.65 (3579)	**32.68** (3579)	**33.87** (3579)	- - -
Jodhpur (224)	26° 18'	73° 1'	10 July/0000 to 13 July/2358	30.67 (5759)	**29.53** (5759)	**29.73** (5759)	28.25 (5759)	28.96 (5759)	29.27 (5759)

Inversion levels are shown in bold letters.

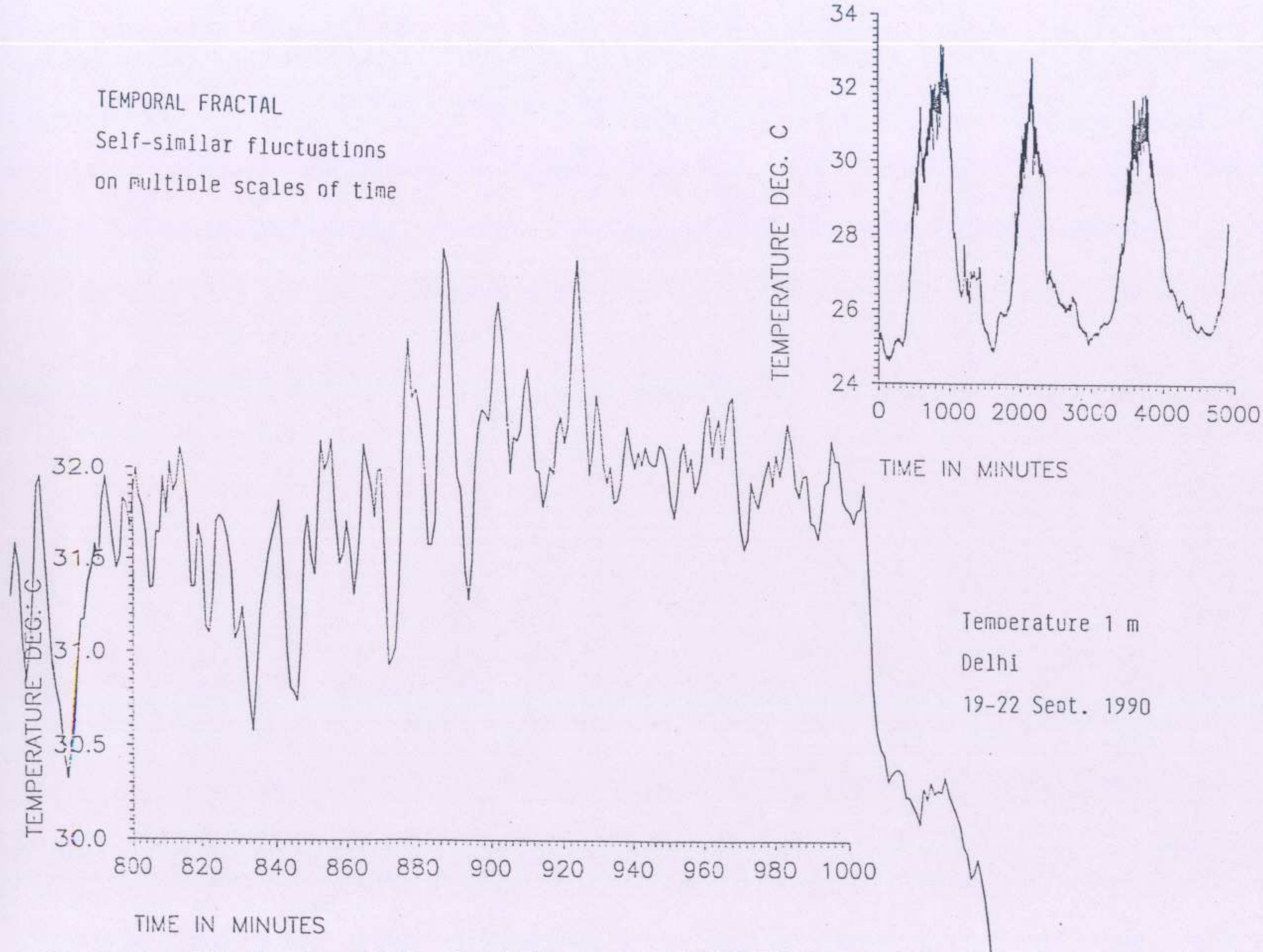

Figure 1 : Illustration of the fractal nature of MONTBLEX temperature time series. Self-similar up and down fluctuations of temperature are seen in long-period (top right-hand corner) and short period (center) record.

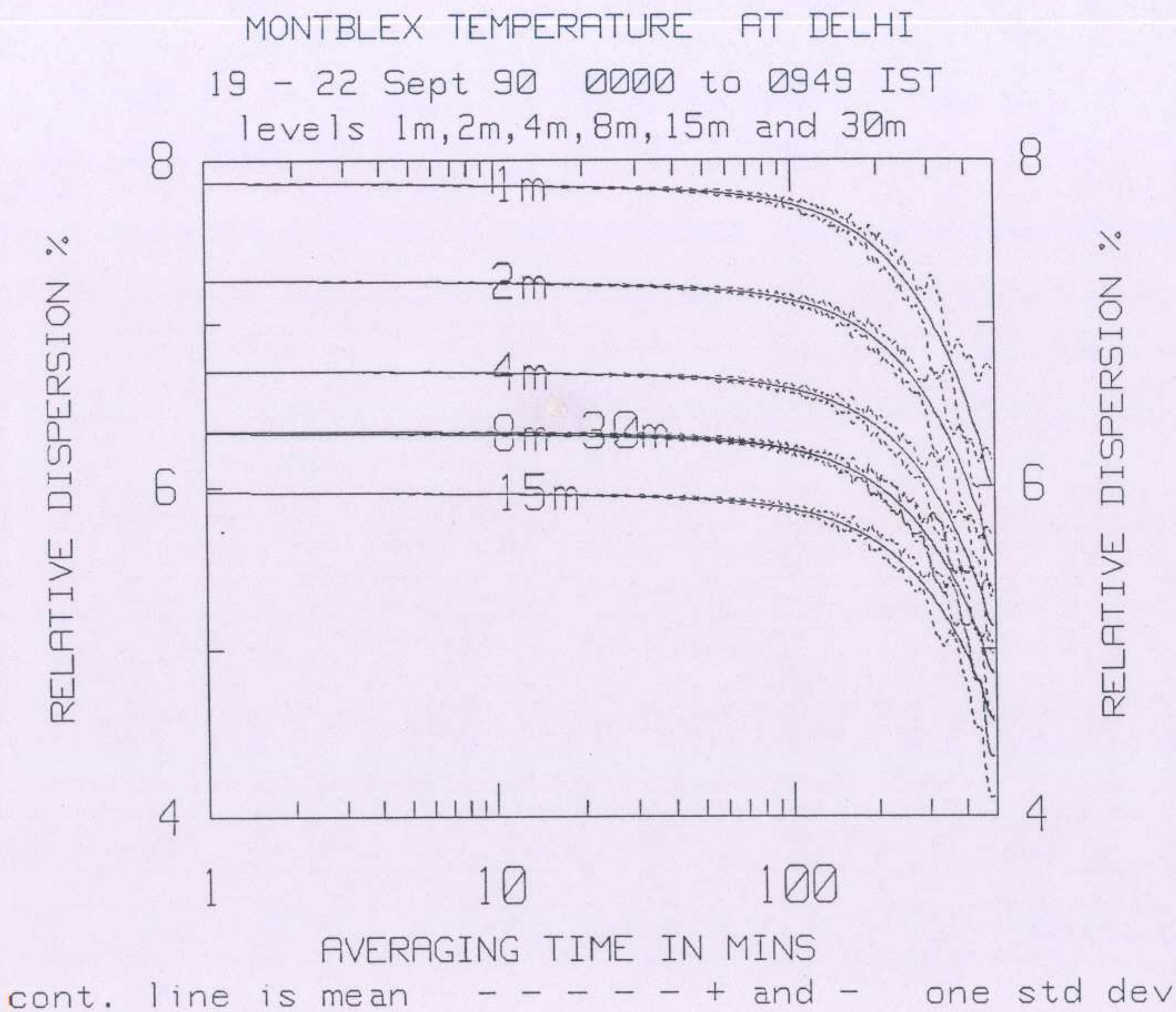

Figure 2 : Variation of relative dispersion RD with averaging-time interval for Delhi.

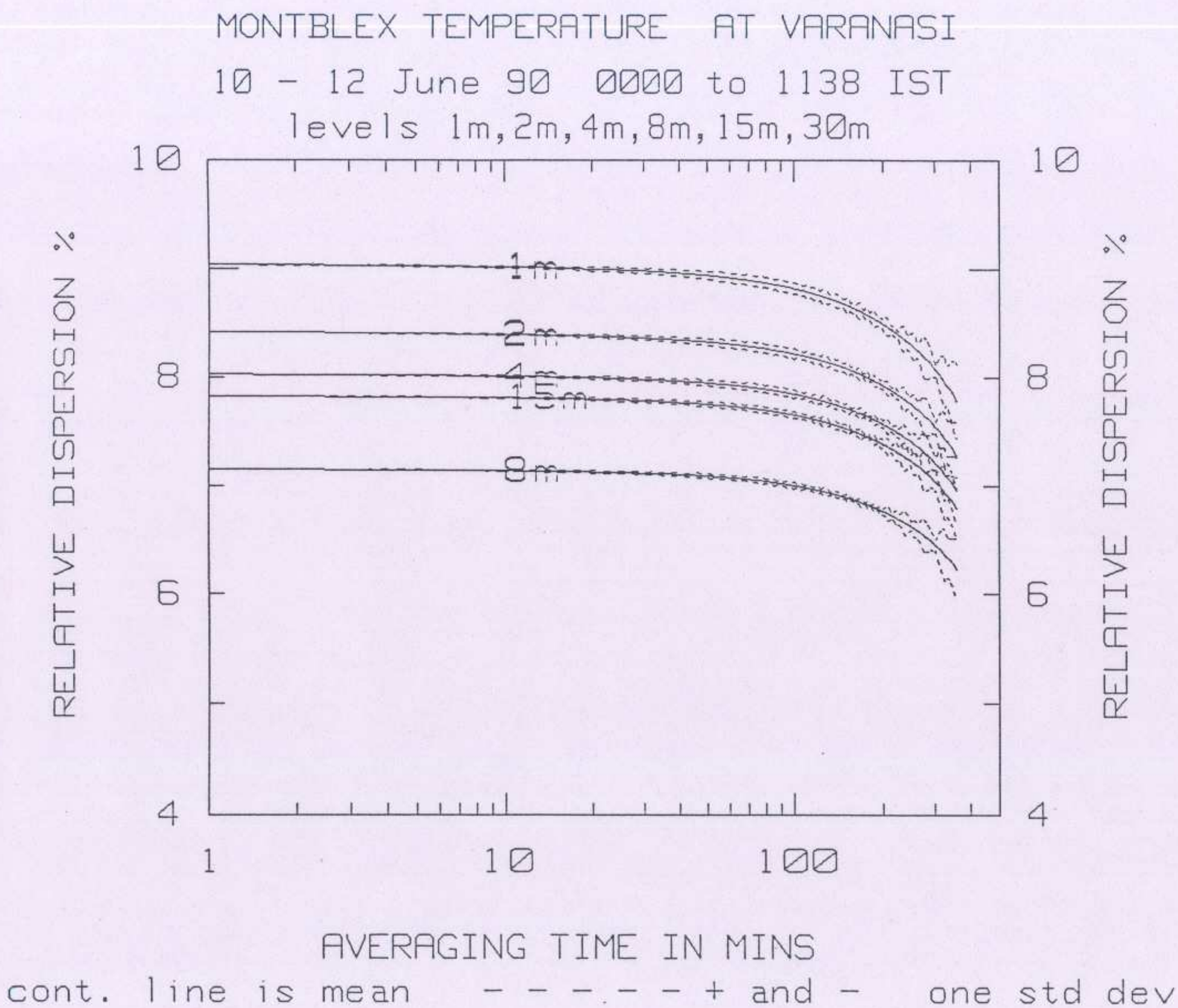

Figure 3 : Same as for Figure 2 for Varanasi

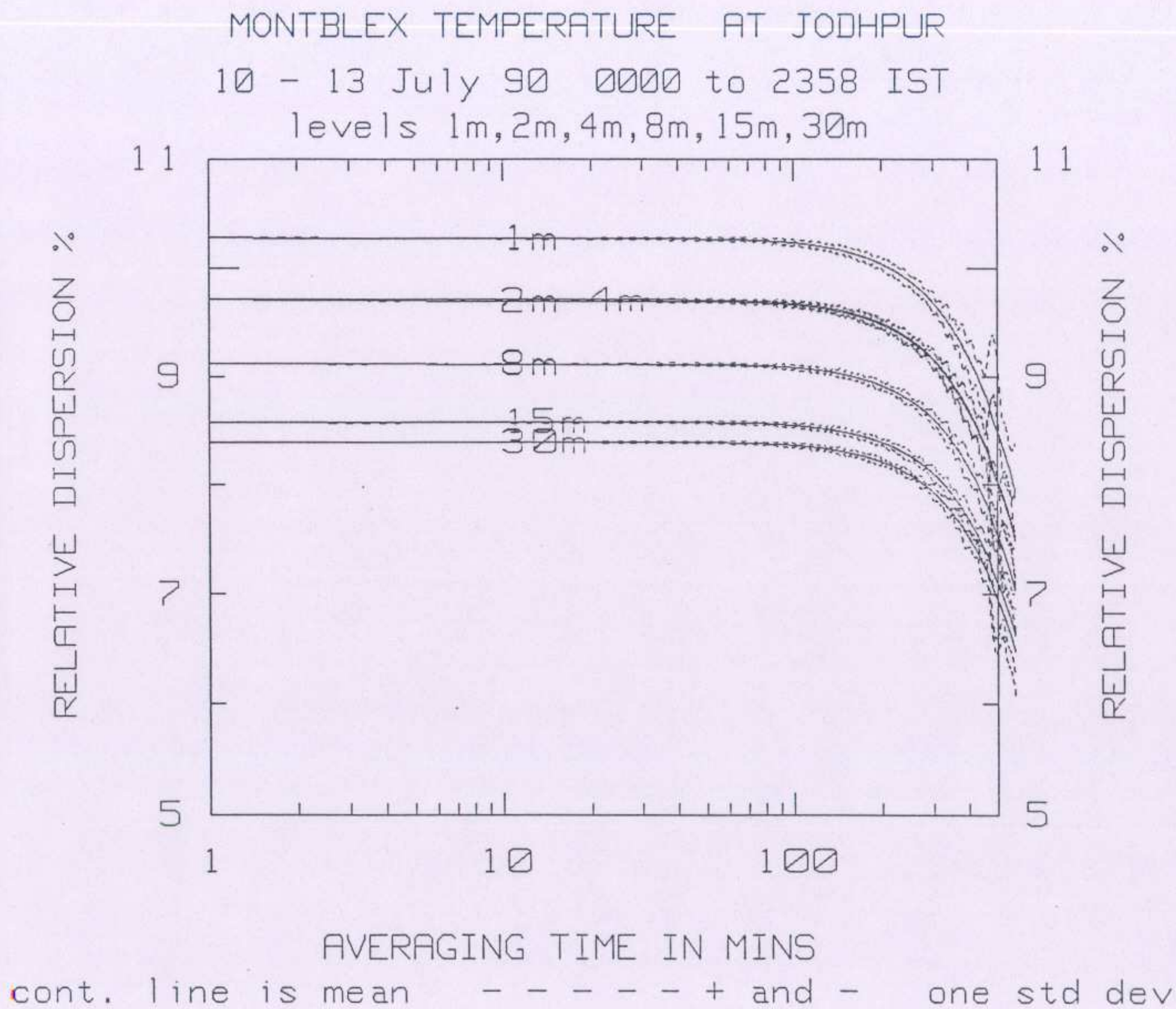

Figure 4 : Same as for Figure 2 for Jodhpur

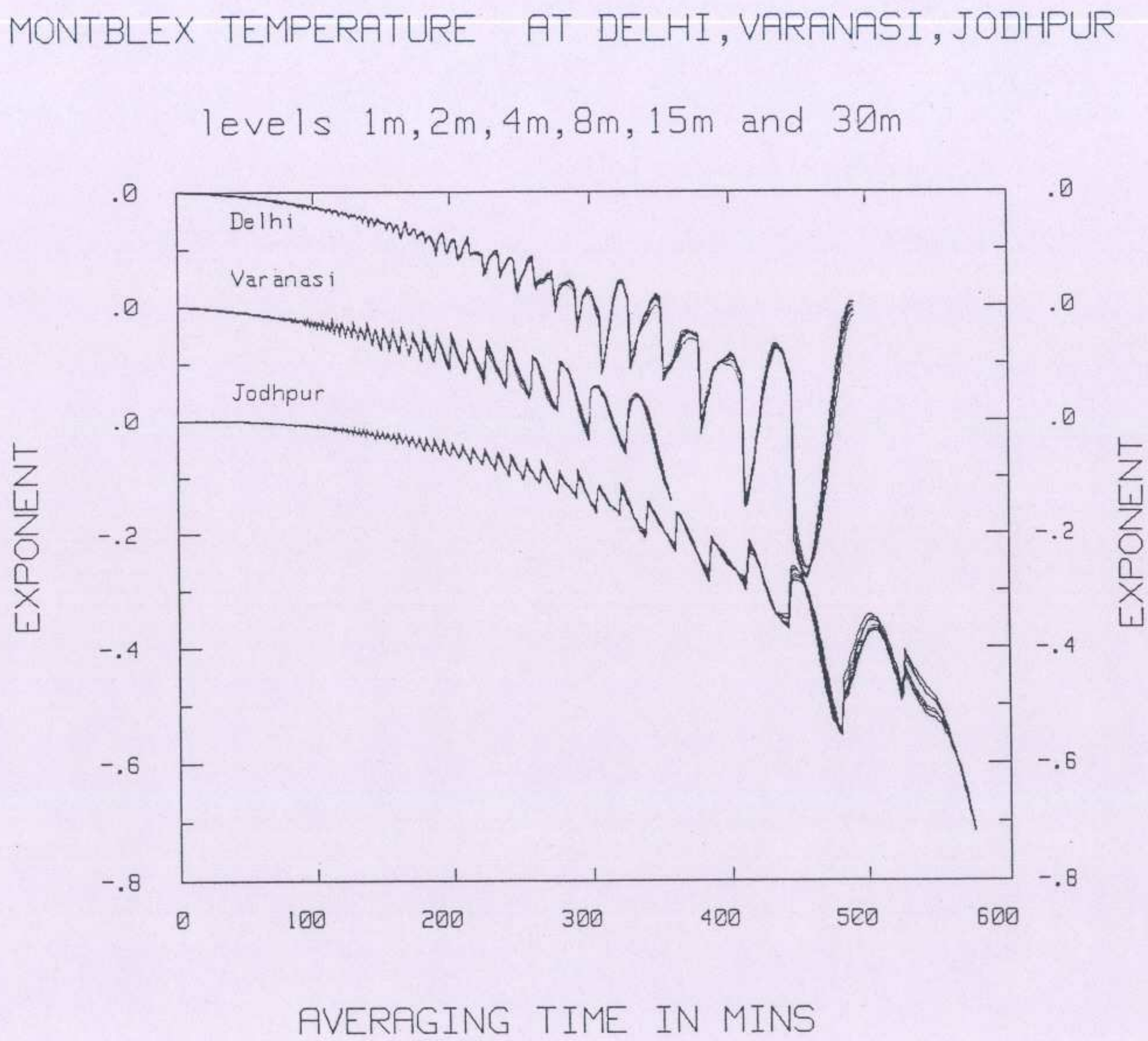

Figure 5 : Variation of exponent 1-D (Eq.1) with averaging-time interval for Delhi, Varanasi and Jodhpur for all available levels.