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UNIVERSAL SPECTRUM FOR SUNSPOT NUMBER VARIABILITY

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Universal Spectrum for Sunspot Number Variability

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1. Introduction

The nonlinear variability of real world dynamical systems such as atmospheric flows, fluid flows, solar activity etc., are conventionally modeled using *Newtonian* continuum dynamics formulated in terms of nonlinear partial differential equations. Finite precision computer realizations of such continuum dynamical systems incorporate the following uncertainties and errors. (1) The continuum dynamical system is modeled as a discrete dynamical system because of finite step size. (2) Exact number representation is not possible at the data input stage itself in the binary form used for number representation. (3) Model approximations and uncertainties in initial conditions. (4) Round-off error of finite precision computer arithmetic amplifies exponentially. The above mentioned uncertainties result in chaotic solutions now identified as deterministic chaos, a field of intensive research in all areas of science (Gleick, 1987). The fidelity of computed model solutions is uncertain in the absence of analytical solutions (Palmore and Herring 1990, Corless 1991). Mary Selvam (1993a) has shown that round-off error of finite precision computations approximately doubles on an average for each iteration of iterative computations. Long-term numerical intergretation schemes used in numerical models incorporate thousands of iterative computations and therefore give unrealistic solutions because of round-off error propagation into the main stream computations. Predictability studies therefore require alternative formulations of governing equations with analytical solutions. The formulation of a model requires identification of essential features of the behavior, finding correlations between the features and then representing them by equations. To understand the behavior of a system it is necessary to keep mathematics in a subservient role to visualize the system itself (Freeman 1990). Long-range spatiotemporal correlations manifested as the self-similar *fractal* geometry to the spatial pattern concomitant with inverse power-law form for the power spectrum of temporal fluctuations is ubiquitous to extended dynamical systems (real) and are recently identified as signatures of **self-organized criticality** (Bak et al. 1988). A recently developed cell dynamical system model for atmospheric flows (Mary Selvam 1990, Mary Selvam et al., 1992) applicable to all dynamical systems predicts such long-range spatio-temporal correlations as intrinsic to quantum like mechanics governing flow dynamics. The model predicts unique quantification for self-organized criticality, namely the inverse power-law form of the statistical normal distribution for the power spectrum of temporal fluctuations.

In this paper a summary of the above cell dynamical system is given in Section 2 followed by results of continuous periodogram analysis of different sets of 5-year means of annual sunspot number for the 275 years period 1700-1974. The spectra follow the universal inverse power-law form of the statistical normal distribution in agreement with model predictions.

2. Cell dynamical system model

In summary, (Mary Selvam 1990, 1993a,b, 1994, 1996; Mary Selvam et al., 1992, 1994, 1995, 1996; Mary Selvam and Radhamani 1994, 1995; Mary Selvam and Joshi 1995) the mean flow at the planetary atmospheric boundary layer (ABL) possesses an inherent upward momentum flux of surface frictional origin. This upward momentum flux is progressively amplified by the exponential decrease of atmospheric density with height coupled with latent heat released during microscale fractional condensation by deliquescence on hygroscopic nuclei even in an unsaturated environment. This mean upward momentum flux generates helical vortex roll (or large eddy) circulations in the ABL seen as cloud rows/streets, mesoscale cloud clusters (MCC) in the global cloud cover pattern. Townsend (1956) has shown that large eddy circulations form as the spatial integration of enclosed turbulent eddies intrinsic to any turbulent shear flow. The relationship between the root mean square (r.m.s) circulation speeds **W** and w_* of large and turbulent eddies of respective radii R and r is then obtained as

$$\mathbf{W}^2 = \frac{\mathbf{2}}{\pi}\frac{\mathbf{r}}{\mathbf{R}} w_*^2 \qquad (1)$$

A continuum of progressively larger eddies grow from the turbulence scale at the planetary surface with two way ordered energy feedback between the larger and smaller scales as given in Eq.(1). Large eddy is visualized as the envelope of enclosed turbulent eddies and large eddy growth occurs in unit length step increments equal to the turbulent eddy fluctuation length **r**. Such a concept is analogous to the non-deterministic cellular automata computational technique where cell dynamical system growth occurs in unit length step increments during unit intervals of time (Oona and Puri 1988). Also, the concept of large eddy growth in length step increments equal to **r**, the turbulence length scale, i.e., length scale doubling is identified as the universal period doubling route to chaos eddy growth process. The large eddy of radius $\mathbf{R_n}$ at the nth stage of growth goes to form the internal circulation for the next stage, i.e., (n+1)th stage of large eddy growth. Such a concept, leads as a natural consequence, to the result that the successive values of the radii **R** and the r.m.s eddy circulation speeds **W** follow the *Fibonnaci* mathematical number series, i.e., the ratio of the successive values of **R** (or **W**) to τ the golden mean = $(1+\sqrt{5})/2 = 1.618$.

The overall envelope of the large eddy traces a logarithmic spiral with the quasi-periodic *Penrose* tiling pattern for the internal structure. Atmospheric circulation structure therefore consists of a nested continuum of vortex roll circulation (vortices within vortices) with a two way ordered energy flow between the larger and smaller scales. Such a concept is in agreement with the observed long range spatiotemporal correlations in atmospheric flow patterns.

The cell dynamical system model also predicts the following logarithmic wind profile relationship in the ABL

$$\mathbf{W} = (\mathbf{w_*}/\mathbf{k}) \ln \mathbf{Z} \qquad (2)$$

where the *Von Karman*'s constant **k** is identified as the universal constant for deterministic chaos and represents the steady state fractional volume dilution of large eddy by turbulent eddy fluctuations. The value of **k** is shown to be equal to $1/\tau^2$ (=*0.382*) where τ is the golden mean The model predicted value of **k** is in agreement with observed values. Since the successive values of the eddy radii follow the *Fibonacci* mathematical number series, the length scale ratio **Z** for the nth step of eddy growth is equal to $\mathbf{Z_n} = \mathbf{R_n}/\mathbf{r} = \tau^n$. Further, **W** represents the standard deviation of eddy fluctuations, since **W** is computed as the instantaneous r.m.s eddy perturbation amplitude with reference to the earlier step of eddy growth. For two successive stages of eddy growth starting from primary perturbation $\mathbf{w_*}$, the ratio of the standard deviations $\mathbf{W_{n+1}}$ and $\mathbf{W_n}$ is given from Eq.(2) as **(n+1)/n**.

Denoting by σ, the standard deviation of eddy fluctuations at the reference level (*n=1*), the standard deviations of eddy fluctuations for successive stages of eddy growth are given as integer multiples of σ , i.e., σ , 2σ , 3σ etc.

The concept of large eddy formation as the spatial integration of enclosed turbulent eddies leads as a natural consequence to the result that the atmospheric eddy energy spectrum follows normal distribution characteristics, i.e., the square of eddy amplitude represents the eddy probability density. Incidentally, the above result, namely that the additive amplitudes of eddies when squared represent the eddy probability density is inherent to the observed sub-atomic dynamics of quantum systems and is accepted as an *ad-hoc* assumption in quantum mechanics (Maddox 1988).

Atmospheric flow structure therefore follows quantum-like mechanical laws where the eddy energy spectrum represents the eddy probability density and the apparent wave-particle duality is physically consistent in the context of atmospheric flows since the bimodal (formation and dissipation) form for energy manifestation in the bi-directional energy flow intrinsic to eddy circulation results in the formation of clouds in updrafts and dissipation of clouds in downdrafts.

The conventional power spectrum plotted as the variance versus the frequency in log-log scale will now represent the eddy probability density on logarithmic scale versus the standard deviation of the eddy fluctuations on linear scale since the logarithm of the eddy wavelength represents the standard deviation i.e, the r.m.s value of eddy fluctuations (Eq.2). The r.m.s value of the eddy fluctuations can be represented in terms of statistical normal distribution as follows. A normalized standard deviation **t**=*0* corresponds to cumulative percentage probability density equal to *50* for the mean value of the distribution. Since the logarithm of the wavelength represents the r.m.s value of eddy fluctuations the normalized standard deviation **t** is defined for the eddy energy distribution as

$$t = (\log L / \log T_{50}) - 1$$

where **L** is the period in years and T_{50} is the period up to which the cumulative percentage contribution to total variance is equal to *50* and $t=0$. Log T_{50} also represents the mean value for the r.m.s eddy fluctuations and is consistent with the concept of the mean level represented by r.m.s eddy fluctuations.

In the following section it is shown that continuous periodogram analyses of the sunspot number data exhibit the signatures of quantum-like mechanics, namely, the cumulative percentage contribution to total variance, computed starting rom the high frequency end of the spectrum, follows the cumulative normal distribution.

3. Data and analysis

Annual mean sunspot number for the *275* years *1700-1974* (SCOSTEP, 1975) was used for this study. The broadband power spectrum of sunspot number time series can be computed accurately by an elementary but very powerful method of analysis developed by Jenkinson (1977) which provides a quasi-continuous form of the classical periodogram allowing systematic allocation of the total variance and degrees of freedom of the data series to logarithmically spaced elements of the frequency range (*0.5, 0*). The periodogram is constructed for a fixed set of *10000*(**m**) periodicities which increase geometrically as $L_m = 2 \exp (Cm)$ where $C=.001$ and $m=0,1,2.....m$. The data series Y_t for the **N** data points was used. The periodogram estimates the set of $A_m \cos (2\pi \nu_m t - \varphi_m)$ where A_m, ν_m and φ_m denote respectively the amplitude, frequency and phase angle for the mth periodicitiy. The cumulative percentage contribution to total variance was computed starting from the high frequency side of the spectrum. The period T_{50} at which *50%* contribution to total variance occurs is taken as reference and the normalized standard deviation t_m values are computed as

$$t_m = (\log L_m / \log T_{50})-1$$

The cumulative percentage contribution to total variance and the corresponding **t** values are plotted as continuous lines in Fig.1 for the sunspot number. The cumulative normal probability density distribution corresponding to the normalized standard deviation **t** values are shown as crosses in Fig.1. It is seen that the cumulative percentage contribution to total variance closely follows the cumulative normal probability density distribution. The "*goodness of fit*" was tested using the standard statistical *chi-square test* (Spiegel 1961). The short horizontal lines in the lower part of Fig.1 indicate the lower limit above which the fit is good at *95%* confidence level.

It is seen from Fig.1 that the power spectra of *5*-year means of sunspot number exhibit universal characteristics of the statistical normal distribution.

4. Conclusion

The cell dynamical system model summarized in this paper provides an alternative exact method for quantifying and predicting the nonlinear variability of *5*-year mean sunspot number time series. It is shown that the power spectra of sunspot number is the same as the normal probability density distribution. The normal probability distribution follows the inverse power-law form t^{-B} where B, the exponent approaches *1* for small values of **t.** It is therefore consistent that the power spectra of sunspot number follow t^{-B} power-law which is identified as the temporal signature of **deterministic chaos** or **self-organized criticality** in atmospheric flows (Mary Selvam, 1990, Mary Selvam et al., 1992) applicable to fluid flows in general. Temporal (years) fluctuations in sunspot number contribute to form a self-organized unique pattern, namely, that of the statistical normal distribution with the square of the eddy amplitude representing the normal probability density corresponding to the normalized standard deviation **t** equal to [$\log L / \log T_{50}$)-1] where **L** is the period length in years and T_{50} the period up to which the cumulative percentage contribution to total variance is equal to *50* and **t**=*0*. Quantification of the non-linear variability of sunspot number in terms of the unique and universal characteristics of the statistical normal distribution implies predictability of the total pattern fluctuations.

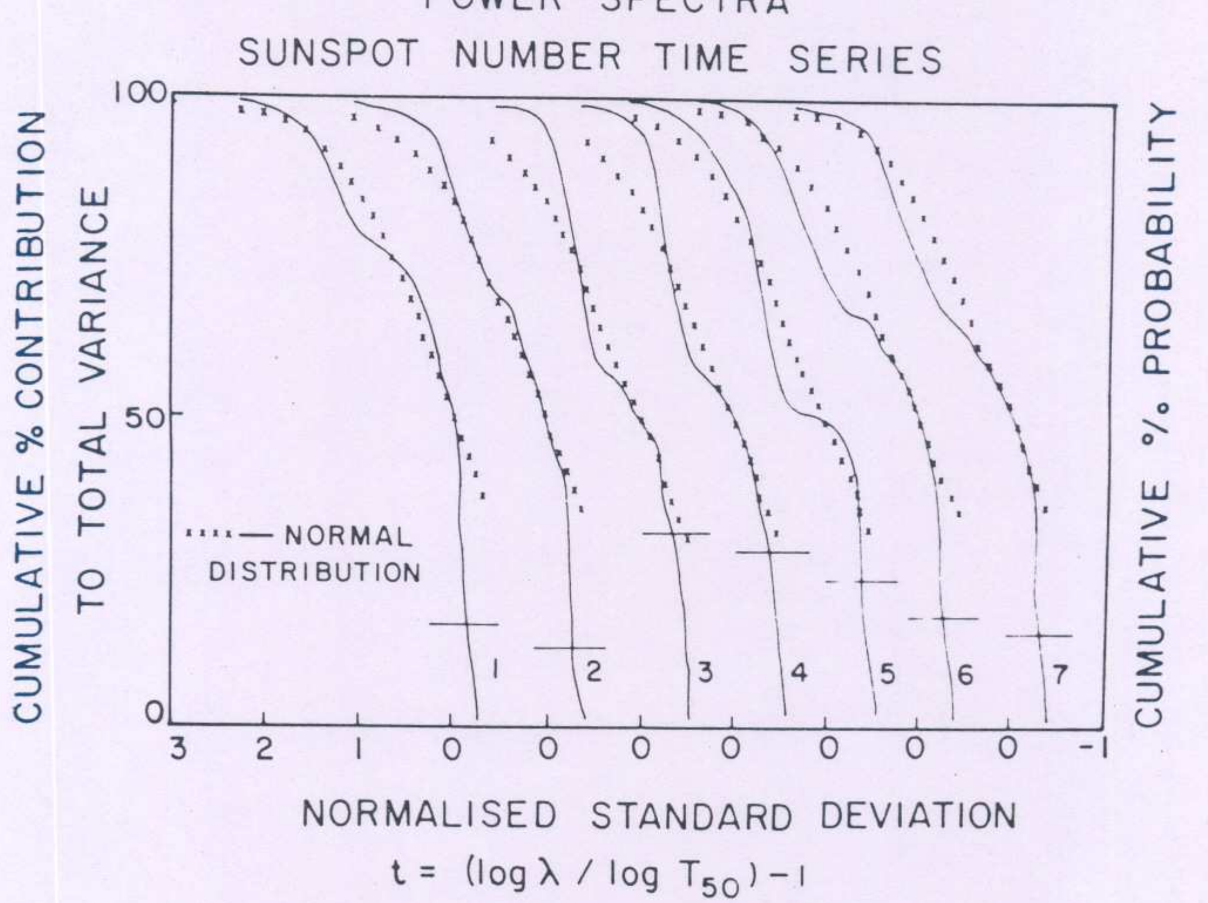

Figure 1. Power Spectra of 5 year means of sunspot number time series. The continuous lines represent the cumulative percentage contribution to total variance corresponding to the normalized standard deviation t values. The cumulative normal distribution corresponding to t values are plotted as crosses. The short horizontal lines in the lower part of the figure represent the limit above which the spectra are the same as the normal distribution as determined by the chi-square test at 95% of confidence level. The number 1 to 7 in the lower part of the figure refer to the following time periods.

(1) 1725-1799 (2) 1730-1804 (3) 1780-1854 (4) 1785-1859
(5) 1724-1827 (6) 1784-1884 (7) 1780-1879

REFERENCES

Bak, P.C. C.Tang and K. Wiesenfeld (1988) Self organized criticality. *Phy. Rev. A* **38**, 364-374.

Corless, R.M. C. Essex and M.A.H. Nerenberg (1991) Numerical methods can suppress chaos, *Phys. Lett. A* **157**, 27-36.

Freeman, G. R. (1990) KNP89: Kinetics of Nonhomogenous Processes (KNP) and nonlinear dynamics, *Can. J. Phys.*, **68**, 655-659.

Gleick, J. (1987) *Chaos: making a new science.* (Viking Press Inc., New York) pp 1-252.

Jenkinson, A.F. (1977) *A powerful elementary method of spectral analysis for use with monthly, seasonal or annual meteorological time series,* (U.K. Meteorol. Office) Met. O 13 Branch Memorandum No. 57, 1 23.

Maddox, J. (1988) License to slang Copenhagen?, *Nature* **332**, 581.

Mary Selvam, A. (1990) Deterministic chaos, fractals and quantum like mechanics in atmospheric flows. *Can. J. Phys.* **68**, 831-841.

Mary Selvam, A., J.S. Pethkar and M.K. Kulkarni (1992) Signatures of a universal spectrum for atmospheric interannual variability in rainfall time series over the Indian region, *Int'l. J. Climatol* **12** 137-152.

Mary Selvam, A. (1993a) Universal quantification for deterministic chaos in dynamical systems, *Applied Math. Modelling*, **17**, 642-649.

Mary Selvam, A.(1993b) A universal spectrum for interannual variability of monsoon rainfall over India. *Adv. Atmos Sci.* **10**, 221-226.

Mary Selvam, A., and Radhamani, M.(1994) Signature of a universal spectrum for nonlinear variability in daily columnar total ozone content. *Adv. Atmos. Sci.* , **11(3)** , 335-342.

Mary Selvam, A., Joshi R.R., and Vijaykumar,R.(1994) Self organized criticality in *COADS* temperature time series : Implication for climate prediction. In *Global Climate Change:Science,Policy and Mitigation Strategies*, Mathai,C.V.,and Stensland,G.(Eds), Proc.of the Air and Waste Management Association, Int'l. Specialty Conf.,Phoenix,Arizona April ,196-205.

Mary Selvam, A. (1994) The physics of deterministic chaos : Implications for global climate model predictions . In *Global Climate Change : Science, Policy and Mitigation Strategies*, Mathai,C.V.,and Stensland,G.(Eds) ,Proc.of the Air and Waste Management Association, Int'l. Specialty Conf.,Phoenix, Arizona April , 412 - 417.

Mary Selvam , A., Pethkar, J. S., and Kulkarni ,M.K.(1995)Some unique characteristics of atmospheric interannual variability in rainfall time series over India and the United Kingdom. *Adv. Atmos. Sci.*, **12(3)** ,377 - 385.

Mary Selvam, A., and Radhamani , M.(1995) Universal spectrum for short period(days) variability in atmospheric total ozone.*Mausam* , **46(3)** , 297 - 302.

Mary Selvam, A., and Joshi R.R. (1995) Universal spectrum for interannual variability in COADS global air and sea surface temperatures. *Int'l. J.Climatol.*,**15** , 613 -624.

Mary Selvam, A., Pethkar J.S., Kulkarni M.K., and Vijayakumar R.(1996) Signatures of a universal spectrum for atmospheric interannual variability in COADS surface pressure time series. *Int'l. J. Climatol.*, **16** , 1 - 11.

Mary Selvam, A.(1996) Quasi crystalline pattern formation in fluid substrates and phyllotaxis.In *Symmetry in Plants*, Barabe,D., and Jean , R.V. (Eds.), No.4.,World Scientific Series in Mathematical Biology and Medicine, World Scientific, Singapore, (in Press).

Oona, Y. and S. Puri (1988) Study of phase separation dynamics by use of cell dynamical systems. I Modelling. *Phys. Rev. A* **38(1)**, 434-453.

Palmore, J. and C. Herring (1990) Computer arithmetic, chaos and fractals. *Physica* D 42 ,99-110.

SCOSTEP (1975) *Solar Terrestrial Physics and Meteorology*: A working document. Nat'l Acad. Sci., Washington, D.C. USA.

Spiegel, M.R. (1961) *Statistics*, (McGraw-Hill, New York), 359 pp.

Townsend, A.A. (1956) *The Structure of Turbulent Shear Flow* Cambridge University Press, U.K. pp. 130.

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