

ISSN 0252-1075
Research Report No. RR-080

Contributions from
Indian Institute of Tropical Meteorology

COMPUTATION OF THERMAL PROPERTIES OF
SURFACE SOIL FROM ENERGY BALANCE
EQUATION USING FORCE-RESTORE METHOD

by

S. SINHA



PUNE - 411 008
INDIA

FEBRUARY 1999

CONTENTS

	SECTIONS	PAGE NOS.
	Abstract	1
1.	Introduction	1
2.	Physics of the model	2
3.	Computational Methodology (Method A) (a) to (c)	3
4.	Solution of Energy Balance equation	5
5.	Computational Methodology (Method B)	5
6.	Results and Discussion	6
7.	Conclusion	7
	Acknowledgement and References	8
	Legeneds of the Figures	9
	Figures (1) to (5)	5 pages

Computation of thermal properties of surface soil from energy balance equation using force-restore method

by

Subroto Sinha

(Indian Institute of Tropical Meteorology, Pashan, Pune 411008, India)

Abstract

In this study, the soil temperature data at different depths, recorded at regular intervals during a 24 – hour period, is utilised to develop and test two methods of computing various soil properties, such as, “thermal diffusivity”, “damping depth” and “thermal conductivity”, which are then used to compute the soil heat flux and soil temperature at the surface, applying the force-restore method. In the first method, only the diurnal component of the surface temperature variation is assumed, whereas in the second method, the other harmonic components of the surface temperature variations are also considered. The surface temperatures computed by the second method are found to be relatively in good agreement with those observed by a sensitive sensor at Anand (Gujrat), during the pilot phase of the Land – Surface – Processes – Field – Experiment.

1. Introduction

The force-restore method has been widely used to simulate the diurnal variations of ground temperature, based on the heat budget of a slab of unspecified thickness, which is heated from above by a single component periodic energy source and supported from below by a substrata at constant temperature. This method is particularly useful for computing the soil heat flux in the surface energy balance equation. Normally, the computation of the soil heat flux requires the time-dependent solution for soil temperatures within six or more layers of soil for reasonably good accuracy. The force-restore method dispenses with the requirement of solving a complex soil-temperature model in order to arrive at the equilibrium surface temperature.

The data from the micrometeorological observing tower, which was set up at Anand for the ‘Land-surface Processes Experiment’, was used for this study. The data for 26 April 1995, which had clear sky, and 27 April 1995, which was cloudy, were selected. Since the humidity data was not available, the latent heat flux was taken as zero in the energy balance equation.

2. Physics of the model

It is assumed that the slab is either infinitely thin or has a temperature θ_g , which is independent of depth, so that the rate of change of temperature of the slab is dictated by the imbalance of fluxes between a forcing term ‘H’ and a restoring term ‘Y’. Thus, the predictive equation for surface soil temperature can be written as :

$$C_g \frac{\partial \theta_g}{\partial t} = H - Y \quad (2.1)$$

where $C_g = C_s \Delta Z$, C_s being the volumetric heat capacity of the soil and ΔZ is the thickness of the slab. C_g is also called the "thermal inertia". The forcing term 'H' is the ground heat flux G_0 , obtained from the energy balance equation and the restoring term is interpreted as the heat flux across the lower boundary of the slab, which is transmitted in total to the upper boundary. We get the forcing term as :

$$H = G_0 = R_N - H_0 - \lambda E_0 \quad (2.2)$$

where R_N , H_0 , λE_0 are the net radiation, sensible heat flux and latent heat flux at the surface, λ being the latent heat of vapourisation

The restoring term 'Y' is given by : $Y = -\mu (\theta_g - \bar{\theta})$ (2.3)
 where ' μ ' is the coefficient of heat transfer and $\bar{\theta}$ is the mean substrata constant temperature.

We consider a thin layer of soil of thickness, ΔZ . The time rate of change in the temperature of the soil layer, neglecting the horizontal conduction of heat in the soil, is given by :

$$\rho c \Delta Z \frac{\partial \theta_g}{\partial t} = \frac{\partial G}{\partial Z} \Delta Z \quad (2.4)$$

where ' ρ ' is the density of the soil and ' c ' is its specific heat ($J.Kg^{-1}.K^{-1}$). The R.H.S. of Eq.(2.1) represents the difference in the heat fluxes into and out of the layer. If the soil layer is very close to the surface, then the R.H.S. can be expressed as :

$$\frac{\partial G}{\partial Z} = G_0 - G_1$$

where G_1 is the heat flux at depth ΔZ from the surface. Eq.(2.1) now becomes :

$$C_s \frac{\partial \theta_g}{\partial t} = G_0 - G_1 \quad (2.5)$$

where $C_s = \rho c$, is the volumetric heat capacity of the soil ($J.m^{-3}.K^{-1}$). The subsurface heat flux at any depth 'Z' can be evaluated from Fourier's law for heat conduction in a homogeneous body, as :

$$G = -k_s (\partial \theta_g / \partial Z) \quad (2.6)$$

Where ' k_s ' is the thermal conductivity of the soil ($W.m^{-1}.K^{-1}$). $k_s = C_s K_s$, where ' K_s ' is the thermal diffusivity ($m^2.s^{-1}$) Substituting in Eq.(2.1), we get :

$$\frac{\partial \theta_g}{\partial t} = K_s \frac{\partial^2 \theta_g}{\partial Z^2} \quad (2.7)$$

In the case of diurnal forcing, we take the upper boundary condition at $Z=0$, as :

$$\theta_g(0,t) = \theta_0(t) + A_0 \sin(\Omega t)$$

where $\Omega=2\pi/P$, 'P' being the time period of the diurnal temperature wave, and θ_0 is the surface temperature at $Z=0$. ($2 A_0$) is the difference between the maximum and minimum temperature during the day. For the lower boundary we have : $\theta_g(\infty,t) = \bar{\theta}$, the deep soil temperature. Combining the two boundary conditions, the solution of Eq.(2.7) is given by :

$$\theta_g(Z,t) = A_0 \exp(-C_1 Z) \sin(\Omega t - C_1 Z) + \bar{\theta} \quad (2.8)$$

We define a "damping depth" (d), as the depth of soil across which the temperature difference is $(1/e)$ times the diurnal amplitude. The constant C_1 , is identified as the reciprocal of the damping depth, i.e. $C_1 = 1/d$. Substituting Eq.(2.8) in Eq.(2.7) we get : $(C_1)^2 = \Omega / (2 K_s)$, or $d = \sqrt{2 K_s / \Omega}$

$$\theta_g(Z,t) = A_0 \exp(-Z/d) \sin(\Omega t - Z/d) + \bar{\theta} \quad (2.9)$$

The soil heat flux at a depth 'Z' can be written from Eq.(2.6) as :

$$G(Z,t) = -(\Omega C_s k_s / 2)^{1/2} [\theta_g(Z,t) - \bar{\theta} + 1/\Omega (\partial \theta_g / \partial t)] \quad (2.10)$$

$$G_0 = G(0,t) \text{ and } \theta_g(0,t) = (1/\delta) \int_0^\delta \theta_g(Z,t) dZ, \text{ when } \Delta Z \text{ is very small and equal to } \delta$$

Substituting in Eq.(2.10), we get :

$$\alpha \frac{\partial \theta_g}{\partial t} = \frac{2}{C_s d} G_0 - \Omega (\theta_g - \bar{\theta}) \quad (2.11)$$

Here $\alpha = (1 + 2\delta/d)$. This is the force-restore formulation.

3. Computational Methodology (Method A)

Eq.(2.11) contains four unknowns, viz., (a) d (b) G_0 (c) θ_g (d) $\bar{\theta}$. We now give the methodology of evaluating these unknowns from the observed temperatures.

(a) **Damping depth 'd' and $\bar{\theta}$** : For evaluating 'd', we need the observed temperatures ' θ_{on} ' at a certain depth ' Z_1 ', at time 'n'. The computed temperature at that depth and time is given by Eq.(2.7) as :

$$\theta_g(Z,n) = \Delta T_1 \sin(\Omega t - \phi - Z_1/d) + \bar{\theta} \quad (3.1)$$

where $\Delta T_1 = A_0 \exp(-Z_1/d)$ and ' ϕ ' is the phase of the temperature wave. Our aim is to make the error between the computed and observed temperature at a depth Z_1 , as small as possible, i.e. to make the quantity $\sum \{ \theta_g(Z,n) - \theta_{on} \}^2$ a minima. Denoting the soil temperature $\theta_g(Z,n)$ by θ_{gn} , we make the normalized quantity given by :

$\sum \{ \theta_{gn} / \theta_{on} - 1 \}^2$, summed over all times 'n', a minima. It is possible to express the above parameter as follows :

$$\sum \{ \theta_{gn} / \theta_{on} - 1 \}^2 = A + B(X_2)^2 + C(X_1)^2 - DX_1X_2 + 2EX_2 - 2FX_1 \quad (3.2)$$

where $X_1 = \Delta T_1 \sin(Z_1/d)$; $X_2 = \Delta T_1 \cos(Z_1/d)$; $Z_1/d = \tan^{-1}(X_1/X_2)$ (3.3)

$$\begin{aligned} A &= \sum (\bar{\theta}/\theta_{on} - 1)^2; & B &= \sum \{ \sin^2(\Omega t_n - \phi) / (\theta_{on})^2 \} \\ C &= \sum \{ \cos^2(\Omega t_n - \phi) / (\theta_{on})^2 \}; & D &= \sum [\sin\{2(\Omega t_n - \phi)\} / (\theta_{on})^2] \\ E &= \sum (\bar{\theta}/\theta_{on} - 1) \{ \sin(\Omega t_n - \phi) / \theta_{on} \}; & F &= \sum (\bar{\theta}/\theta_{on} - 1) \{ \cos(\Omega t_n - \phi) / \theta_{on} \} \end{aligned} \quad (3.4)$$

The problem now reduces to finding the values of X_1 and X_2 in Eq.(3.2), such that the R.H.S. becomes minimum. The simplex method was applied in two stages, viz.

- (i) The minimum value of R.H.S. is obtained from a set of values of $\bar{\theta}$, keeping X_1 and X_2 constant.
- (ii) The value of $\bar{\theta}$ for which the R.H.S. of Eq.(3.2) is minimum, gave the appropriate value of $\bar{\theta}$. With this value of $\bar{\theta}$, the appropriate values of X_1 and X_2 are obtained for which the R.H.S. of Eq.(3.2) is a minima.

Having obtained the values of X_1 and X_2 , the value of the damping depth can be obtained from Eq.(3.3).

(b) **Soil heat flux G_{0n}** : From Eq.(2.10) we get:

$$G_{0n} = -(\Omega C_s k_s/2)^{1/2} [\theta_{gn} - \bar{\theta} + 1/\Omega (\partial \theta_{gn} / \partial t)]$$

From Eq.(3.1), $\theta_{gn} - \bar{\theta} = \Delta T_1 \sin(\Omega t_n - \phi)$, for $Z_1=0$

$$\partial \theta_{gn} / \partial t = \Omega \Delta T_1 \cos(\Omega t_n - \phi)$$

$$\therefore G_{0n} = (\Omega C_s k_s/2)^{1/2} \Delta T_1 [\sin(\Omega t_n - \phi) + \cos(\Omega t_n - \phi)] \quad (3.5)$$

(c) **Surface temperature, θ_{gn}** : From Eq.(3.1) we get :

$$\theta_{gn} = \Delta T_1 \sin(\Omega t_n - \phi - Z_1/d) + \bar{\theta} \quad (3.6)$$

Net Radiation, R_N : Having computed the soil parameters, we now focus our attention on the net radiation, which is an important parameter in the energy balance equation. The net radiation is defined as follows:

$$R_N = (1-a) S T_k \sin \psi - \sigma [\epsilon_s (\theta_g)^4 - \epsilon_a (\theta_a)^4] \quad (3.7)$$

Where 'a' is the soil albedo, 'S' is the solar constant, 'T_k' is the net sky transmissivity, 'ψ' is the solar elevation angle, 'ε_s' is the emissivity of the surface layer of soil, 'ε_a' is the emissivity of air, 'θ_a' is the air temperature close to the surface, and 'θ_g' is the surface temperature. The solar elevation angle is given as :

$$\sin \psi = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H \quad (3.8)$$

Where 'φ' is the latitude, 'δ' is the declination of the sun, and 'H' is the hour angle.

Sensible heat flux H_0 : The sensible heat flux at the surface is given by :

$$H_0 = -\rho c_p C_H u_a (\theta_g - \theta_a) \quad (3.9)$$

Where ' ρ ' is the density of air, ' c_p ' is the specific heat of air under constant pressure, ' C_H ' is the transfer coefficient for heat, and ' u_a ' is the wind speed at a height of 1m. above the surface.

4. Solution of Energy Balance equation

The energy balance equation at the surface is given by :

$$R_N - G_0 = H_0 + \lambda E_0 \quad (4.1)$$

R_N and H_0 are computed from Eqs.(3.7) and (3.9) respectively, in terms of surface temperature θ_g and air temperature θ_a . The first guess value of the surface temperature is computed from Eq.(3.6) and a guess value is taken for the air temperature. The ground heat flux is obtained from Eq.(3.5). The root of Eq.(4.1) is obtained by the simplex method, which gave the appropriate values for θ_g and θ_a .

5. Computational Methodology (Method B)

In this method, we consider the following parameters :

- (a) $T_m(Z, t_0)$ is the maximum temperature at a depth ' Z ', occurring at time ' t_0 '. Then an equation analogous to Eq.(3.1) can be written as :

$$T_m(Z, t_0) = \bar{T} + \Delta T_0 \exp(-Z/d) \quad (5.1)$$

Where ΔT_0 is the diurnal temperature amplitude and \bar{T} is the mean temperature

- (b) $T_n(Z, t_n)$ is the temperature at a depth ' Z ' and time ' t_n '. It is represented in terms of all its harmonic components as follows :

$$T_n(Z, t_n) = \bar{T} + \sum \Delta T_k \sin(\Omega_k t_n - \phi_k - Z/d_k) \quad (5.2)$$

Where ΔT_k is the amplitude of the k -th harmonic component, ϕ_k is its phase and d_k is its 'damping depth'. The summation is over all the harmonic components ' k '. The maximum temperature $T_m(Z, t_0)$ can also be represented by all its harmonic components as follows :

$$T_m(Z, t_0) = \bar{T} + \sum \Delta T_k \sin(\Omega_k t_0 - \phi_k - Z/d_k) \quad (5.3)$$

$$(c) Y_n = (T_m / \bar{T}) - (T_n / \bar{T})$$

- (d) $X_n = (T_{om} / \bar{T}) - (T_{on} / \bar{T})$, where T_{om} and T_{on} are the maximum observed temperature and the observed temperature at time t_n , respectively, at depth Z .

Thus we have two time series, X_n and Y_n . The computed time series Y_n can be expressed as follows :

$$Y_n = (2/\bar{T}) \sum (X_2)_k \cos\{\Omega_k(t_n + t_m)/2 - \phi_k\} \sin\{\Omega_k(t_n - t_m)/2\} + (2/\bar{T}) \sum (X_1)_k \sin\{\Omega_k(t_n + t_m)/2\} \sin\{\Omega_k(t_m - t_n)/2\} \quad (5.4)$$

where $(X_1)_k = \Delta T_k \sin(Z/d_k)$ and $(X_2)_k = \Delta T_k \cos(Z/d_k)$

The three unknown parameters are X_1 , X_2 and \bar{T} , for each of the harmonic components. The time series Y_n is correlated with the time series X_n and the correlation coefficient obtained as follows :

$$r^2 = \frac{[\sum (Y_n - \bar{Y})(X_n - \bar{X})]^2}{[\sum (Y_n - \bar{Y})\sum (X_n - \bar{X})]^2} \quad (5.5)$$

where 'r' is the correlation coefficient and the summation is over all the time intervals for a particular harmonic component.

We use the simplex method to evaluate X_1 , X_2 and \bar{T} , by minimizing the quantity $(1/r^2)$, i.e. applying the condition that the correlation coefficient is maximum. The damping depth for the k-th harmonic component is obtained from the expression :

$d_k = Z / \tan^{-1}[(X_1)_k/(X_2)_k]$ and the temperature amplitude for the k-th component is given by : $\Delta T_k = [\{(X_1)_k\}^2 + \{(X_2)_k\}^2]^{1/2}$. The temperature amplitude at the surface is given by : $(\Delta T_o)_k = \Delta T_k \exp(Z/d_k)$. The surface temperature can now be evaluated from Eq.(5.2). The remaining procedure is the same as in method A, for each harmonic component.

6. Results and Discussion

Two days' temperature data (26 and 27 April 1995), from the site of the micrometeorological tower at Anand were used to test the methods developed herein. The soil temperatures were recorded at depths varying from the surface to 60 cms. below. Figure 1 shows the amplitudes of the various harmonic components of the temperature wave recorded at a depth of 5 cms., and 20 cms., respectively, on each of the aforesaid days. It may be seen that on 26 April, the amplitude of wave number one was much larger than its amplitude on 27 April. It may be attributed to the clear and cloudy conditions during the respective days. However, the amplitudes of the other components, notably wave numbers 3,5 and 7, were larger on 27 April, suggesting that on a cloudy day the components other than the diurnal, also make significant contributions and should not be neglected. More than 80% of the total contributions to the temperature amplitude, is made by the diurnal and semi-diurnal components.

Figure (2a) shows the simulation of the temperature at the surface and at a depth of 5 cms. for 26 April. The solid line curve represents the simulated temperature, which is computed by considering only the diurnal component, while the curve joining the asterisk represents the simulated temperature computed by considering the first two components, i.e. diurnal and semi-diurnal components. The damping depths for the two cases are indicated in the figure. It is seen that the simulated temperatures are closer to the observed values in the latter case, where two components are considered. Figure (2b) shows the same simulations for 26 April, but at a depth of 5 cms.. In this figure also, it is seen that the simulated temperatures obtained by considering two components are much closer to the observed values.

Figure 3 shows the simulation of the temperatures at the surface and at a depth of 5 cms. for 27 April. Here also the simulated temperatures are closer to the observed

temperatures when two components are considered, but the deviations are larger than those obtained in figure 2.

Figure 4 shows the simulation of temperatures at the surface and at a depth of 5 cms., under the following conditions, viz., (i) All the harmonic components are considered and the resultant damping depth computed and (ii) only the diurnal component is considered, but the same damping depth as computed in (i) is used. It is seen that the simulated temperatures are much closer to the observed values in case (i). Even the simulation done with only the diurnal component deviates less from the observed values as compared with figure (2b), because a more realistic value of the damping depth is used.

Figure 5 shows the computed hourly variations of the different components of the energy balance equation. It may be seen that net radiation, given by the difference between the curves (a) and (b), minus the sensible heat flux, given by curve (d), does not balance the ground heat flux, given by the curve (c). This is because the latent heat flux and the heat storage terms are not considered.

Once the damping depth 'd' is obtained, the thermal diffusivity, 'K_s', of the soil can be computed from the relation: $K_s = d^2 \Omega / 2$. If the specific heat 'c' of the soil, its density and the volumetric soil moisture 'θ_i' is known, then the volumetric heat capacity of the soil 'C_s' can be computed from the relation of Lehtveer and Int (1977) and mentioned in Mihailovic et al (1991): $C_s = \rho (c + 4187 \theta_i)$ (7.1)

Alternatively, if the volumetric heat capacity of the dry soil is known, the volumetric soil moisture can be computed from Eq.(7.1). Assuming the following values for 'c', 'ρ' and 'θ_i': $c = 840 \text{ J.Kg}^{-1}$; $\rho = 1.29 \times 10^3 \text{ Kg.m}^{-3}$; $\theta_i = 0$, we obtain the values of the various soil parameters as :

$$K_s = 8.45 \times 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$$

$$C_s = 1.08 \times 10^6 \text{ J.m}^{-3} \cdot \text{K}^{-1}$$

$$k_s = 0.92 \text{ W.m}^{-1} \cdot \text{K}^{-1}$$

Dickinson (1988), found the typical values for the soil parameters as follows :

Damping depth "d", for a diurnal temperature wave is equal to 0.1m. within a factor of 2 in either direction. The thermal conductivity, k_s , is equal to 1-2 $\text{W.m}^{-1} \cdot \text{K}^{-1}$, for soils with some moisture and the ratio $(k_s/d) \approx 15 \pm 5 \text{ W.m}^{-2} \cdot \text{K}^{-1}$. The values computed in this study agree well with these typical values, taking into consideration the fact that moisture was not considered.

7 Conclusions

The methods described in this paper provide a simple method of computing the surface temperature, which is difficult to measure accurately. It is also shown that the accuracy of the simulated temperature can be improved significantly, by including the other harmonic components of the temperature wave, particularly during cloudy conditions. The use of a more realistic damping depth improves the accuracy of the simulated temperatures. The damping depth increases for the higher components of the temperature wave, indicating more penetrative power for these components. Thus, even though the amplitudes of these components are small, they can penetrate deeper into the soil.

Acknowledgement

The author acknowledges the assistance given by the staff members of the BL & LSPS Division in various stages of the project and the Director of the Institute for providing the necessary facilities.

REFERENCES

Dickinson, R. E. (1988) : The Force – Restore model for surface temperature and its generalizations. *Jour. Of Clim.* (1), pp. 1086-1097

Lin, J. D. (1980) : On the Force – Restore method for prediction of ground surface temperature. *Jour. of Geophy. Res.* (85). Pp3251-3254

Mihailovic, D. T. and F. Acs (1991) : A coupled soil moisture and temperature prediction model. *Jour. App. Meteor.*, (30), pp. 812-822.

Yee, S. Y. K. (1988) : The Force Restore method revisited. *Bound. Layer Meteor.*, (43), pp. 85-90

Legends of the Figures

- Figure (1a)** : Amplitudes of the different components of the observed soil temperature wave at Anand on 26 April 1995, at depths of 5 cms. and 20 cms., respectively
- Figure (1b)** : Same as in figure (1a) for 27 April 1995
- Figure (2a)** : Simulated and observed temperatures over a period of 24 hours, at the surface on 26 April 1995
- Figure (2b)** : Same as in figure (2a), at a depth of 5 cms.
- Figure (3a)** : Simulated and observed temperatures over a period of 24 hours, at the surface on 27 April 1995
- Figure (3b)** : Same as in figure (2a), at a depth of 5 cms.
- Figure (4a)** : Simulated and observed temperatures over a period of 24 hours, at the surface on 27 April 1995, with (i) diurnal component only, but using a more realistic value for damping depth and (ii) all the harmonic components.
- Figure (4b)** : Simulated and observed temperatures over a period of 24 hours, at a depth of 5 cms. on 27 April 1995, with (i) all components but using a constant value for damping depth as used in figure (4a) and (ii) all the harmonic components and computing the values of the damping depths separately for each component
- Figure (4c)** : Same as in figure (4b) with (i) only diurnal component and the value of damping depth appropriate to this component and (ii) only diurnal component but with the value of the damping depth as used in figure (4a)
- Figure 5** : Hourly variations of the different components of the energy balance equation

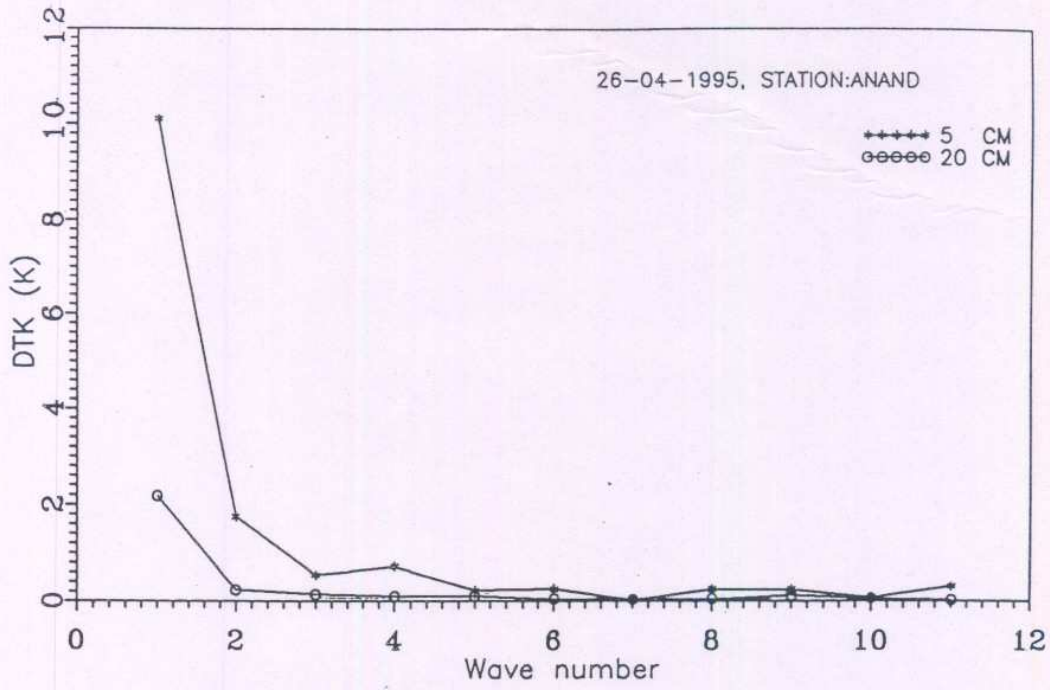


Fig. (1a)

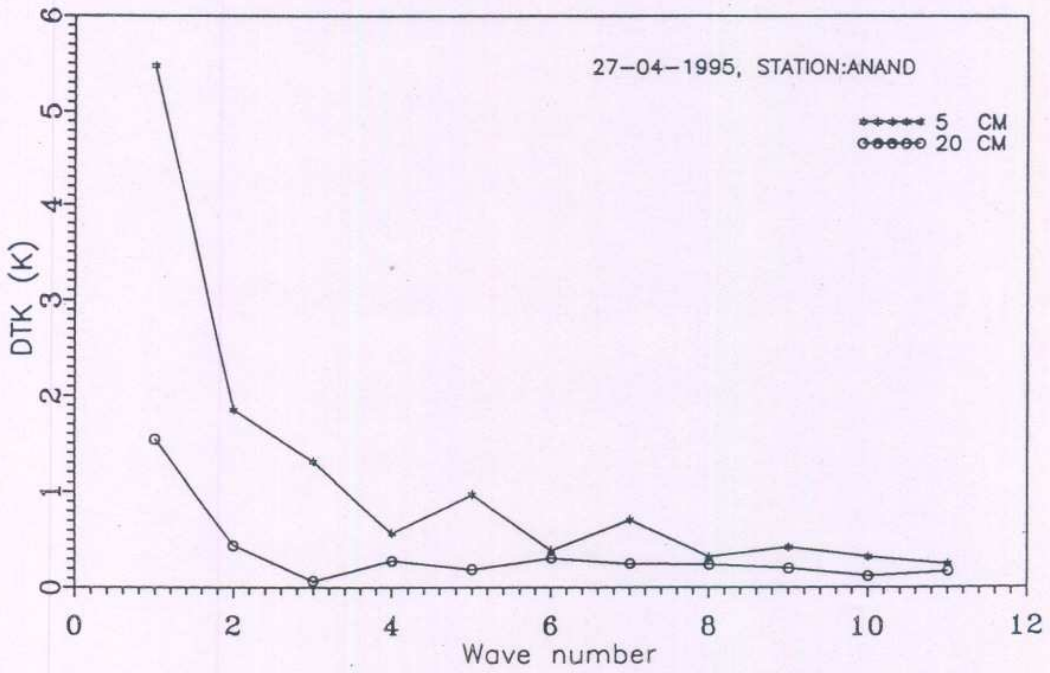


Fig. (1b)

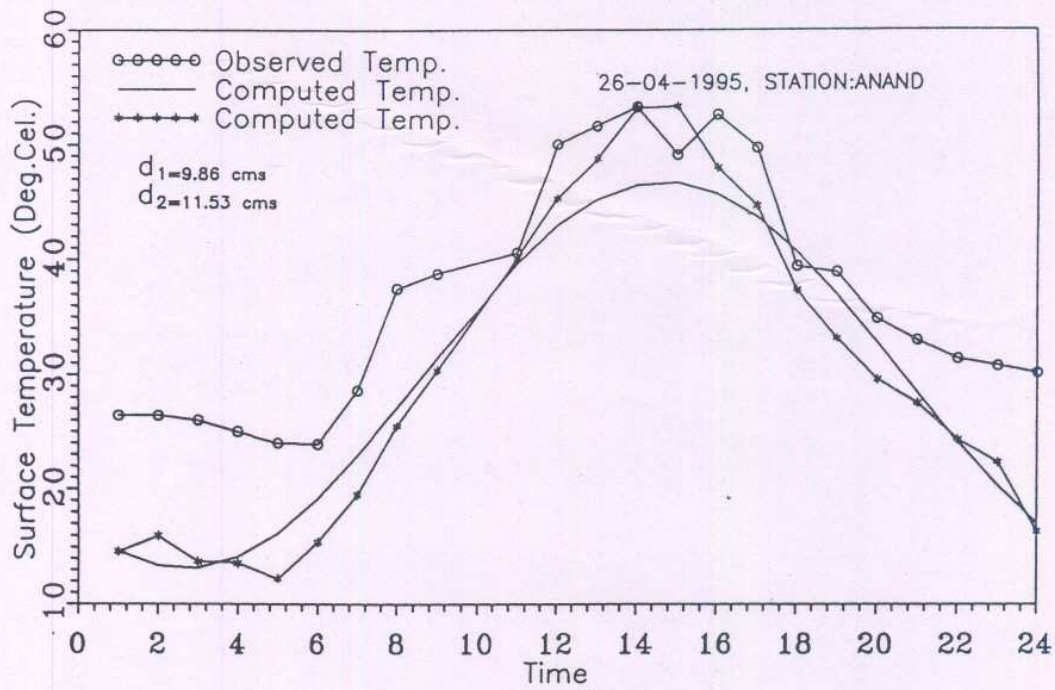


Fig. (2a)

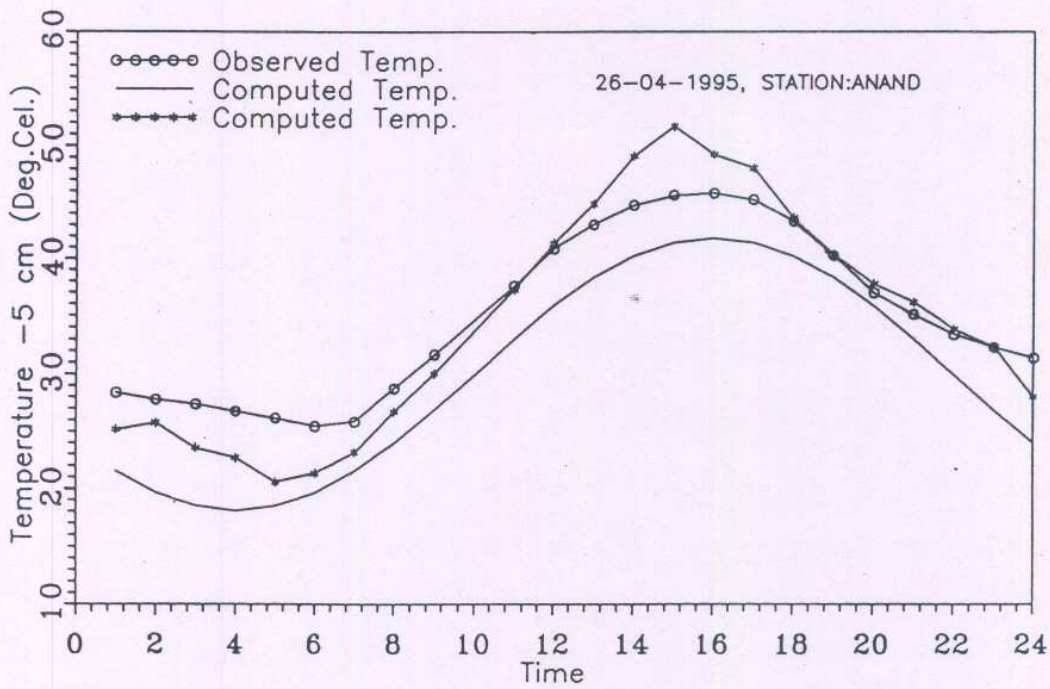


Fig. (2b)

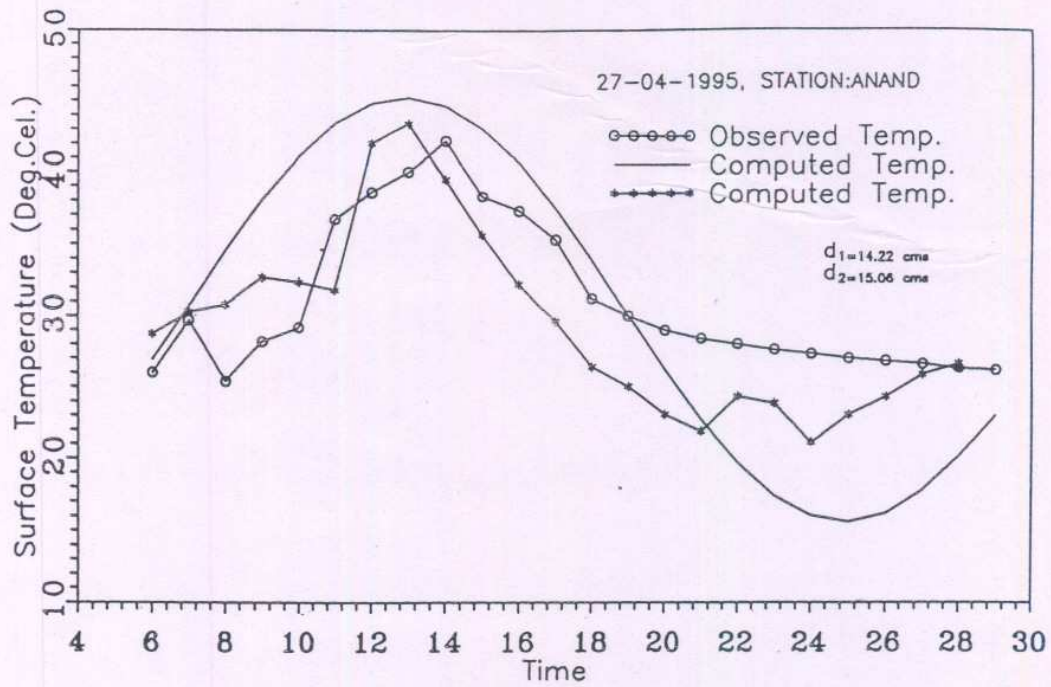


Fig. (3a)

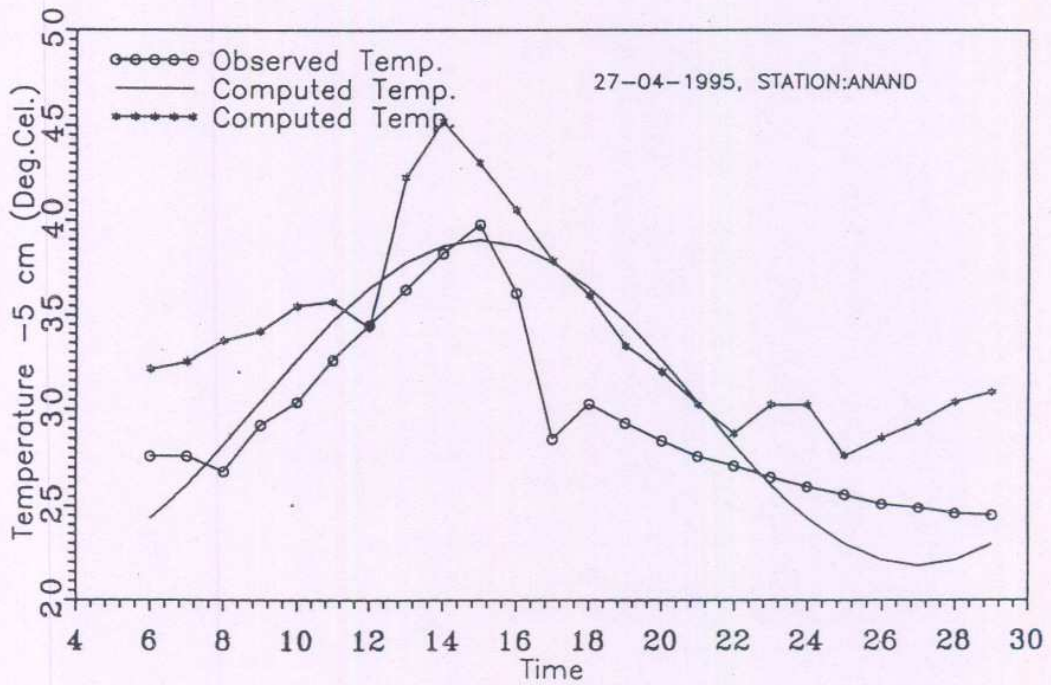


Fig. (3b)

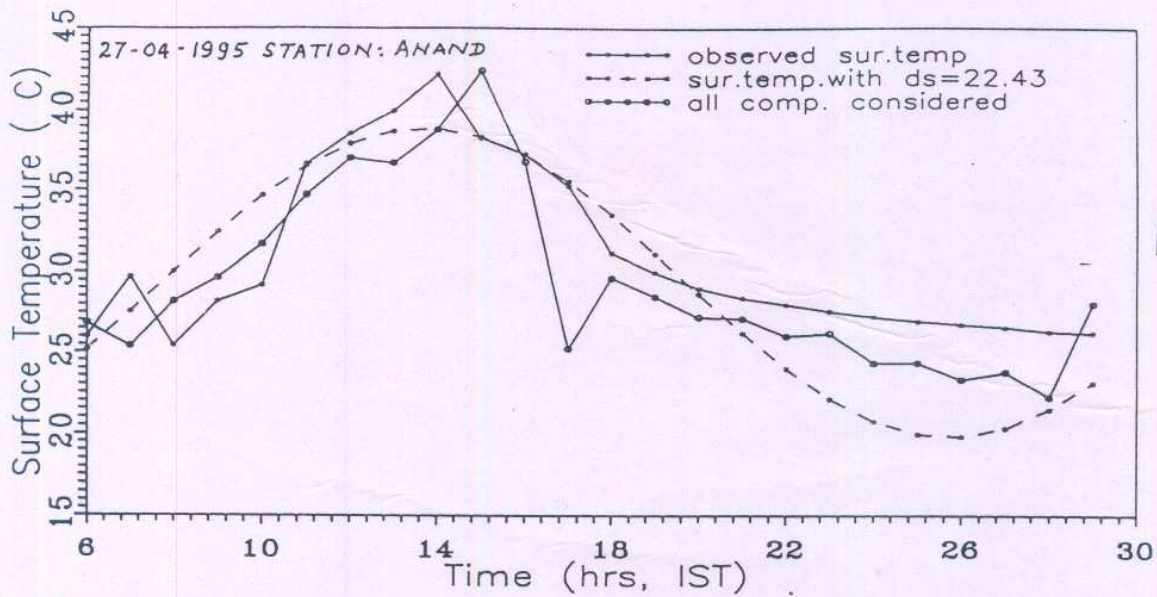


Fig.(4a)

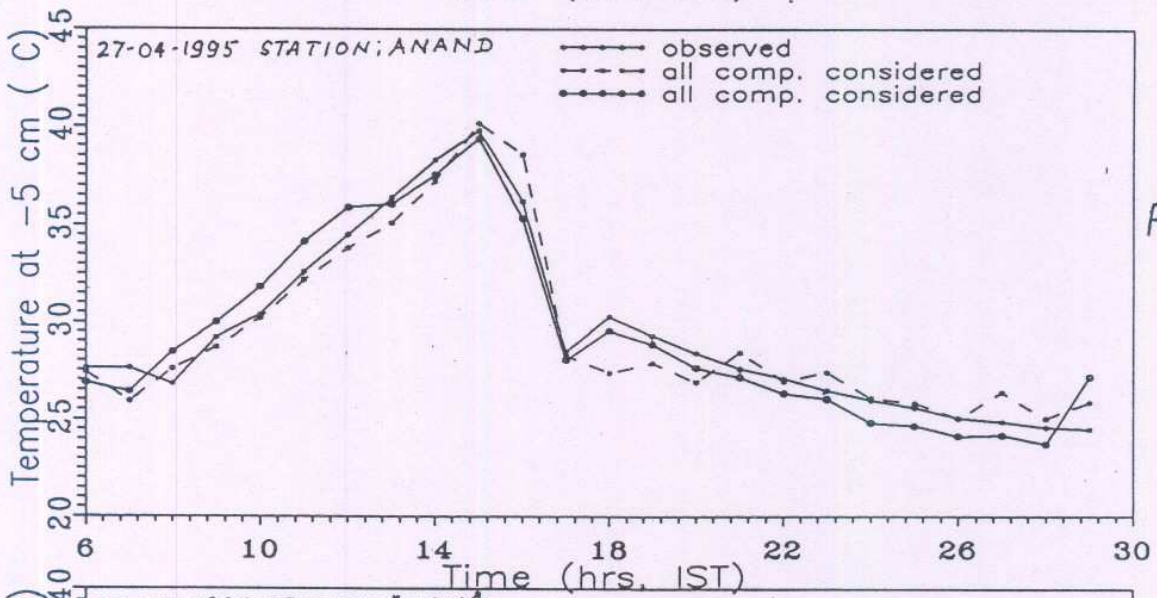


Fig.(4b)

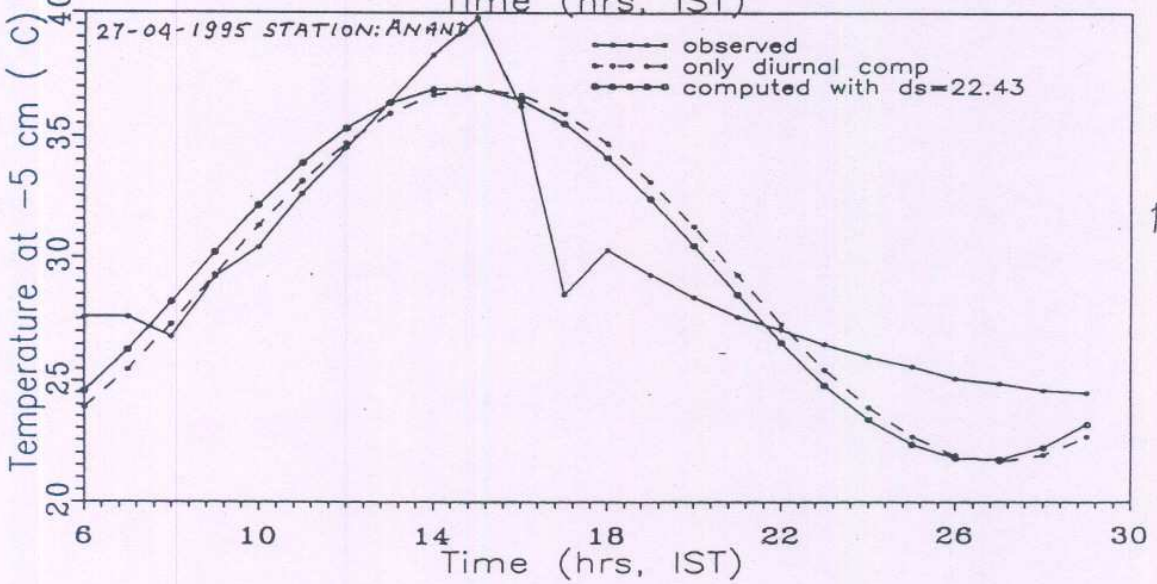


Fig.(4c)

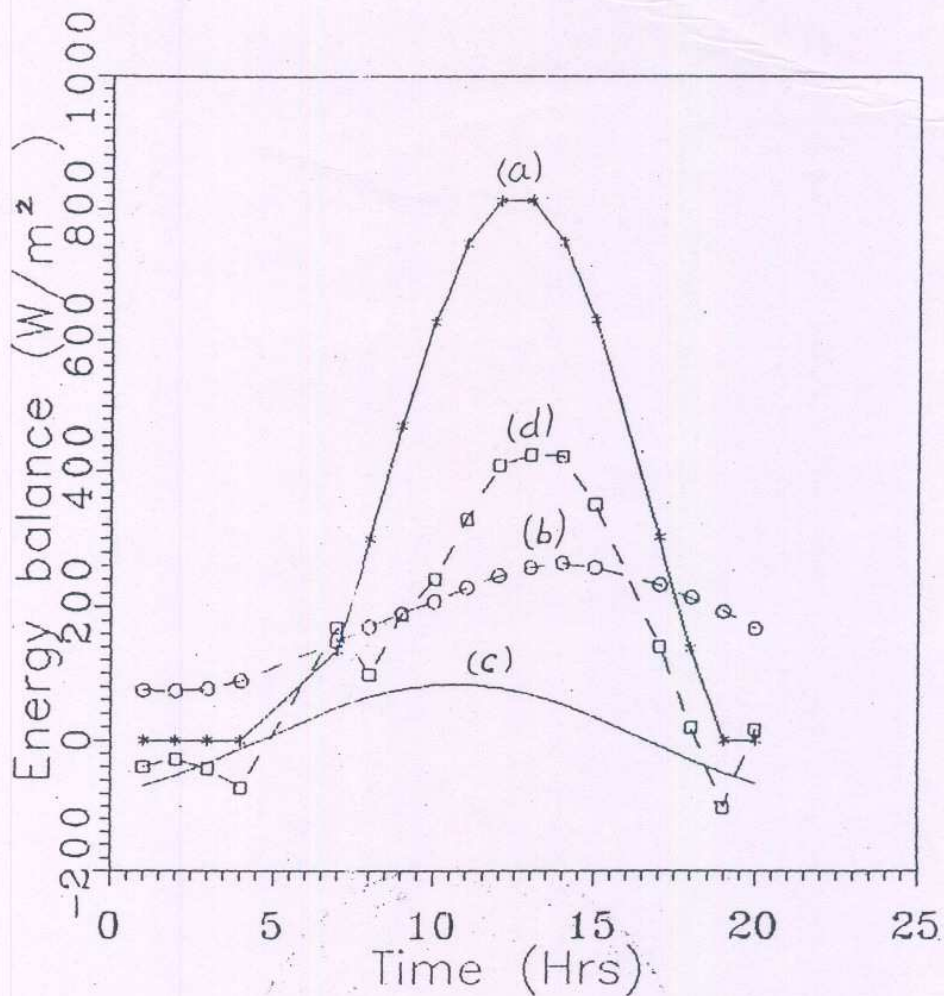


Figure 5: Computed hourly variation of (a) short wave radiation ($\leftarrow \rightarrow$), (b) long wave radiation ($\circ \text{---} \circ$), (c) ground heat flux (---), (d) sensible heat flux ($\square \text{---} \square$).