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FRACTAL NATURE OF TOGA TEMPERATURE TIME  
SERIES

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**Fractal Nature of TOGA Temperature Time Series**  
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**Abstract**

Atmospheric flows as recorded in meteorological parameters such as temperature, pressure, etc., are characterized by turbulent (irregular or nonlinear) fluctuations in space and time. Noisy or irregular signals are common to dynamical systems in nature and are usually thought of as being composed of, or driven by random processes. However, power spectral analyses of such irregular fluctuations reveal inverse power-law form for power spectra implying long-range spatiotemporal correlations. Such non-local connections are ubiquitous to dynamical systems and are recently identified as signatures of self-organized criticality. Long-range correlations imply selfsimilar or fractal fluctuations in space and time. The apparently irregular or nonlinear fluctuations may therefore be described and quantified in terms of the newly developed concept of *fractals*. One common statistical measure of variability is relative dispersion (*RD*) which is equal to the standard deviation divided by the mean. The *RD* decreases with increase in measurement time or space scale. Standard statistical theory does not provide satisfactory explanation for such resolution dependent decrease in *RD*. In this paper, it is shown that the resolution (time) dependent *RD* can be quantified in terms of the *fractal dimension D*. Global TOGA (Tropical Ocean Global Atmosphere) daily temperatures at 850, 500 and 200 hPa for the 5-year period 1986 - 1990 were used for the study.

**Keywords :** Fractals, Self-organized criticality, Relative dispersion, Atmospheric temperature.

## **1. Introduction**

Atmospheric flows as recorded in meteorological parameters such as temperature, pressure, etc. are characterized by turbulent (irregular or nonlinear) fluctuations in space and time. Noisy or irregular signals are common to dynamical systems in nature and are usually thought of as being composed of, or driven by random processes [1]. However, power spectral analysis of such irregular fluctuations reveal inverse power-law form for power spectra implying long-range spatiotemporal correlations. Such non-local connections are



ubiquitous to dynamical systems in nature and are recently identified as signatures of self-organized criticality [2]. Long-range correlations imply selfsimilar or fractal fluctuations in space and time. The apparently irregular or nonlinear fluctuations may therefore be described and quantified in terms of the newly developed concept of *fractals*. Lovejoy and his group [3-5] have documented the selfsimilar fractal geometry of the space-time fluctuations in atmospheric flows as manifested in the fractal geometry of cloud shapes and the inverse power law form for power spectra of temporal fluctuations of meteorological parameters. The fractal shape of clouds over the Indian region has been identified [6-7]. Self-organized criticality in the temporal evolution of rainfall, atmospheric total ozone and temperature have been identified [8-12]. In this paper it is shown that the fractal nature of meteorological time series enables quantification of the relation between variances at different time resolutions, i.e. the variability at shorter time resolutions may be inferred from more convenient measurements at larger time-averaged measurements. Standard statistical procedures [13-14] do not provide for quantifying the observed increase in variance associated with higher resolution (space-time) measurements of the *fractal* (self-similar) time series with long-range spatiotemporal correlations. The fractal description of the relationships between variances of the time-averaged variable at different time-resolutions will help to identify suitable time-interval for predictability studies [15].

## 2. Data and analysis

TOGA (Tropical Ocean Global Atmosphere) daily temperature data [16] for the region 50° N to 50° S at 5 degree intervals for the 1800-day period from 1 Jan 1986 to 1990 was used for the study.

In Statistics, the *relative dispersion RD* analysis compares the variance of a variable as the measurement resolution increases. The coefficient of variation, namely *RD*, is an index of variability or heterogeneity within the domain. *RD equals the standard deviation divided by the mean*. In general, the observed degree of heterogeneity increases as resolution of the method increases. When these increases are proportional, the relation can be *fractal* or at least describable by a fractal relation, if it holds true over a sizable range of observation unit sizes. Given that a fractal relation exists between the observed *RD* of temperature and  $n$ , the averaging time interval in days, the relation can be expressed by the equation [14]

$$RD(n) = RD(n_0) \left[ \frac{n}{n_0} \right]^{1-D} \quad (1)$$

where  $n_0$  is the available time resolution (1 day in the present study). The reason for calling this a fractal relationship rather than just a power law relationship is that the possible slopes of the relationship are bounded by limits, and the fractal dimension  $D$ , gives insight into the nature of the data. These



equations, describing the relation between a measured quantity and the temporal resolution of measurement, are also found in mathematical constructs and natural phenomena having fractal geometry [17-19].

Traditional statistical methods deal with perfectly random variations where any one set of observations at a particular size of observed unit serves to characterize it completely such that

$$RD(n) = \frac{RD(n_0)}{\sqrt{n/n_0}} \quad (2)$$

A general approach to describing variances, in particular for the ubiquitous self-similar (correlated) time series is not provided in conventional statistics. There is no law that says that  $\log RD$  versus  $\log n$  for the number  $n$  of aggregates of non-random variables should show self-similarity. However, the grouping of neighbors must give a monotonically decreasing  $RD$ , since the averaging smoothes out extreme fluctuations. The *fractal dimension*  $D$  gives a measure of the temporal correlation  $r$  between defined time intervals. Van Beek et. al. [20] have derived the general expression

$$r = 2^{3-2D} - 1 \quad (3)$$

for precise description of heterogeneity of regional myocardial blood flows over a wide range of domain sizes. Selvam and Sapre [15] have shown that the above relationship is applicable for meteorological time series.

The above equation is an important derivation since it summarizes the properties of the data set. If there is no temporal correlation,  $r=0$ , so that when the temperature fluctuations are completely random the fractal dimension  $D=1.5$  and Eq.(1) reduces to Eq.(2) consistent with traditional statistics. This gives a maximal slope in the plots of  $RD$  versus the time interval for averaging. At the opposite extreme, with perfect correlation,  $r=1$ , temperature fluctuations are uniform and the fractal dimension is  $1.0$ . Regions of averaging time intervals for which  $D$  is a constant have averaging-time independent correlation coefficient between neighbors. Eq.(1) and (3) can be applied to all temporal fractals. If a fractal relationship is a reasonably good approximation, even over a decade or so, then it will prove useful in considerations of temporal functions and might be useful in initiating research for the underlying basis for correlation.

In the present study the relative dispersion  $RD$  was computed for different averaging-time intervals  $n$  which are factors of the total number of days  $N$  (1800 days). The fractal dimension  $D$  was computed for the time intervals  $n$  from Eq. (1) and is shown in Figs.1(a-c) for all grid points in each latitude belt for the three tropospheric levels (850 hPa, 500 hPa, 200 hPa).

In an earlier work Mary Selvam et. al [7] have shown that spatial integration of enclosed small scale fluctuations give rise to large scale circulations which follow logarithmic spiral trajectory. Therefore, the relative



dispersion  $RD$  for the various averaging time intervals  $n$  on logarithmic scale should follow normal distribution according to the *Central Limit Theorem* in Statistics. A graph of  $RD$  versus the normalized time scale (logarithmic)  $n$  is given as

$$t = \frac{\log n}{\log t_{50}} - 1 \quad (4)$$

should follow normal distribution characteristics. In Eq.(4)  $t_{50}$  is the time scale where the  $RD$  is equal to 50% of its value at the highest resolution. Fig.2 shows the mean normalized percentage  $RD$  with respect to the normalized time scale  $t$  (Eq.4) for each latitude belt for the three levels 850 hPa, 500 hPa, and 200 hPa. The statistical normal distribution also is shown in Fig.2.

### 3. Discussion and conclusion

The observed decrease in relative dispersion  $RD$  of temperature with increase in averaging time interval exhibits the following universal characteristics at all the grid points for the three levels ( 850 hPa, 500 hPa, 200 hPa) used in this study.

1. The fractal dimension  $D$  of  $RD$  remains almost constant, close to a value equal to 1.1 for averaging time intervals greater than 10-days and increases rapidly, particularly in the higher latitudes ( greater than 10-degrees) for averaging time intervals less than 10-days. (Figs. 1(a-c)).
2. The  $RD$  follows universal statistical normal distribution with respect to normalized averaging-time scale  $t$  (Fig.2).

The above fractal analyses technique provides unique quantification for resolution dependent relative dispersion  $RD$ .

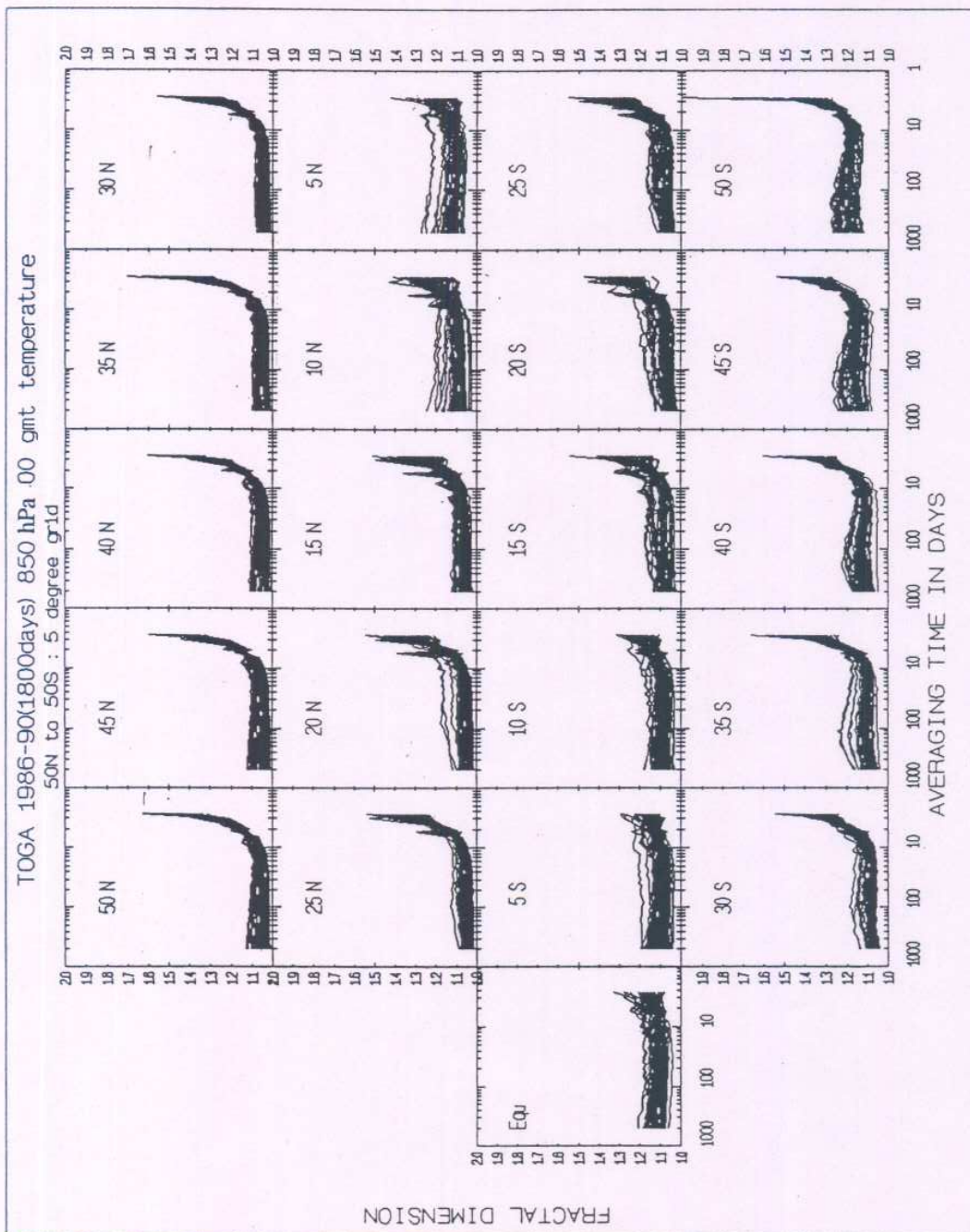
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**Fig. 1a :** Fractal dimension D versus averaging time interval quantifying the resolution dependent value of relative dispersion RD of temperature at 850 hPa at all grid points for each latitude belt from 50° N to 50° S.



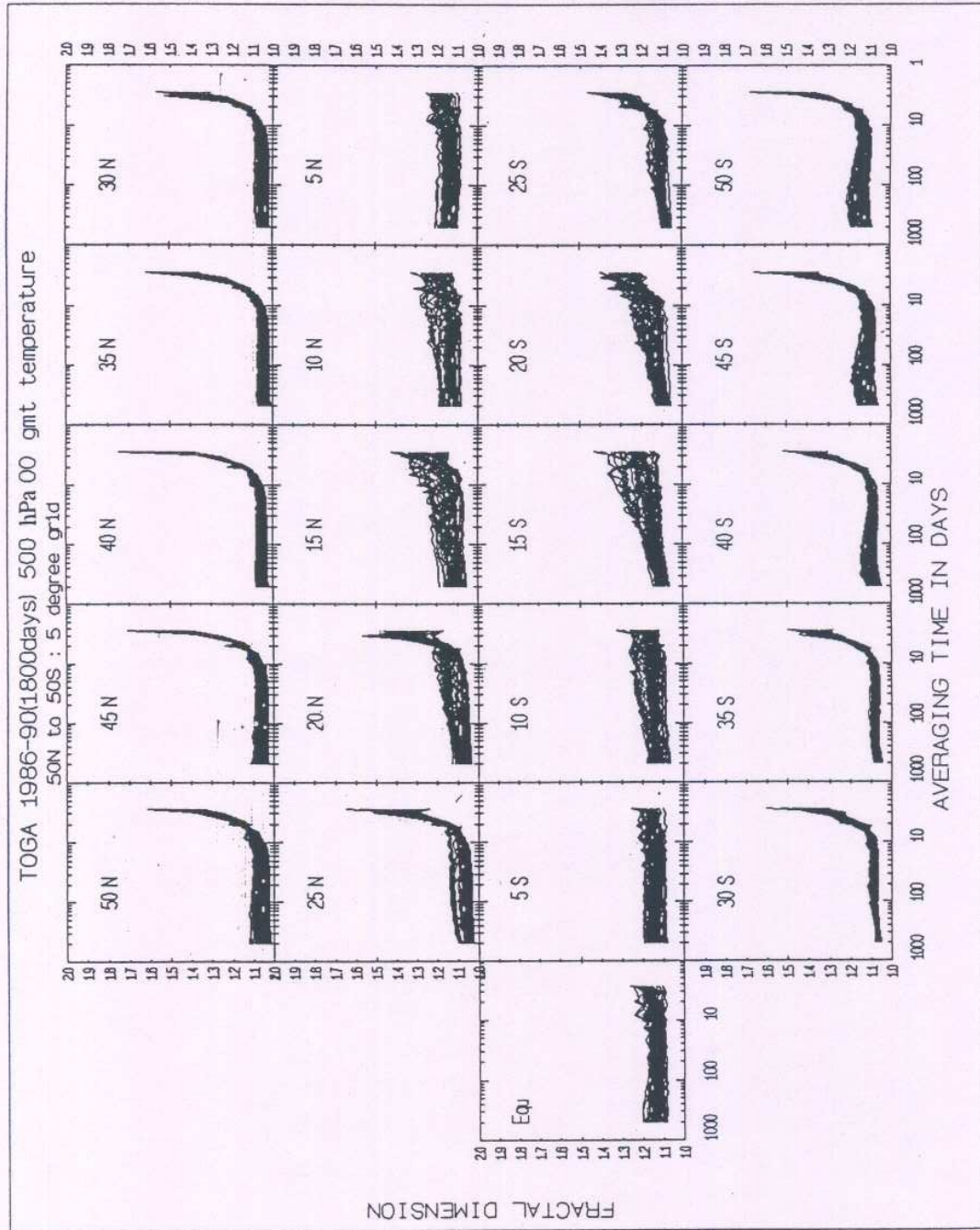


Fig. 1b : Same as for Figure 1a for 500 hPa

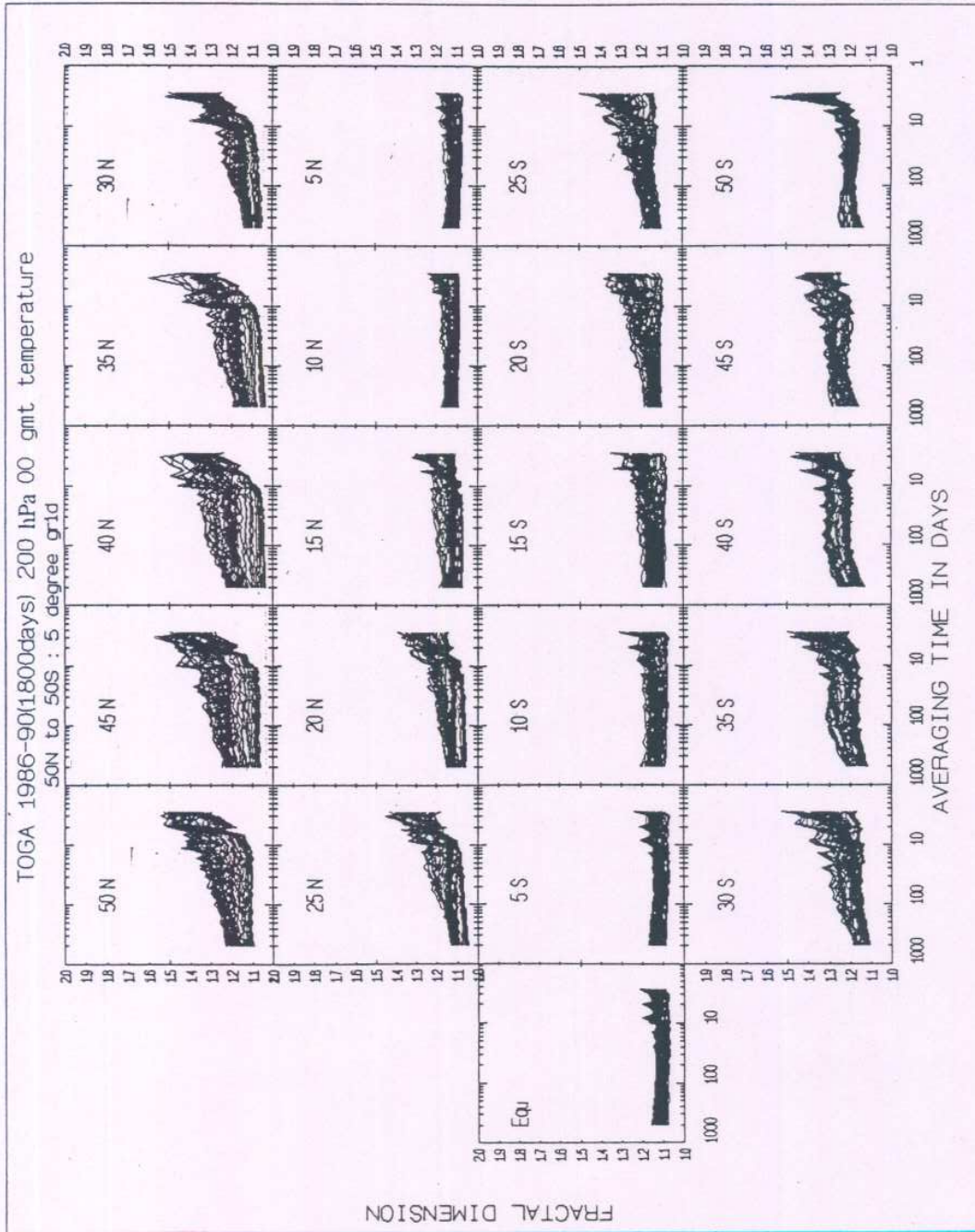


Fig. 1c : Same as for Figure 1a for 200 hPa



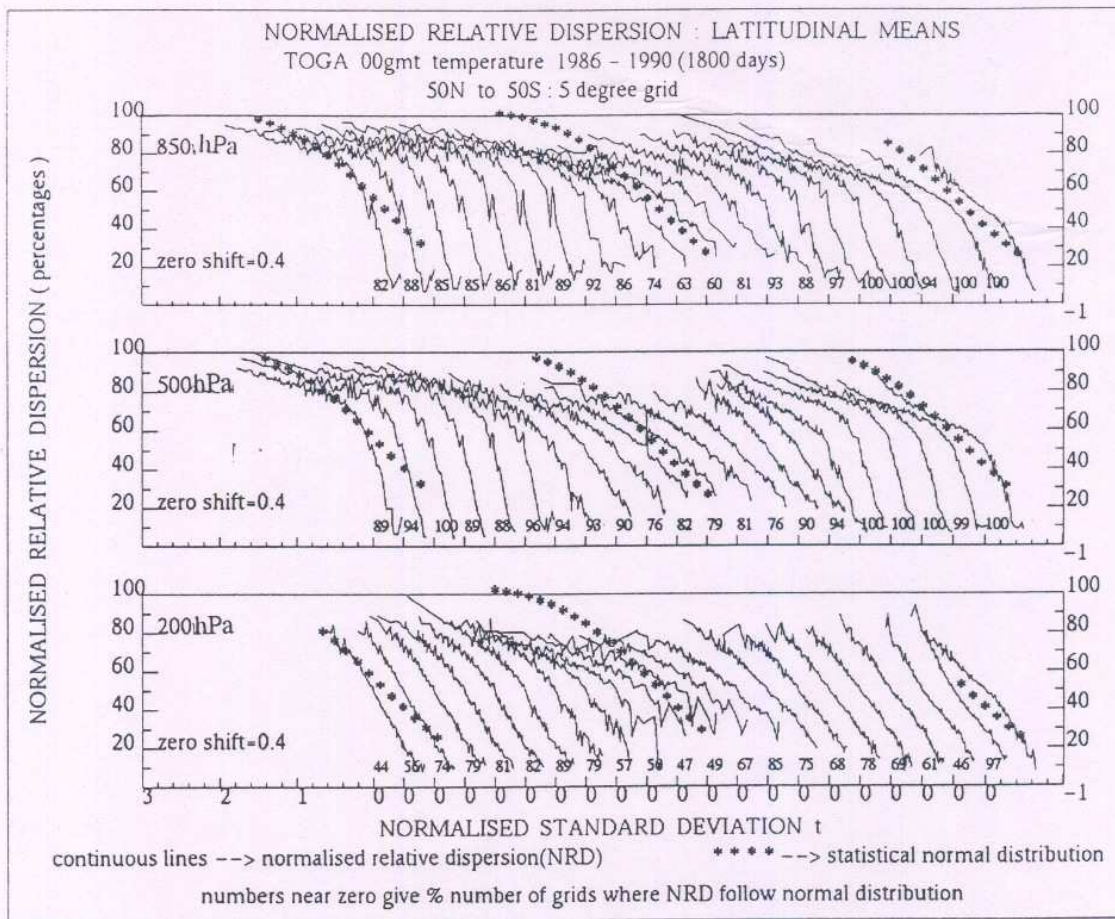


Fig. 2 : Latitudinal mean graphs of normalized relative dispersion versus normalized averaging time interval. The statistical normal distribution is also shown in the figure

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