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ESTIMATION OF NONLINEAR HEAT AND  
MOMENTUM TRANSFER IN THE FREQUENCY  
DOMAIN BY THE USE OF  
FREQUENCY CO-SPECTRA AND CROSS-BISPECTRA

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# Estimation of Nonlinear Heat and Momentum Transfer in the Frequency Domain by the Use of Frequency Co-Spectra and Cross-Bispectra

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## ABSTRACT

In order to isolate the important time scale as Madden Julian and the El Nino which have bearing on the performance of Indian Monsoon, computations of energy fluxes in the frequency domain is a powerful tool. The mathematical technique proposed here is to compute fluxes of Latent Heat, Sensible Heat and u-momentum in the frequency domain at the surface layer and PBL by the use of cross spectral technique based on Monin-Obukhov similarity theory, bulk aerodynamic formulas and K-Theory. Here, cospectra and coherence of the time series of nonlinearities constitute the main computations. An example of the application of this method to compute the fluxes through nonlinear interactions over Central Pacific at surface layer and PBL is given. These interactions involve both the atmosphere and the ocean indicating the influence of Atmospheric Oceanic coupling.

Key words: Cospectra, K-Theory, Cross-bispectra, Richardson number, Monin-Obukhov length



## 1. Introduction

The atmospheric oscillations on the time scale of 30 to 50 days are not entirely forced by internal atmospheric instability, they are related to coupling between ocean and atmosphere. Most of the atmospheric processes on different spatial and temporal scales depend directly or indirectly on the transport of mass energy and momentum through the Planetary Boundary Layer (PBL). The information about the boundary layer and the interfacial layer at the earth-atmosphere boundary is essential for the understanding of the energy exchange processes in the wavenumber and frequency domain. The heat balance adjustment in the upper layers also alters the oceanic circulation in different time scales and so the heat transported to the atmosphere in the form of fluxes on different temporal scales is to be generally calculated using the ocean feed back as well.

Computations of the energy fluxes in the frequency domain is a powerful tool for isolating the role of an important time scale such as Madden Julian and the El-Nino. Pioneering work in casting the frequency domain came from the studies such as those of Hayashi (1980) and Sheng and Hayashi (1990 a,b). When the energy budgets are examined in this context, one asks questions on the maintenance of a certain time window from it's interaction with other time scales. This entails the computations of the time series of non-linearities (generally quadratic and triple product). Here cospectra and coherence of the time series of non-linearities constitute the main computations. In the present study, we shall propose a mathematical technique of computing non-linear transfer of fluxes of latent heat, sensible heat and u momentum using the cross-spectral technique by generalizing the grid method. Here we have derived expressions for just the nonlinear flux transfers into a frequency from it's interaction with other frequencies. This kind of intrafrequency flux transfer arises from triad interactions in the frequency domain, which follow certain selection rules for permissible transfers. The nonlinear transfer to a frequency  $n$  arises from triple products.

In Section 2 following Krishnamurti et. al. (1988) Surface Similarity Theory is described. The formulas for computing nonlinear transfer of fluxes of latent heat, sensible heat and u-momentum in the frequency domain are given in Section 3. Section 4 deals with the formulations in the frequency domain regarding dispositions of the fluxes at the PBL. In Section 5 an example of their applications are given.

## 2. Monin – Obukhov Similarity Theory

The basis of the similarity analysis are the planetary boundary layer observations, e.g., Businger et. al. (1971). According to these observations nondimensionalized vertical gradients of large scale quantities such as the wind, potential temperature and specific humidity can be expressed as universal functions of a nondimensional height ( $z/L$ ) where  $z$  is the height above the earth's surface and  $L$  is the Monin-Obukhov length.

Monin-Obukhov length is defined by

$$L = u^*{}^2 / k\beta\theta^*$$

Here  $\beta = g/\theta_0$  where  $\theta_0$  is a reference temperature,  $u^*$  is the friction velocity and  $\theta^* u^*$  is a measure of the surface heat flux.

We express these by the relations:

$$\frac{kz}{u^*} \frac{\partial \bar{u}}{\partial z} = \phi_m(z/L) \quad (1)$$

$$\frac{kz}{\theta^*} \frac{\partial \bar{\theta}}{\partial z} = \phi_h(z/L) \quad (2)$$

$$\frac{kz}{q^*} \frac{\partial \bar{q}}{\partial z} = \phi_q(z/L) \quad (3)$$

The symbols are explained in Table 1. The empirical fits of the boundary layer observations are usually separated in terms of stability. Stability is usually expressed in terms of the sign of Monin-Obukhov length  $L$  or that of the Richardson number  $R_{iB}$ .

$$R_{iB} = \beta \frac{\frac{d\bar{\theta}}{dz}}{\left[\frac{du}{dz}\right]^2} \quad (4)$$

For the stable surface layer

$$L > 0$$

or

$$R_{iB} > 0.$$

For the unstable layer

$$L < 0$$

or

$$R_{iB} < 0.$$

Since a priori the Monin-Obukhov length is an unknown quantity, stability is assessed from the sign of the bulk Richardson number  $R_{iB}$ .

Empirical fits on the boundary layer observations (Businger et. al., 1971) for the stable and unstable cases, are expressed, following Chang(1978), by the nondimensional relations,

(i) Unstable case:

$$kz/u^* \frac{\partial \bar{u}}{\partial z} = (1 - 15z/L)^{-1/4} \quad (5)$$

$$kz/\theta^* \frac{\partial \bar{\theta}}{\partial z} = 0.74(1 - 9z/L)^{-1/2} \quad (6)$$

$$kz/q^* \frac{\partial \bar{q}}{\partial z} = 0.74(1 - 9z/L)^{-1/2} \quad (7)$$



(ii) Stable case:

$$kz/u^* \frac{\partial \bar{u}}{\partial z} = 1.0 + 4.7z/L \quad (8)$$

$$kz/\theta^* \frac{\partial \bar{\theta}}{\partial z} = 0.74 + 4.7z/L \quad (9)$$

$$kz/q^* \frac{\partial \bar{q}}{\partial z} = 0.74 + 4.7z/L \quad (10)$$

For both the stable and unstable cases the definition of stability yields a fourth equation,

$$z/L = \frac{zk\beta\theta^*}{u^{*2}}. \quad (11)$$

The four equations (for the stable or the unstable case) need to be solved for the four variables  $u^*$ ,  $\theta^*$ ,  $q^*$  and the Monin-Obukhov length  $L$ . The surface fluxes of momentum, heat and moisture are respectively given by,

$$F_M = u^{*2} \equiv -\overline{u'w'}_0 \quad (12)$$

$$F_H = -u^*\theta^* \equiv \overline{\theta'w'}_0 \quad (13)$$

$$F_q = -u^*q^* \equiv \overline{q'w'}_0 \quad (14)$$

The solution procedure, followed by different investigators, for the aforementioned equations vary somewhat, we have followed a method developed in Florida State University with the global spectral model, (Krishnamurti et. al., 1983, 1984). A two (vertical) level surface layer representation is convenient for the evaluation of surface fluxes.

Let  $z_1$  and  $z_2$  denote two levels where  $z_1 = z_0$  the roughness length,  $z_2$  is the top of a surface layer and  $\Delta z = z_2 - z_1$  is roughly 10 meters. At these two levels the respective winds, potential temperature and the specific humidity may be denoted by  $(\bar{u}_1, \bar{v}_1), (\bar{u}_2, \bar{v}_2); \bar{\theta}_1, \bar{\theta}_2; \bar{q}_1, \bar{q}_2$ .

We shall next redefine the bulk Richardson number

$$R_{iB} = \beta \frac{(\bar{\theta}_2 - \bar{\theta}_1)\Delta z}{(u_2^2 + v_2^2)}. \quad (15)$$

Following Chang (1978) we shall express the two level representation of the flux relations for the unstable case by the following equations

$$\frac{z_2}{L} = R_{iB} \frac{z_2}{\Delta z} \left[ \frac{\ln(z_2/z_1) - \psi_1}{0.74[\ln(z_2/z_1) - \psi_2]} \right] \quad (16)$$

where  $\psi_1$  and  $\psi_2$  are complex functions of  $L$  and are defined in Table 1.

Upon integration of Eq. (5) from  $z_1$  and  $z_2$  we obtain

$$\int_{u_1}^{u_2} \frac{k}{u^*} d\bar{u} = \int_{z_1}^{z_2} \frac{1}{z} \left[ 1 - \frac{15z}{L} \right]^{-1/4} dz$$

or

$$\frac{K}{u^*} (\bar{u}_2 - \bar{u}_1) = \int_{z_1}^{z_2} \left[ \frac{1}{z} - \frac{1}{z} + \frac{1}{z} \left( 1 - \frac{15z}{L} \right)^{-1/4} \right] dz$$

$$\begin{aligned}
&= \int_{z_1}^{z_2} \frac{1}{z} dz - \int_{z_1}^{z_2} \frac{1}{z} \left\{ 1 - \left( 1 - \frac{15z}{L} \right)^{-1/4} \right\} dz \\
&= \ln \frac{z_2}{z_1} - \psi_1 \left( \frac{z_2}{L}, \frac{z_2}{z_1} \right).
\end{aligned} \tag{17}$$

Similar vertical integration of Equation (6) and (7) results in

$$\frac{k(\bar{\theta}_2 - \bar{\theta}_1)}{0.74\theta^*} = \ln \frac{z_2}{z_1} - \psi_2 \left( \frac{z_2}{L}, \frac{z_2}{z_1} \right) \tag{18}$$

$$\frac{k(\bar{q}_2 - \bar{q}_1)}{0.74q^*} = \ln \frac{z_2}{z_1} - \psi_2 \left( \frac{z_2}{L}, \frac{z_2}{z_1} \right) \tag{19}$$

Here,

$$\begin{aligned}
\psi_1 &= \int_{z_1}^{z_2} \frac{1}{z} \left[ 1 - \left( 1 - \frac{15z}{L} \right)^{-1/4} \right] dz, \\
\psi_2 &= \int_{z_1}^{z_2} \frac{1}{z} \left\{ 1 - \left( 1 - \frac{9z}{L} \right)^{-1/2} \right\} dz;
\end{aligned}$$

Equations (16), (17), (18) and (19) are solved for the variables  $L^*$ ,  $u^*$ ,  $\theta^*$  and  $q^*$  rather simply. The variation of the Monin-Obukhov length is monotonic with respect to  $u^*$  and  $\theta^*$ , (Businger et. al., 1971). A simple linear incremental search of  $L$  in Eq. (16) provides a rapid solution to a desired degree of accuracy. Upon substitution of  $L$  on the right hand side of (17), (18) and (19) one obtains the corresponding solution for  $u^*$ ,  $\theta^*$  and  $q^*$ .

The solution for the stable case requires a sequential solution of four linear algebraic equations, these are  $\frac{z_2 - z_1}{L} = \ln(z_2/z_1) \left[ \frac{9.4R_{iB} - 0.74 + \sqrt{4.89R_{iB} + 0.55}}{9.4 - 44.18R_{iB}} \right]$ . (20)

Equation (20) is obtained from an elimination of  $u^*$  and  $\theta^*$  from vertically integrated form Eqs. (8), (9) and (11).

$$\frac{k(\bar{u}_2 - \bar{u}_1)}{u^*} = \ln(z_2/z_1) + 4.7 \frac{(z_2 - z_1)}{L} \tag{21}$$

$$\frac{k(\bar{\theta}_2 - \bar{\theta}_1)}{\theta^*} = 0.74 \ln(z_2/z_1) + 4.7 \frac{(z_2 - z_1)}{L} \tag{22}$$

$$\frac{(z_1 - z_2)}{L} = \frac{(z_2 - z_1)k\beta\theta^*}{u^{*2}}. \tag{23}$$

The lower level  $z_l$  is identified as the roughness length  $z_0$ ; over the oceans  $z_0$  is determined from Charnocks formula, (Charnock, 1955). At these two levels one needs to define the winds, potential temperature and the specific humidity in order to carry out the desired computations. The procedure we follow is identical to what is done in numerical weather prediction models. At the lower level  $z_l$ , we set all the winds ( $u_l$ ,  $v_l$  to zero); the temperature  $T_l$  at the lower level is set to the sea surface temperature; we furthermore set



$\theta_1 = T_1$  since the surface pressure is close to 1000mb. The specific humidity  $q_1$  at the lower level, over the ocean, is set to the saturation value at the temperature  $T_1$ . At the upper level, i.e.,  $z_2$ , it is set to its value 10 meters above the ocean. The winds  $U_2$  and  $V_2$  are interpolated using a log-linear profile while the temperature  $T_2$  and specific humidity  $q_2$  are linearly interpolated between 1000 and 850 mb from the FGGE IIb analysis. The surface ship data for the FGGE period enters into the 1000 mb analysis. The numerical model algorithm requires information at 950 mb which is obtained by interpolation between 1000 and 850 mbs. The extrapolation downwards from 950 implicitly thus carry fully the 1000 mb and the surface data set. Detailed check of the fluxes for the Soviet ship data was used to verify the procedure. The Von Karman constant,  $k$  has a value of 0.35;  $\beta$  stands for  $g/\theta_0$ . Since the  $R_{IB}$  is known from large scale data sets the sequence of solution yields  $L^*, u^*, \theta^*, q^*$ .

### 3. Fluxes in the Frequency Domain

#### 3.1. Quadratic and Triple Product Nonlinearities and Computation of Co-spectra

We assume that a given time series of data for two variables  $u(t)$  and  $v(t)$  are cyclic and discrete in time. Since the FSU coupled model has run for two years, we have assumed a fundamental periodicity over this length of period, our aim is to examine the role of the MJO within this period. These series are represented by temporal Fourier series with discrete frequency ( $n$ ) and Nyquist frequency ( $N$ ) such as:

$$u(z) = \sum_{n=0}^N [C_n^u \cos nt + S_n^u \sin nt] \quad (24)$$

In particular,  $C_0^u = u_0$  (time mean) and  $S_0^u = 0$ ;

The sample frequency cospectra  $P_n(u, v)$  are defined as  $P_n(u, v) = (C_n^u C_n^v + S_n^u S_n^v)$  (25)

In the wave number domain the cospectra would be defined by

$$P_n(u, v) = \frac{1}{2\pi} \int_0^{2\pi} \left[ \sum_n (C_n^u \cos n\lambda + S_n^u \sin n\lambda) \sum_n (C_n^v \cos n\lambda + S_n^v \sin n\lambda) \right] d\lambda$$

However over the frequency domain the cospectra would be expressed by

$$P_n(u, v) = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[ \frac{\sum_n (C_n^u \cos nt + S_n^u \sin nt) \sum_n (C_n^v \cos nt + S_n^v \sin nt)}{\int_{-\frac{T}{2}}^{\frac{T}{2}} dt} \right] dt$$

Thus in principle the frequency cospectra can be formally replaced by a wavenumber cospectra (Hayashi, 1980).



When we have triad interaction in the frequency domain, then  $v(t)$  in equation (25) can be expressed as a product of two sets of  $b(t)$  and  $c(t)$  as

$$v(t) = b(t)c(t)$$

Now,  $P_n(u, v)$  takes on the form :

$$P_n(u, v) = P_n(u, bc) = L(n) = \sum LN(n, r, s) \quad (26)$$

$LN(n, r, s)$  in (26) denotes a contribution to the  $L(n)$  due to specified combination of  $r$  and  $s$ , where frequencies  $r$  and  $s$  satisfy either of the selection rules  $r+s=n$  or  $|r-s|=n$ . The explicit expression for  $L(n)$  in the frequency domain is given below using Fourier transforms of  $u$ ,  $b$  and  $c$  in time domain.

Thus,

$$L(n) = \frac{\int_{-t/2}^{t/2} \left[ \sum_n (C_n^u \cos nt + S_n^u \sin nt) \sum_r (C_r^b \cos rt + S_r^b \sin rt) \sum_s (C_s^c \cos st + S_s^c \sin st) \right] dt}{\int_{-t/2}^{t/2} dt} \quad (27)$$

Thus we may write  $L(n) =$

$$\begin{aligned} 1/t \int_{-t/2}^{t/2} [ & \sum_n \sum_r \sum_s C_n^u C_r^b C_s^c \cos nt \cos rt \cos st + C_n^u S_r^b C_s^c \cos nt \sin rt \cos st \\ & + S_n^u C_r^b C_s^c \sin nt \cos rt \cos st + S_n^u S_r^b C_s^c \sin nt \sin rt \cos st + C_n^u C_r^b S_s^c \cos nt \cos rt \sin st \\ & + C_n^u S_r^b S_s^c \cos nt \sin rt \sin st + S_n^u C_r^b S_s^c \sin nt \cos rt \sin st + S_n^u S_r^b S_s^c \sin nt \sin rt \sin st ] dt \end{aligned}$$

Following Chakraborty (1995), the nonlinear triad interaction in the explicit form is expressed by

$$\begin{aligned} L(n) = 1/2 & \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] C_n^u S_r^b C_s^c + 1/2 \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] C_n^u S_r^b S_s^c + 1/2 \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] S_n^u C_r^b C_s^c \\ & + 1/2 \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] S_n^u S_r^b C_s^c \quad (28) \end{aligned}$$

This is used to calculate the net gain or loss of energy (in our problem the fluxes) by a frequency  $n$  as it interacts with frequencies  $r$  and  $s$ .

### 3.2. Fluxes of Latent Heat, Sensible Heat and U-Momentum in the Frequency Domain

Following Fairall et. al.,(1996a) the bulk aerodynamic expressions for latent heat flux, sensible heat flux and U-momentum are given by :

Latent heat flux:

$$L_h = \mu C_{LH} |V| L (Q_s - Q_a) \quad (29)$$

$\mu$  = Density of air = 1.275 kg/m<sup>3</sup>

$C_{LH}$  = Drag co-efficient related to latent heat flux

$L = 4.1868 \times (597.0 - 0.6 \times T_s) \times 1000$  (J/kg)

$Q_s$  = sp. humidity at sea surface

$Q_a$  = sp. humidity of air at surface

$V$  = Wind speed (m/sec) =  $\sqrt{u^2 + v^2}$

$T_s$  = Sea surface temperature in degrees Kelvin

Sensible heat flux is given by:

$$L_s = \mu C_p C_{LS} (T_s - T_a) |V| \quad (30)$$

$C_p$  = specific heat of air at constant pressure

$C_{LS}$  = Drag co-efficient related to sensible heat flux

$T_a$  = temperature of air near surface of sea in Kelvin

U-momentum flux is given by:

$$\tau^x = \mu C_{LV} u (U_{atm} - U_{sea}) \quad (31)$$

where  $u = \sqrt{(u_{atm} - u_{sea})^2 + (v_{atm} - v_{sea})^2}$

These are the respective fluxes on the top of the constant flux layer. These exchange coefficients are functions of stability and as such these are space time dependent.

Using Fourier time series for  $C_{LH}$ ,  $|V|$ , and  $L (Q_s - Q_a)$  and by taking cospectrum among them in the r.h.s. of expression (29) with the help of formula (28), latent heat flux in the frequency domain can be written as :

$$\begin{aligned} \langle L_h(n) \rangle = & \frac{\mu}{2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [\text{LHC}(n). \text{MVC}(r). \text{DMC}(s)] \\ & + \frac{\mu}{2} \left[ \begin{aligned} & - \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [\text{LHC}(n). \text{MVS}(r). \text{DMS}(s)] \end{aligned}$$



$$\begin{aligned}
& + \frac{\mu}{2} \left[ \begin{array}{c} + \sum_{r+s=n} \\ - \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] [\text{LHS}(n). \text{MVC}(r). \text{DMS}(s)] \\
& + \frac{\mu}{2} \left[ \begin{array}{c} + \sum_{r+s=n} \\ + \sum_{r-s=n} \\ - \sum_{r-s=-n} \end{array} \right] [\text{LHS}(n). \text{MVS}(r). \text{DMC}(s)]
\end{aligned} \tag{32}$$

where (LHC,LHS), (MVC,MVS) and (DMC,DMS) are the cos and sine coefficients of  $C_{LH}$ ,  $|V|$ , and  $L(Q_s-Q_a)$  associated with the frequencies  $n$ ,  $r$  and  $s$  respectively.

Using Fourier time series for  $C_{LS}$ ,  $|V|$  and  $(T_s-T_a)$  and by taking cospectrum among them in the r.h.s. of the expression (30) with the help of formula (28), sensible heat flux in the frequency domain can be written as:

$$\begin{aligned}
\langle L_s(n) \rangle = & \frac{\mu C_p}{2} \left[ \begin{array}{c} + \sum_{r+s=n} \\ + \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] [\text{LHC}(n). \text{MVC}(r). \text{DTC}(s)] \\
& + \frac{\mu C_p}{2} \left[ \begin{array}{c} - \sum_{r+s=n} \\ + \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] [\text{LHC}(n). \text{MVS}(r). \text{DTS}(s)] \\
& + \frac{\mu C_p}{2} \left[ \begin{array}{c} + \sum_{r+s=n} \\ - \sum_{r-s=n} \\ + \sum_{r-s=-n} \end{array} \right] [\text{LHS}(n). \text{MVC}(r). \text{DTS}(s)] \\
& + \frac{\mu C_p}{2} \left[ \begin{array}{c} + \sum_{r+s=n} \\ + \sum_{r-s=n} \\ - \sum_{r-s=-n} \end{array} \right] [\text{LHS}(n). \text{MVS}(r). \text{DTC}(s)]
\end{aligned} \tag{33}$$

where (DTC,DTS) are the cos and sine coefficients of  $(T_s - T_a)$  associated with frequency  $s$ .

Using Fourier time series for  $C_{LV}$ ,  $u$  and  $(u_{atm} - u_{sea})$  and taking cospectrum among them in the r.h.s. of expression (31) with the help of formula (28), U-momentum flux in the frequency domain can be written as :

$$\begin{aligned}
 \langle \tau^x(n) \rangle = & \frac{\rho_{atm}}{2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [LHC(n). UC(r). DUC(s)] \\
 & + \frac{\rho_{atm}}{2} \left[ \begin{aligned} & - \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [LHC(n). US(r). DUS(s)] \\
 & + \frac{\rho_{atm}}{2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & - \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [LHS(n). UC(r). DUS(s)] \\
 & + \frac{\rho_{atm}}{2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & - \sum_{r-s=-n} \end{aligned} \right] [LHS(n). US(r). DUC(s)] \quad (34)
 \end{aligned}$$

where (UC, US) and (DUC,DUS) are the cos and sine coefficients of  $u$  and  $(u_{atm} - u_{sea})$  associated with frequencies  $r$  and  $s$ , respectively.

At each and every transform grid point of the spectral model, where the physical processes are evaluated, we construct a time series for these triple product variables.

#### 4. Dispositions of the Fluxes in the Frequency Domain at PBL

The vertical disposition of eddy fluxes of momentum, heat and moisture above the surface layer is based on K-Theory. This theory uses an eddy diffusion coefficient  $K$  that depends on the mixing length  $\ell$ , the vertical wind shear, and the stability of the atmosphere as determined by the bulk Richardson number  $R_{iB}$ .



The eddy diffusion coefficient for heat and moisture is

$$K_H = K_q = \ell^2 \frac{\partial |v|}{\partial z} F_h R_{iB} \quad (35)$$

and for momentum it is given by

$$\text{and } K_M = \ell^2 \frac{\partial |v|}{\partial z} F_M R_{iB} \quad (36)$$

where  $F_h$  denotes the heat flux,  $\ell$  is the mixing length expressed by  $\ell = \frac{k_z}{1 + k_z / \lambda}$  where  $z$  is

the height of the relevant computational level in the PBL and  $\lambda$  is a constant which clearly denotes an asymptotic mixing length whose values for heat and moisture fluxes are set to 450 meters and equal to 150 meters for momentum exchange following Louis(1979).

The bulk Richardson number  $R_{iB}$  is defined over an atmosphere layer is given by

$$R_{iB} = \beta \frac{\overline{\partial \theta / \partial z}}{\left( \frac{\partial u}{\partial z} \right)^2}.$$

The bar denotes average over smallest resolvable scales. It can be shown that  $R_{iB} \leq 0.212$  is a physically valid value.

Following Louis (1979),  $F_h$  and  $F_m$  are given as:

$$F_h = F_m = \frac{1}{(1 + 5R_{iB})^2}, R_{iB} \geq 0$$

For the stable case and

$$F_h = \frac{1 + 1.286 |R_{iB}|^{\frac{1}{2}} - 8R_{iB}}{1 + 1.286 |R_{iB}|^{\frac{1}{2}}}, R_{iB} < 0, \quad (37)$$

$$F_m = \frac{1 + 1.746 |R_{iB}|^{\frac{1}{2}} - 8R_{iB}}{1 + 1.746 |R_{iB}|^{\frac{1}{2}}}, R_{iB} \leq 0 \quad (38)$$

for the unstable case.

The heat flux is given by :

$$F_H = p C_p K_H \frac{\partial \theta}{\partial z} = p C_p \ell^2 \frac{\partial |v|}{\partial z} F_h R_{iB} \frac{\partial \theta}{\partial z} \quad (39)$$

$$= p C_p \frac{k^2 z^2}{(1 + k_z / \lambda)^2} \frac{\partial |v|}{\partial z} \frac{R_{iB}}{(1 + 5R_{iB})^2} \frac{\partial \theta}{\partial z} \text{ for the stable case}$$

$$= p C_p \frac{k^2 z^2}{(1 + k_z / \lambda)^2} \frac{\partial |v|}{\partial z} \frac{(R_{iB} + 1.286 |R_{iB}|^{\frac{1}{2}} - 8R_{iB}^2)}{(1 + 1.286 |R_{iB}|^{\frac{1}{2}})^2} \frac{\partial \theta}{\partial z} \quad (40)$$

for the unstable case.

Using Fourier series for the term involving  $R_{iB}$ ,  $\frac{\partial \theta}{\partial z}$  and  $\frac{\partial |v|}{\partial z}$  and by taking cospectrum among them in the r.h.s. of expression (40) with the help of the formula (28), the heat flux in the frequency domain is given by :

$$\begin{aligned}
 \langle F_H(n) \rangle = & \frac{pC_p}{2} \frac{k^2 z^2}{(1+kz/\lambda)^2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [\text{RIC}(n). \text{PTC}(r). \text{VTC}(s)] \\
 & + \frac{pC_p}{2} \frac{k^2 z^2}{(1+kz/\lambda)^2} \left[ \begin{aligned} & - \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [\text{RIC}(n). \text{PTS}(r). \text{VTS}(s)] \\
 & + \frac{pC_p}{2} \frac{k^2 z^2}{(1+kz/\lambda)^2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & - \sum_{r-s=n} \\ & + \sum_{r-s=-n} \end{aligned} \right] [\text{RIS}(n). \text{PTC}(r). \text{VTS}(s)] \\
 & + \frac{pC_p}{2} \frac{k^2 z^2}{(1+kz/\lambda)^2} \left[ \begin{aligned} & + \sum_{r+s=n} \\ & + \sum_{r-s=n} \\ & - \sum_{r-s=-n} \end{aligned} \right] [\text{RIS}(n). \text{PTS}(r). \text{VTC}(s)] \quad (41)
 \end{aligned}$$

Where (RIC, RIS), (PTC, PTS) and (VTC, VTS) are the cos and sine coefficients of the term involving  $R_{iB}$ ,  $\frac{\partial \theta}{\partial z}$  and  $\frac{\partial |v|}{\partial z}$  associated with the frequencies  $n$ ,  $r$  and  $s$  respectively. For stable and unstable case values of RIC( $n$ ) and RIS( $n$ ) will be different. The moisture flux is given by:

$$\begin{aligned}
 F_Q &= p K_Q \frac{\partial q}{\partial z} = p \ell^2 \frac{\partial |v|}{\partial z} F_h R_{iB} \frac{\partial q}{\partial z} \\
 &= p \frac{k^2 z^2}{(1+kz/\lambda)^2} \frac{\partial |v|}{\partial z} \frac{R_{iB}}{(1+5R_{iB})^2} \frac{\partial q}{\partial z} \text{ for the stable case} \\
 &= p \frac{k^2 z^2}{(1+kz/\lambda)^2} \frac{\partial |v|}{\partial z} \frac{(R_{iB} + 1.286 R_{iB} |R_{iB}|^{\frac{1}{2}} - 8R_{iB}^2)}{1 + 1.286 |R_{iB}|^{\frac{1}{2}}} \frac{\partial q}{\partial z} \quad (42)
 \end{aligned}$$

for the unstable case.



Using Fourier time series involving  $R_{iB}$ ,  $\frac{\partial q}{\partial z}$  and  $\frac{\partial |v|}{\partial z}$  and by taking cospectrum among them in the r.h.s. of expression (42) with the help of formula (28), the expression of moisture flux in the frequency domain,  $F_Q(n)$  is the same as  $F_H(n)$  except PTC and PTS are to be replaced by PQC and PQS, the fourier temporal coefficients of  $\frac{\partial q}{\partial z}$ .

The momentum flux is given by:

$$\begin{aligned}
 F_M &= p K_M \frac{\partial v}{\partial z} = p \ell^2 \frac{\partial |v|}{\partial z} F_M R_{iB} \frac{\partial v}{\partial z} \\
 &= p \frac{k^2 z^2}{(1 + kz/\lambda)^2} \frac{\partial |v|}{\partial z} \frac{R_{iB}}{(1 + 5R_{iB})^2} \frac{\partial v}{\partial z} \text{ for the stable case} \\
 &= p \frac{k^2 z^2}{(1 + kz/\lambda)^2} \frac{\partial |v|}{\partial z} \frac{(R_{iB} + 1.746R_{iB} |R_{iB}|^{\frac{1}{2}} - 8R_{iB}^2)}{1 + 1.746 |R_{iB}|^{\frac{1}{2}}} \frac{\partial v}{\partial z} \quad (43)
 \end{aligned}$$

for the unstable case.

Using Fourier time series for the terms involving  $R_{iB}$ ,  $\frac{\partial v}{\partial z}$  and  $\frac{\partial |v|}{\partial z}$  and by taking cospectrum among them in the r.h.s. of expression (43) with the help of formula (28), the expression of momentum flux in the frequency domain,  $F_M(n)$  is the same as  $F_H(n)$  except PTC and PTS are to be replaced by PVC and PVS, the Fourier temporal coefficients of  $\frac{\partial v}{\partial z}$ .

## 5. Example of Application

### 5.1. Data and Computation

The computation is carried out using one year daily forecast values of  $u$ ,  $v$ ,  $T$  and  $q$  (moisture) based on one year numerical integration of the Florida State University Coupled Global spectral Model (FSUCGSM) from April 1996 to March 1997 using global atmospheric analysis from ECMWF and global SST analysis from NCEP. Using similarity theory daily values of fluxes of sensible heat, latent heat and  $u$  momentum are obtained at the surface layer. Daily values of Bulk coefficients are obtained at the surface layer using the daily values of fluxes based on the Aerodynamic formulas (29), (30) and (31). To separate the high frequency transients, the low frequency transients from the daily total values of  $u$ ,  $v$ ,  $T$ ,  $q$  and bulk coefficients, a temporal Fourier decomposition is employed. The fluxes of latent heat, sensible heat and  $u$ -momentum in the frequency domain are computed over Central Pacific ( $11.2^\circ\text{S} - 11.2^\circ\text{N}$ ,  $160^\circ\text{W} - 180^\circ\text{W}$ ) (Surface, 950 hpa) at the surface layer, between surface layer and PBL (990hpa, 900hpa) and between PBL and free atmosphere (900hpa, 800hpa) with the help of the formulas (32), (33), (34) and (41).



## 5.2. Results

Fig. 1(a, b, c) and Fig.2(a, b, c) illustrate the area average values of fluxes of sensible heat and latent heat respectively associated with MJO time scale 40-60 day period over Central Pacific which corresponds to frequencies 6-9. The Madden Julian Oscillations (MJO) interacted nonlinearly through triad interaction with the other frequencies including annual cycle and Nyquist frequency of period 2 day. It is noticed that fluxes of sensible heat and latent heat are more strong in the off equatorial latitudes. Possibly full MJO events may occur only when the off equatorial atmosphere over Central Pacific has been destabilized through the build up of moist static energy. Thus we notice systematic strengthening of fluxes on MJO time scale from surface layer to free atmosphere through PBL. The time scale of the MJO associated with sensible heat flux might be more important to interactions involving oceanic Kelvin waves. Warm air advection seems to be important for the sensible heat flux over the Central Pacific on this time scale (Bergman et. al. 2001). The maximum values of sensible heat flux assume the values about  $30 \text{ Wm}^{-2}$ ,  $52 \text{ Wm}^{-2}$  and  $65 \text{ Wm}^{-2}$  over surface layers, PBL and at free atmosphere respectively. The corresponding values of latent heat are  $58 \text{ Wm}^{-2}$ ,  $70 \text{ Wm}^{-2}$  and  $80 \text{ Wm}^{-2}$ .

U momentum fluxes associated with 40-60 day time scale Fig. 3(a, b, c) are too found to be positive at all latitudes except at one/two where those are negative indicating downward flux at those latitudes. Of course, magnitude associated with downward transport is very small. In this case too systematic increase of positive u-momentum flux on the MJO time scale have been noticed from surface layer to free atmospheric layer. The results show in all the cases, particularly for fluxes of latent heat and sensible heat the contributions of high frequency oscillations to MJO time scale is quite significant. It is about 40-50% to the total flux of sensible and latent heat associated with low frequency oscillations on Madden Julian time scale. Thus given the sea surface temperature  $T_w$  (and the surface specific humidity) if they carry a small signal on the MJO time scale, that frequency can amplify through triad interactions at the top of the constant flux layer. Further, these fluxes in the frequency domain build up within the PBL thus enhancing the amplitude of the Madden Julian time scale via these triad interactions. The selection rules of intrafrequency interaction tells us that higher frequency tropical waves can provide such a possibility from the frequency spectra of the wind field.

## 6. Remarks

The mathematical method is proposed for computing nonlinear transfer of fluxes of latent heat, sensible heat and u momentum in the frequency domain at the surface layer, PBL and free atmosphere by use of cross spectral technique based on Monin-Obukhov Similarity Theory, Bulk aerodynamic formulas and K-theory. Computation of energy fluxes in the in the frequency domain is a powerful tool for isolating the role of an important time scale such as the Madden-Julian and the El-Nino. This method is applied over Central Pacific to compute fluxes in the frequency domain at the surface layer, between surface layer and PBL, PBL and



free atmosphere. If nonlinear interactions allowed variations on high frequency tropical waves to affect these on MJO time scale in strengthening it from surface layer to free atmosphere through PBL over the Central Pacific, then these interactions involve both the atmosphere and the ocean indicating the influence of Atmospheric Oceanic coupling.

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Table 1. List of useful symbols

Symbol	Meaning
$F_H$	Surface heat flux
$F_m$	Surface momentum
$F_q$	Surface moisture flux
$K$	von Karman's constant
$L$	Monin-Obukhov length
$R_{iB}$	Bulk Richardson Number
$z$	Height above ground
$\Phi_m(z/L)$	Nondimensional function of vertical momentum gradient
$\Phi_h(z/L)$	Nondimensional function of vertical temperature gradient
$\Phi_q(z/L)$	Nondimensional function of vertical humidity gradient



 Contribution of 2-10 day cycle to MJO  
 MJO

Figure 1 (a)

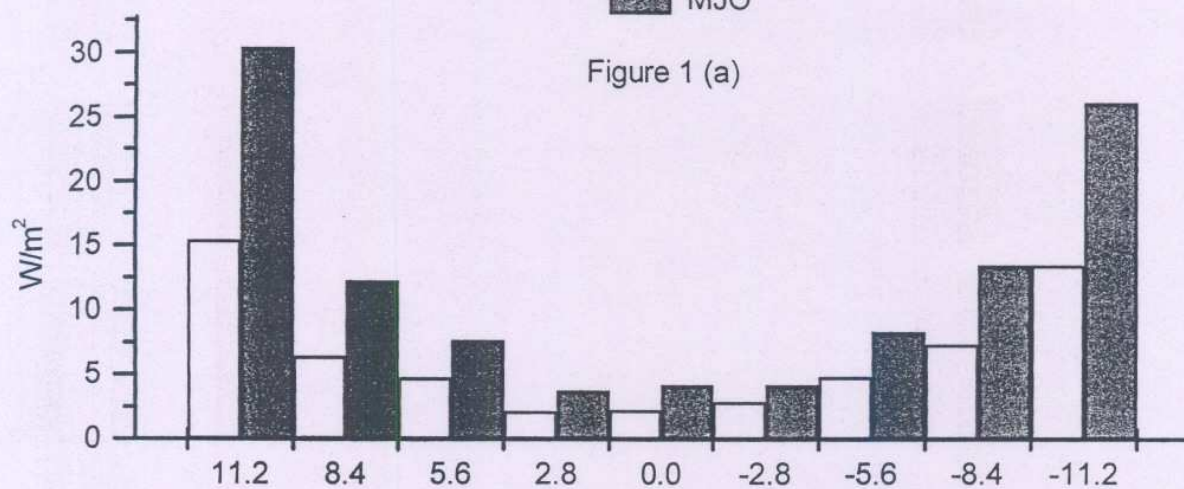


Figure 1 (b)

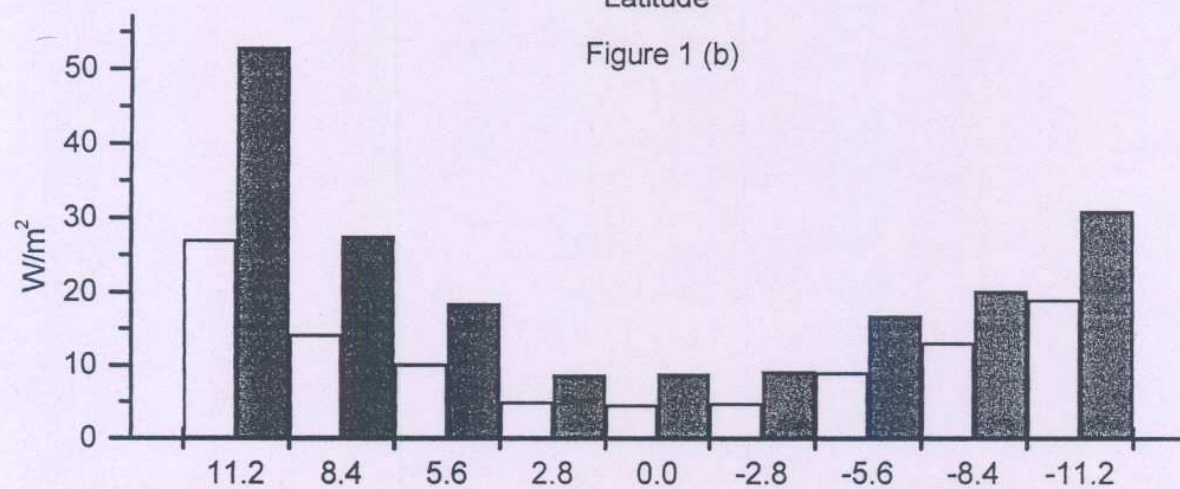


Figure 1 (c)

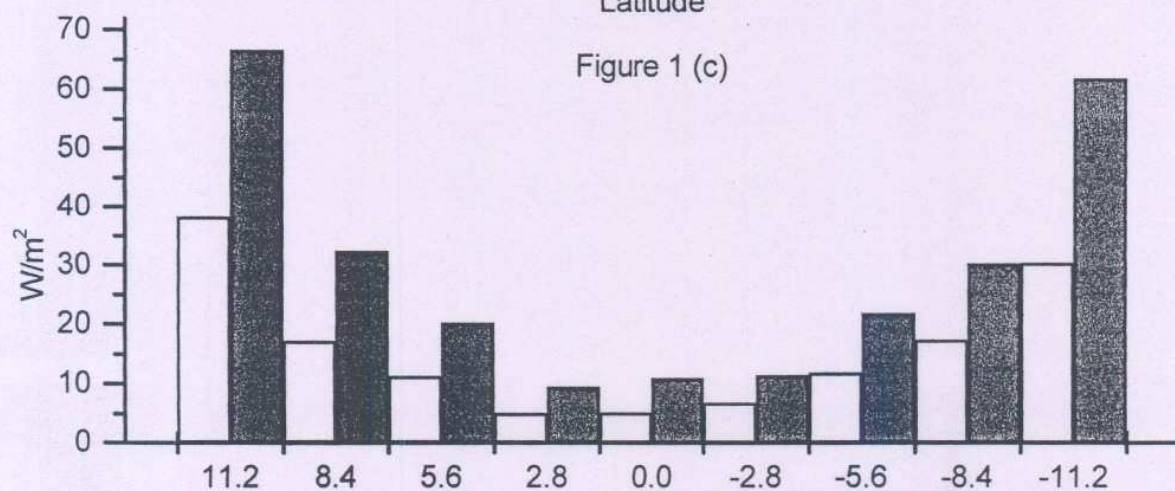


Figure 1. Sensible Heat Flux (a) at Surface Layer, (b) between Surface layer and PBL and (c) between PBL and Free Atmosphere



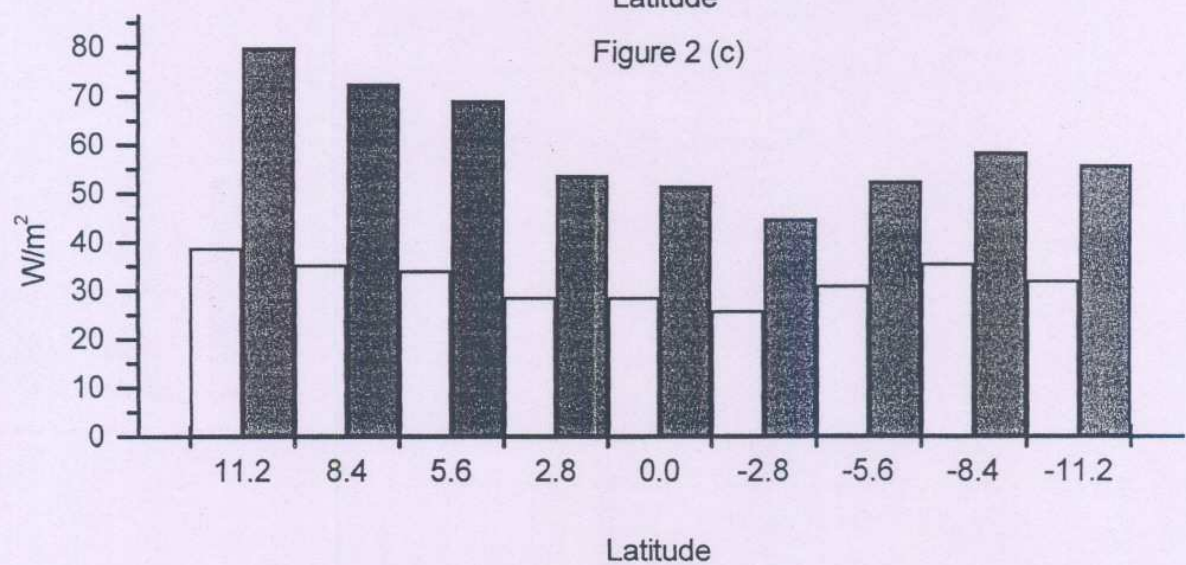
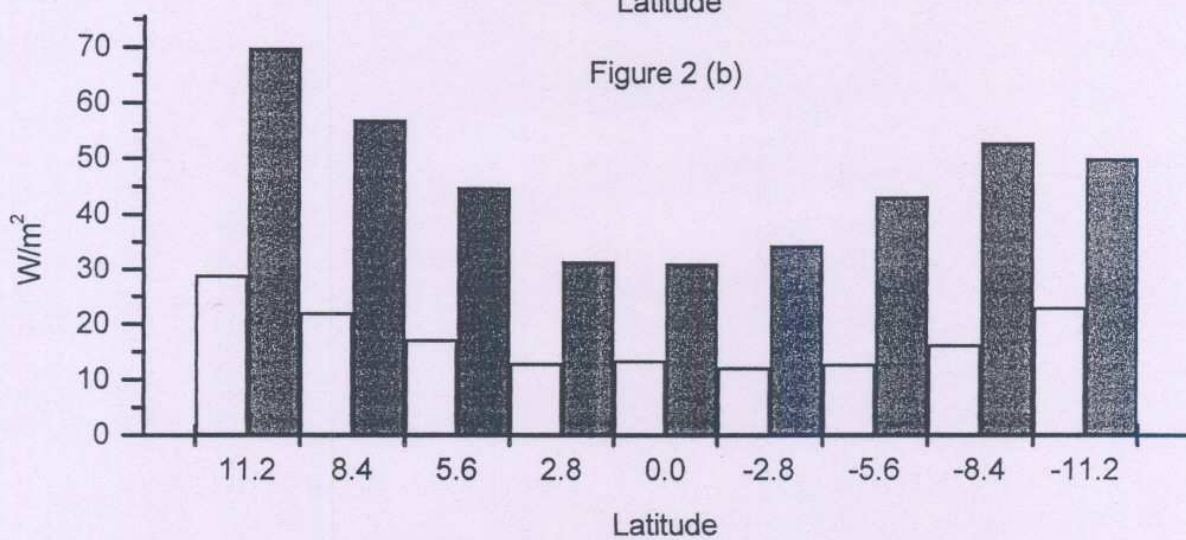
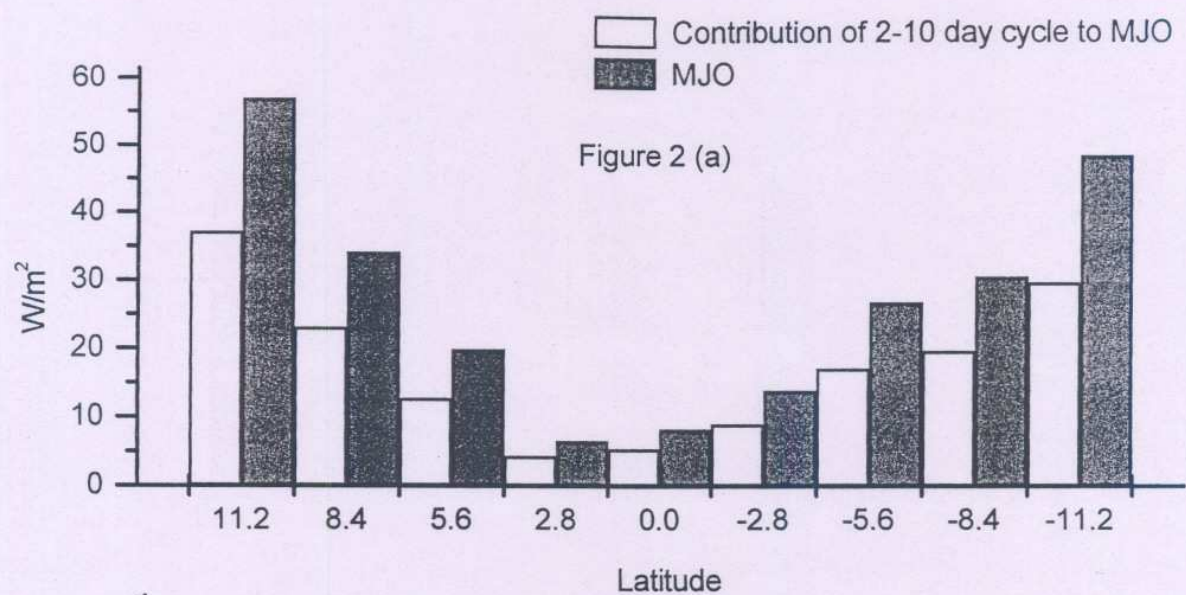


Figure 2. Latent Heat Flux (a) at Surface Layer, (b) between Surface layer and PBL and (c) between PBL and Free Atmosphere

□ Contribution of 2-10 day cycle to MJO  
 ■ MJO

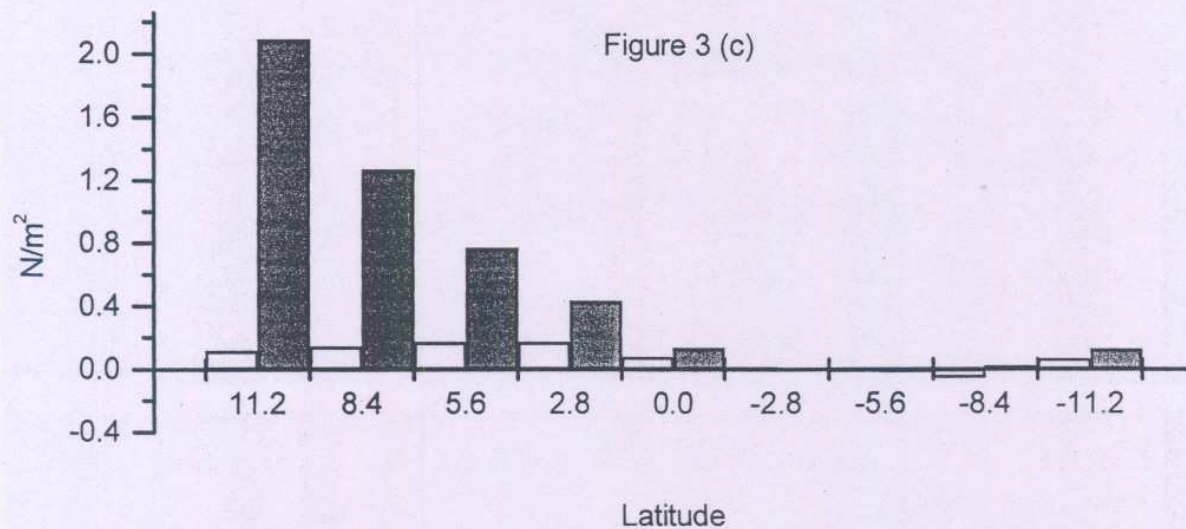
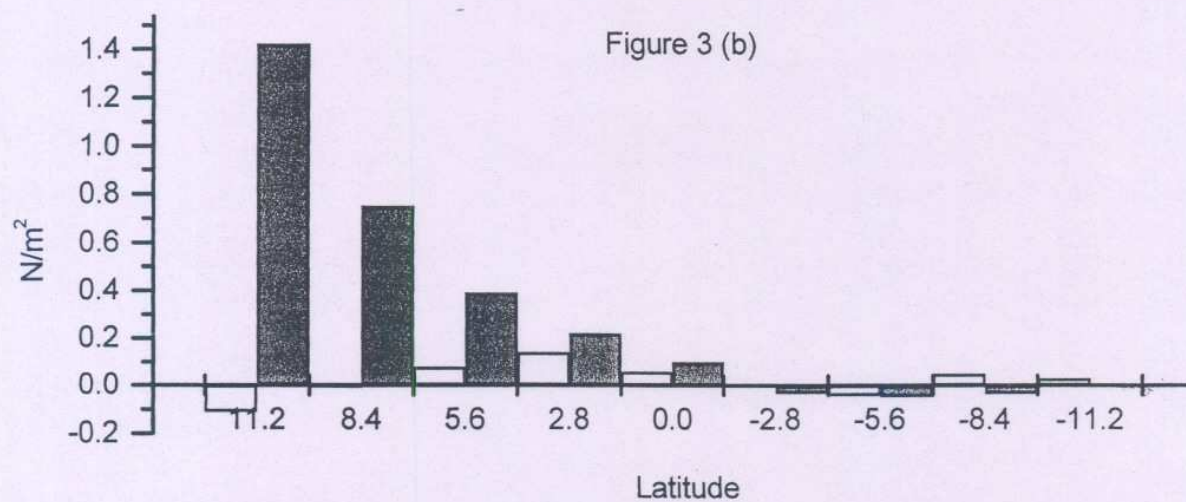
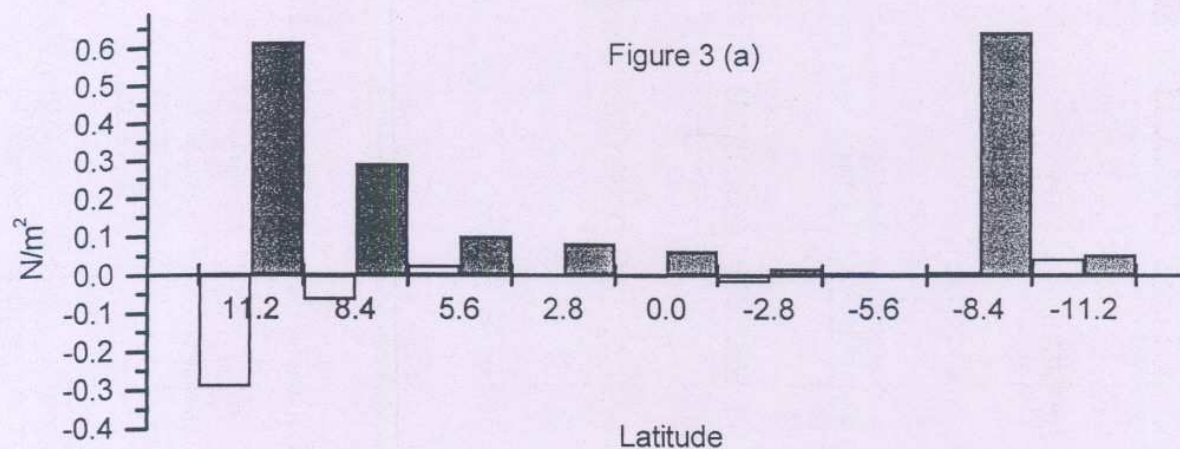


Figure 3. U-Momentum Flux (a) at Surface Layer, (b) between Surface layer and PBL (c) between PBL and Free Atmosphere



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