Organized tropical convection

- Tropical variability is dominated by multiscale convective systems
- Involves propagating waves that are embedded in each other
- Impacts topical rainfall as well as climate and weather patterns in both the tropics and extra-tropics

<table>
<thead>
<tr>
<th>Individual convective cells</th>
<th>Mesoscale cloud systems</th>
<th>Synoptic scale superclusters</th>
<th>MJO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10 km, minutes-hours</td>
<td>50-200 km, hours-2 days</td>
<td>1000-5000 km, 5-10 days</td>
<td>20,000 km, 40-60 days</td>
</tr>
</tbody>
</table>
Convectively Coupled Waves: Interaction between Convection and Large Scale (Wave)Dynamics

OBSERVATIONS OF KELVIN WAVES AND THE MJO
Time–longitude diagram of CLAUS Tb (2.5S–7.5N), January–April 1987

Kelvin waves (15 m s⁻¹)

MJO (5 m s⁻¹)
Three main cloud types above trade wind inversion layer:
Congestus, Deep, and Stratiform

..., which characterize tropical convective systems at multiple scales

Johnson et al. 1999
Multicloud—building block model

- Not based on a column model
- Based on observed features of tropical convective systems
- Convection is integrated in equations of motion: direct feedback
- Convection responds to progressive adjustment of environmental variables
- Allows interactions across scales -- between moisture and precipitation
- Successful in representing CCWs and Tropical Intra-seasonal oscillations in both simple models and in aquaplanet HOMME GCM (MJO and monsoon variability)

Khouider and Majda (2006)
This book is a survey of the research work done by the author over the last 15 years, in collaboration with various eminent mathematicians and climate scientists on the subject of tropical convection and convectively coupled waves. In the areas of climate modelling and climate change science, tropical dynamics and tropical rainfall are among the biggest uncertainties of future projections. This not only puts at risk billions of human beings who populate the tropical continents but it is also of central importance for climate predictions on the global scale. This book aims to introduce the non-expert readers in mathematics and theoretical physics to this fascinating topic in order to attract interest into this difficult and exciting research area. The general thyme revolves around the use of new deterministic and stochastic multi-cloud models for tropical convection and convectively coupled waves. It draws modelling ideas from various areas of mathematics and physics and used in conjunction with state-of-the-art satellite and in-situ observations and detailed numerical simulations. After a review of preliminary material on tropical dynamics and moist thermodynamics, including recent discoveries based on satellite observations as well as Markov chains, the book immerses the reader into the area of models for convection and tropical waves. It begins with basic concepts of linear stability analysis and ends with the use of these models to improve the state-of-the-art global climate models. The book also contains a fair amount of exercises that makes it suitable as a textbook complement on the subject.
Warm Pool Simulation using the stochastic MC Model

CRM (Grabowski et al. 2000)

SMCM acts as a super-parameterization in representing the statistics of sub-grid variability due to organized convection!

Moist Gravity waves!
Implementation of SMCM in a Coupled Climate Model (NCEP’s CFSv2)


NATIONAL MONSOON MISSION
MoES, India
SMCM parameterized Total heating

\[ Q_{\text{tot}}(z) = H_d \phi_d(z) + H_c \phi_c(z) + H_s \phi_s(z) \]

Empirical Heating And Moisture Sink Profiles

\[ H_c = \frac{\sigma_c}{\bar{\sigma}_c} \alpha_c Q_c \]

\[ H_d = \frac{\sigma_d}{\bar{\sigma}_d} Q_d \]

\[ \frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} \left( \frac{\alpha_s \sigma_s H_d}{\bar{\sigma}_s} - H_s \right) \]
Clear improvements in Wheeler-Kiladis-Takayabu Spectra

- **OBS**
  - MJO, Kelvin and 2d waves better in SMCM
  - MRG peak weaker than OBS but inexistent in CFSv2

- **CTL**

- **SMCM**
  - Tropical depressions perhaps exaggerated!
Rainfall-OLR Joint Distribution

Spread of high rainfall events to higher OLR, better captured in SMCM in CFSv2. Precipitation rates are quasi-uniform distributed and locked to low ORL.

Figure 4. Joint distribution (in %) of OLR-rainfall over ISM region (15°S-30°N, 50°E-110°E), during boreal
MJO Propagation
Stochastic Multicloud Plume Model — to modify Zhang-McFarlane Scheme

- Extend SMCM to four cloud types: shallow, congests, deep, stratiform
- Use the SMCM CAF to inform detrainment levels
- Modify calculation of bulk mass flux and bulk entrainment and detrainment rates: previous attempts with varying degrees of success in making entrainment parameter moisture dependent (Takoika, Neale and Mapes, …)
- Key idea: Missing physics for ZM scheme to self-reproduce the multicloud-heating profiles?
Mass-Flux Concept Revisited

- Steady state plume model (Arakawa-Schubert, Zhang-McFarlane, etc.)
  \[
  \frac{1}{M(z)} \frac{\partial M}{\partial z} = \epsilon(z) - \delta(z)
  \]

  \[
  M(z) = \sum_{j, \lambda_j \leq \lambda_z} m_j(z) \approx \frac{N_z}{\lambda_z} \int_0^{\lambda_z} M_b(\lambda) e^{\lambda(z-z_b)} d\lambda
  \]

  \[
  m_\lambda(z) = M_b(\lambda) e^{\lambda(z-z_b)}, \quad z_b \leq z \leq z_\lambda
  \]

- One-to-one between entrainment rate and detrainment level

- Turbulent closure for convection: \((\phi = S, q, \cdots)\)
  \[
  \frac{\partial \bar{\phi}}{\partial t} + \bar{V} \nabla \cdot \nabla \bar{\phi} = -\frac{1}{\rho} \frac{\partial \rho \phi' w'}{\partial z} + \cdots
  \]

  \[
  = -\frac{\partial}{\partial z} [M_u(z)(\phi_u - \bar{\phi}) + M_d(\phi_d - \bar{\phi})] + \cdots
  \]
Use SMCM to inform detrainment levels

ZM plume ensemble: Detrainment levels are uniformly distributed between level of minimum h* and LNB.
Fine lattice overlaid on GCM Grid

Each lattice is either occupied by a cloud of certain type or is clear sky...time varying!
4 cloud-type SMCM: Intuitive Probability Rules

- A clear sky turns into a shallow cumulus site with high probability if there is CIN, or strong large-scale Subsidence
- A shallow site turns into congestus with high probability if there is CAPE and mid troposphere is dry
- A shallow site turns into deep with high probability if there is CAPE and mid troposphere is moist
- A clear sky site turns into a congestus site with high probability if CAPE>0 and mid troposphere is dry.
- A congestus or clear sky site turns into a deep site with high probability if CAPE>0 and mid troposphere is moist.
- A deep site turns into a stratiform site with high probability.
- All three cloud types decay naturally according to prescribed decay rates.

✓ Distribution continuously changes with large scale state
Transition Rate Functions

\[
R_{01} = \frac{1}{\tau_{01}} F(C_N, W)
\]
\[
R_{02} = \frac{1}{\tau_{02}} \Gamma(C_L) \ast \Gamma(D) \ast [1 - F(C_N, W)]
\]
\[
R_{03} = \frac{1}{\tau_{03}} \Gamma(C) \ast (1 - \Gamma(D)) \ast [1 - F(C_N, W)]
\]
\[
R_{12} = \frac{1}{\tau_{12}} \Gamma(C_L) \ast \Gamma(D) \ast [1 - F(C_N, W)]
\]
\[
R_{13} = \frac{1}{\tau_{13}} \Gamma(C) \ast (1 - \Gamma(D)) \ast [1 - F(C_N, W)]
\]
\[
R_{23} = \frac{1}{\tau_{23}} \Gamma(C) \ast (1 - \Gamma(D)) \ast [1 - F(C_N, W)]
\]
\[
R_{34} = \frac{1}{\tau_{34}}
\]

\[
\Gamma(X) = (1 - e^{-X}) \mathbb{1}_{X > 0}, \quad F(X, Y) = \Gamma(X) + \Gamma(Y)
\]
Horizontal Plume Distribution

- On each site a random number of plumes of certain type is launched

\[ \text{Prob}\{P_{k,n,m}^\sigma = j | \sigma_{n,m}\} \]

\[ = \text{Prob}\{j \text{ plumes of type } \sigma \text{ detraining at } z_D \geq z_k \text{ occur over the site } (n, m)\} \]

\[ = \text{Prob}\{j \text{ plumes of type } \sigma \text{ detraining at } z_D \geq z_k \text{ occur over the site } (n, m) | d_{\sigma} \geq z_k\} \]

\[ \times \ldots \times \text{Prob}\{d_{\sigma} \geq z_k\} \]

- Plume number is Poisson distributed (Craig and Cohen, 2006): Mean of each type is set by corresponding cloud area fraction predicted by SMCM

\[ M(z) = \int_0^\Lambda M_b \frac{\lambda}{M(z)} \int_0^\lambda e^{\lambda z} (z - z_b) f(\lambda) d\lambda + \frac{\Lambda M_b}{M(z)} e^{\lambda z} (z - z_b) f(\lambda) \frac{d\lambda}{dz} \]

\[ \epsilon(z) - \delta(z) = \epsilon(z) - \delta(z) \]

\[ = \frac{\Lambda M_b}{M(z)} \int_0^\lambda e^{\lambda z} (z - z_b) f(\lambda) d\lambda + \frac{\Lambda M_b}{M(z)} e^{\lambda z} (z - z_b) f(\lambda) \frac{d\lambda}{dz} \]

Mb is computed using a CAPE-TKE weighted closure according to SMCM CAF predictions.
Transition times Scales are Uncertain Parameters

- Bayesian Inference to learn transition time scales from data
- De La Chevrotiere et al.: Giga-LES, Kharoutidnov et al. (GATE III) (1 day simulation)
- Cardosso-Bihlo et al.: Dynamo, MJO I (1 day suppressed, 2 days initiation, ~4 days active phase)
- Applying the inference method to long-period data set has been a challenge
- Undergraduate Student Isobel Glover has been working on optimizing the code…
- Replace MCMC with Variational Inference ….
Validation in Single Column CAM
Validation test cases

- GATE III: 4 prominent squall lines (Houze & Rappaport, 1984), convective envelopes with trailing stratiform decks
- TOGA-COARE II: squall lines and isolated convection (Rickenbach & Rutledge 1998).
- TWP-ICE (2006): Passage of MJO/Monsoon Active & Break Phases
- ARM95: 3 distinct weather regimes of mid-latitude land-convection: variable cloudiness, clear sky, high cloudiness: meso-scale systems (Xu and Randall, 2000)
- ARM97: 3 weather periods (heavy precipitation, clear sky, moderate precipitation) (Xu et al., 2002)
- MPACE: various weather regimes including boundary layer stratocumulus, cirrus clouds, multilayer stratus (Wang et al. 2016)
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>EXP1</th>
<th>EXP2</th>
<th>EXP3</th>
<th>EXP4</th>
<th>EXP5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear sky to shallow cumulus</td>
<td>$\tau_{01}$</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>0.035</td>
<td>2</td>
</tr>
<tr>
<td>Clear sky to congestus</td>
<td>$\tau_{02}$</td>
<td>5.6</td>
<td>32</td>
<td>32</td>
<td>0.008</td>
<td>2</td>
</tr>
<tr>
<td>Clear sky to deep</td>
<td>$\tau_{03}$</td>
<td>0.1</td>
<td>12</td>
<td>12</td>
<td>0.085</td>
<td>0.75</td>
</tr>
<tr>
<td>Shallow cumulus to congestus</td>
<td>$\tau_{12}$</td>
<td>3</td>
<td>1</td>
<td>12</td>
<td>4.13</td>
<td>2</td>
</tr>
<tr>
<td>Shallow cumulus to deep</td>
<td>$\tau_{13}$</td>
<td>0.14</td>
<td>1</td>
<td>1</td>
<td>0.00045</td>
<td>1</td>
</tr>
<tr>
<td>Congestus to deep</td>
<td>$\tau_{23}$</td>
<td>0.31</td>
<td>0.25</td>
<td>0.25</td>
<td>0.00018</td>
<td>1.5</td>
</tr>
<tr>
<td>Deep to stratiform</td>
<td>$\tau_{34}$</td>
<td>2</td>
<td>0.25</td>
<td>0.25</td>
<td>0.0026</td>
<td>1</td>
</tr>
<tr>
<td>Shallow cumulus to clear sky</td>
<td>$\tau_{10}$</td>
<td>5</td>
<td>2</td>
<td>20</td>
<td>6.667</td>
<td>5</td>
</tr>
<tr>
<td>Congestus to clear sky</td>
<td>$\tau_{20}$</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>0.0001</td>
<td>7</td>
</tr>
<tr>
<td>Deep to clear sky</td>
<td>$\tau_{30}$</td>
<td>14.3</td>
<td>9.5</td>
<td>9.5</td>
<td>6.2</td>
<td>10</td>
</tr>
<tr>
<td>Stratiform to clear sky</td>
<td>$\tau_{40}$</td>
<td>30</td>
<td>1</td>
<td>1</td>
<td>0.018</td>
<td>20</td>
</tr>
</tbody>
</table>

**EXP1:** Dynamo MJO, Active  
**EXP2:** GATEIII  
**EXP3:** Modif. GATEIII  
**EXP4:** Variational, Mean CAF Dynamo  
**EXP5:** Guess
GATE III

Obs/Sim

Conv/Strat.

Precipitation Time Series

ARM95

Obs/Sim

Conv/Strat.

Precip (mm/day) GATEIII

Precip (mm/day) ARM95

GATE III

ARM95
Precipitation Time Series

Obs/Sim

Conv/Strat.

MPACE

TWP-ICE 2006
Mean T

Mean SpHum Bias (g/Kg) MPACE

Mean SpHum Bias (g/Kg) TWP

Mean q
RMSE over levels

Graphs showing relative RMSE over different experiments (CTRL, EXP1, EXP2, EXP3, EXP4, EXP5) for different datasets:

- **Temperature**
  - ARM95
  - GATEIII

- **Sp Humld**
  - ARM95
  - GATEIII

The graphs illustrate the performance of various models over different experiments, with ARM95 and GATEIII highlighted for comparison.
Stochasticity and Model Error

GATEIII

TOGAI2

EXP1

EXP2

EXP3

EXP4

EXP5
Ensemble simulation with 100 members

(Change Seed of Random Number generator)
Stochasticity to account for model error in high dimensional systems

- Ergodicity valid only for Stationary Distribution Systems
- Especially true for Markov process will no or weak mixing!
Imagine we made only one realization of the Brownian motion?
Dash-individual member means
Conclusion

- SMCM mimics organized tropical convection
- Represents missing subgrid variability in GCMs
- Preliminary implementation of SMCM in CFS is very successful in representing tropical modes of variability: MJO, CCWs, Monsoon variability, LPS's …
- Extended to four cloud types and used to build unified mass-flux scheme: stochastic multicloud plume model (SMPM)
- Validated in Single Column CAM on various test cases of tropical and non-tropical convection: Often beats default ZM control
- Ensemble runs used to demonstrate how stochastic model can be used to take into account model error uncertainty
ZM plume ensemble:
Detrainment levels are uniformly distributed between level of minimum $h^*$ and LNB

Saturation Moist Static Energy