Impact of turbulence of cloud droplet growth: from DNS to LES and beyond

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The National Center for Atmospheric Research sponsored by NSF
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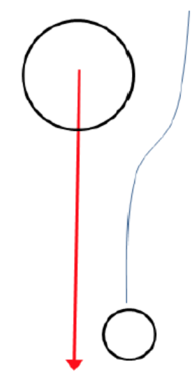
ASR Atmospheric System Research

U.S. DEPARTMENT OF ENERGY Office of Science

UNIVERSITY OF WARSAW

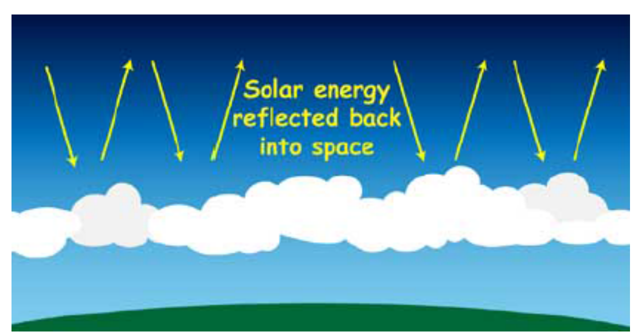
Cloud droplets grow by the diffusion of water vapor (i.e., by condensation) and by collision/coalescence.

For both growth mechanisms, cloud turbulence and cloud dilution ("entrainment") plays a significant and still poorly understood role.



For gravitational collisions, width of the droplet spectrum grown by diffusion is the key…

The width of the droplet spectrum also affects the amount of solar radiation reflected back to space by clouds...



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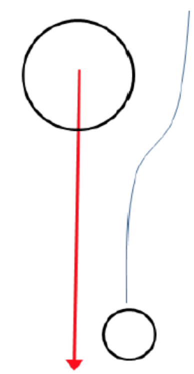

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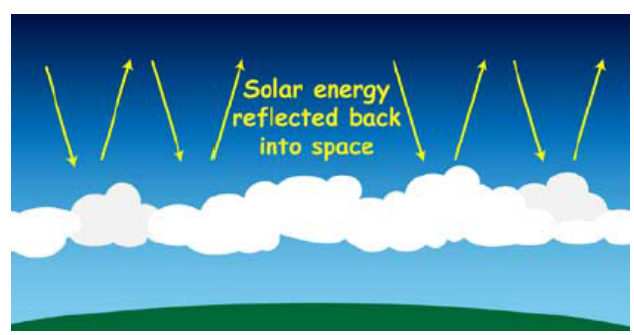
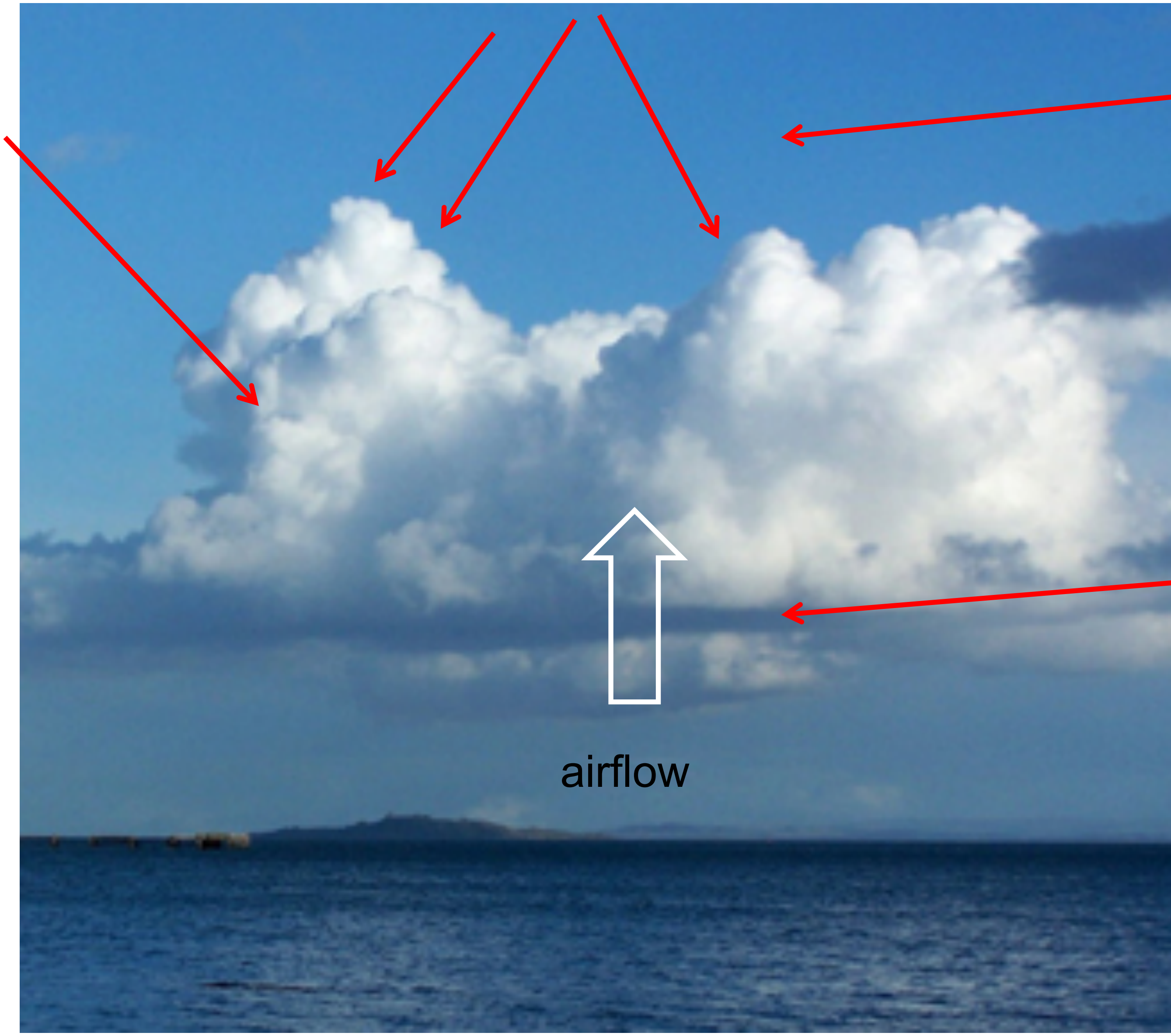


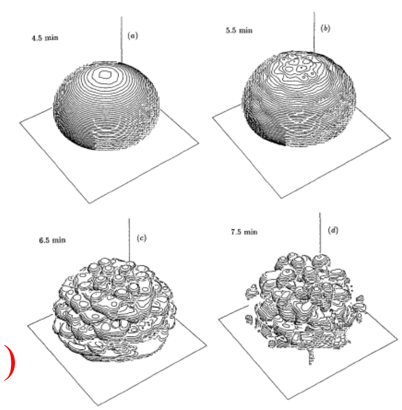
turbulent
cloud

interfacial
instabilities

calm (low-
turbulence)
environment



cloud base
(CCN activation)



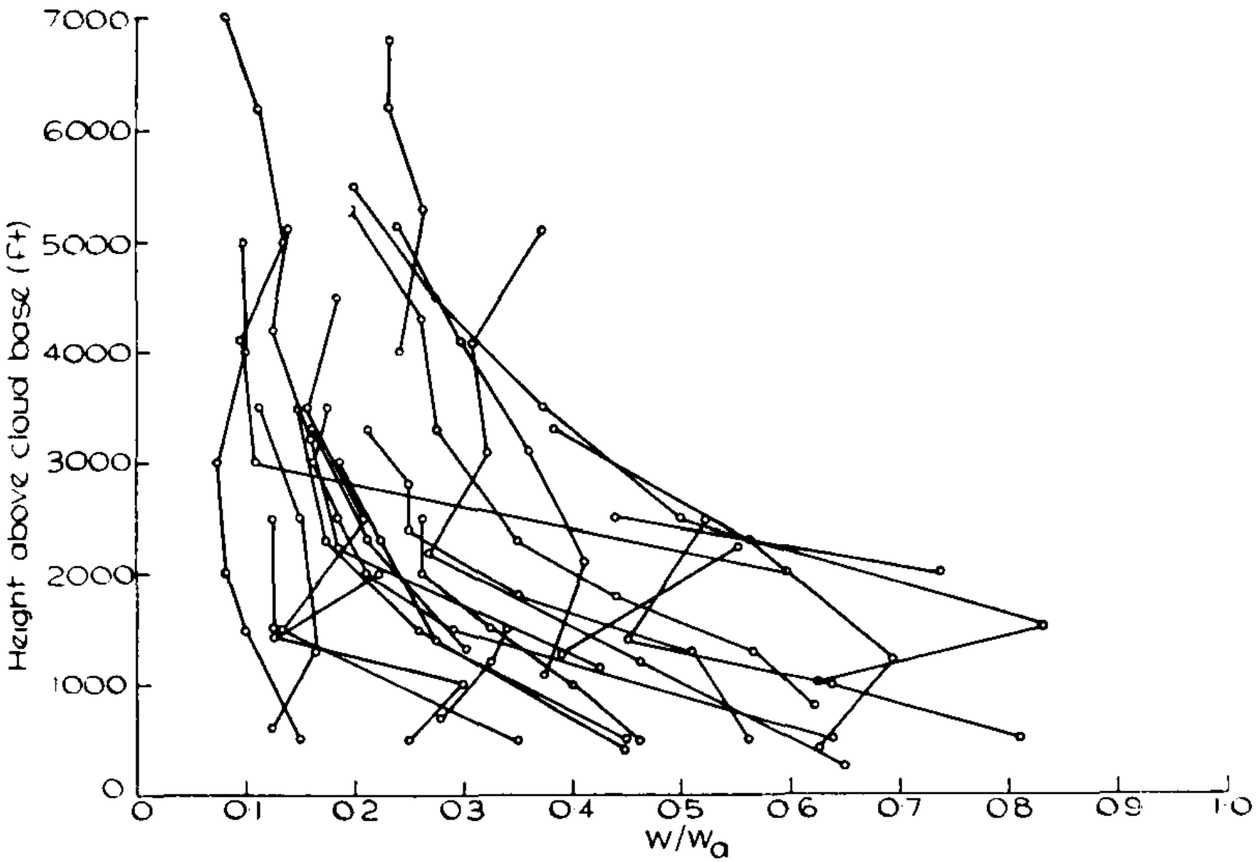
Grabowski and Clark (*JAS* 1993)

The Water Content of Cumuliform Cloud

***Tellus*, 1955**

By J. WARNER, Radiophysics Laboratory, C.S.I.R.O., Sydney

(Manuscript received April 5, 1955)

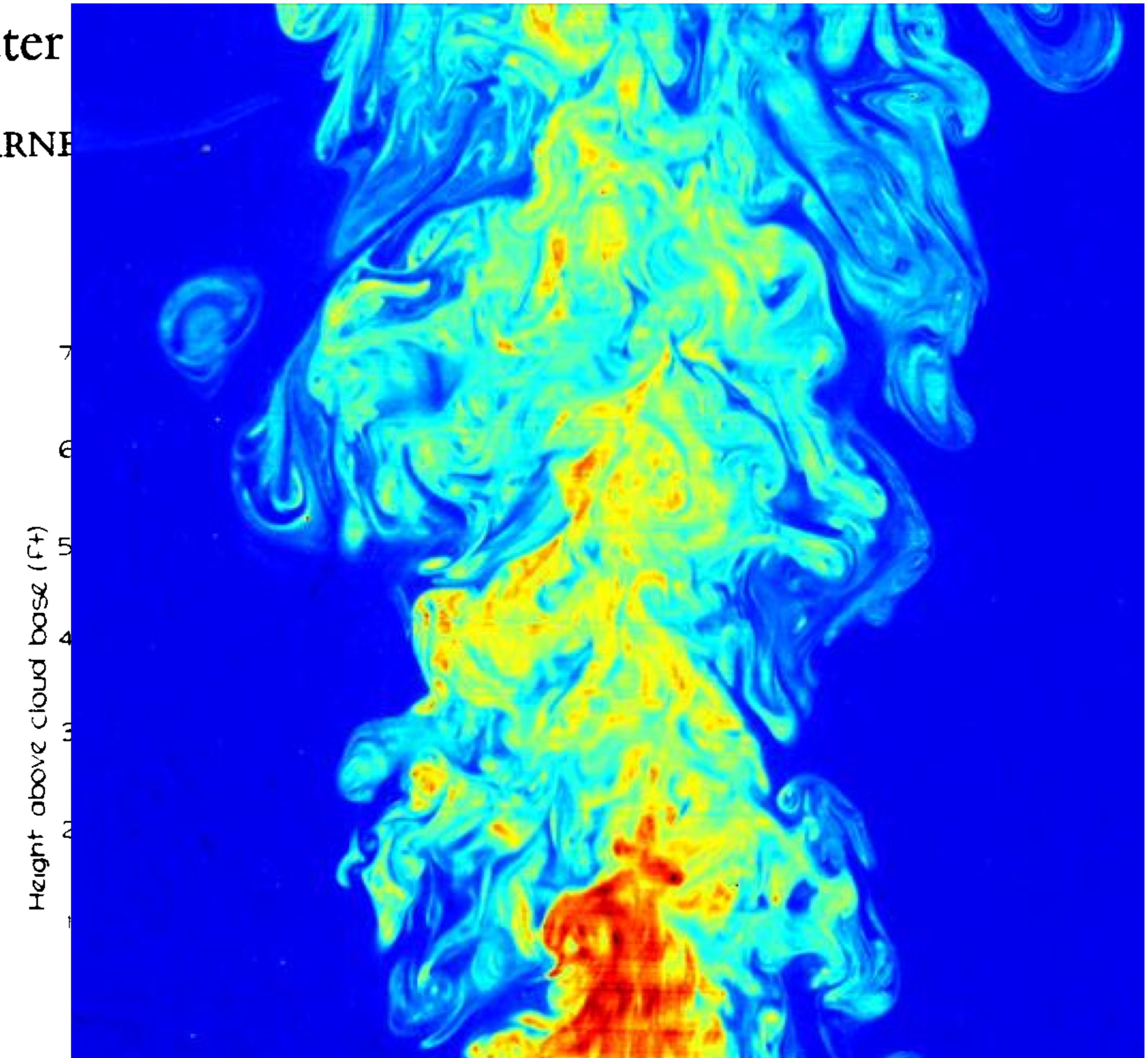


Cumulus clouds are heterogeneous and *on average* strongly diluted...

The Water

By J. WARNE

1955



turbulent laboratory jet

Cumulus clouds are heterogeneous and *on average* strongly diluted...

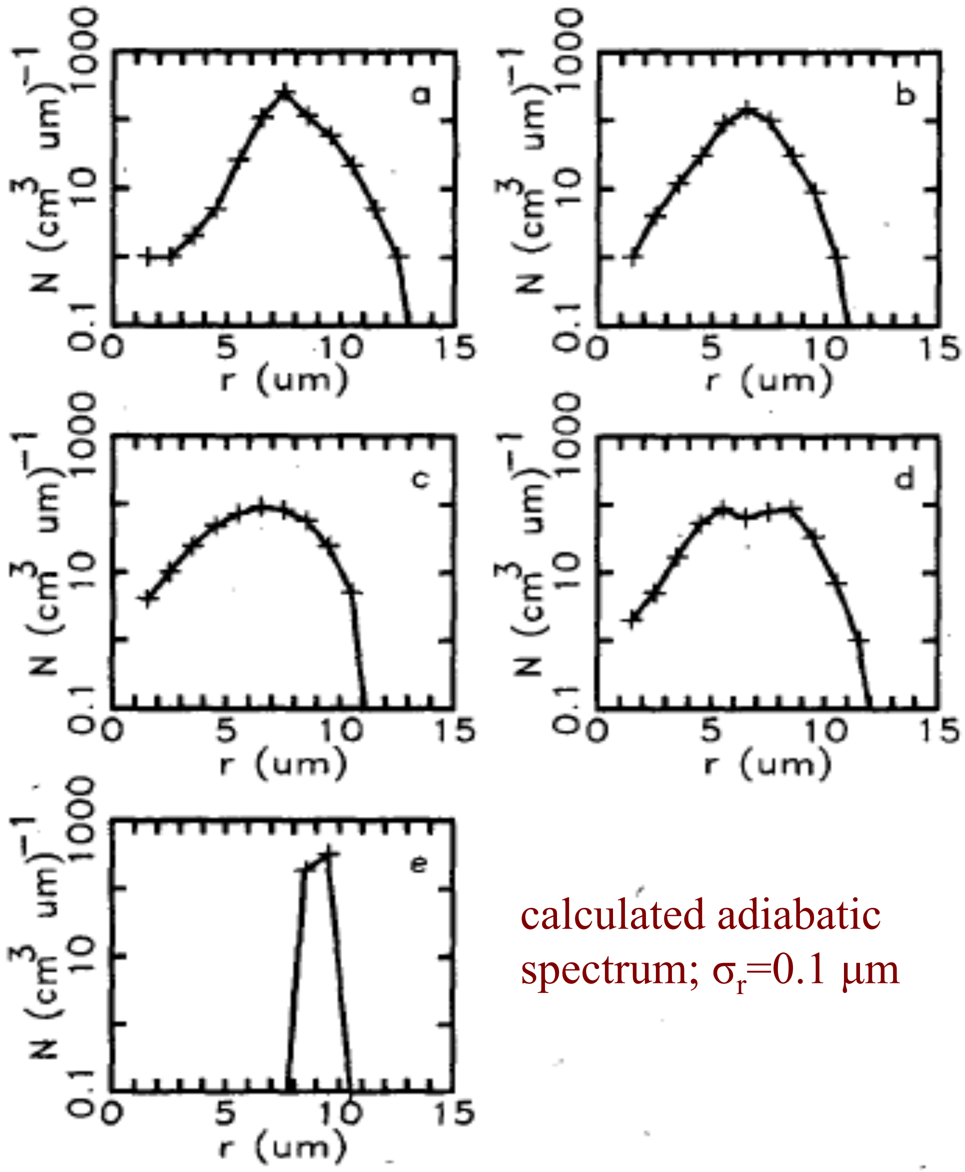
Observed cloud droplet spectra averaged over ~100m:

observed,
adiabatic fraction
AF ≈ 1; σ_r=1.3 μm

observed, AF ≈ 0.8;
σ_r=1.3 μm

observed, AF ≈ 0.8;
σ_r=1.8 μm

observed, AF ≈ 1;
bimodal



calculated adiabatic
spectrum; σ_r=0.1 μm

(Jensen et al. *JAS* 1985)

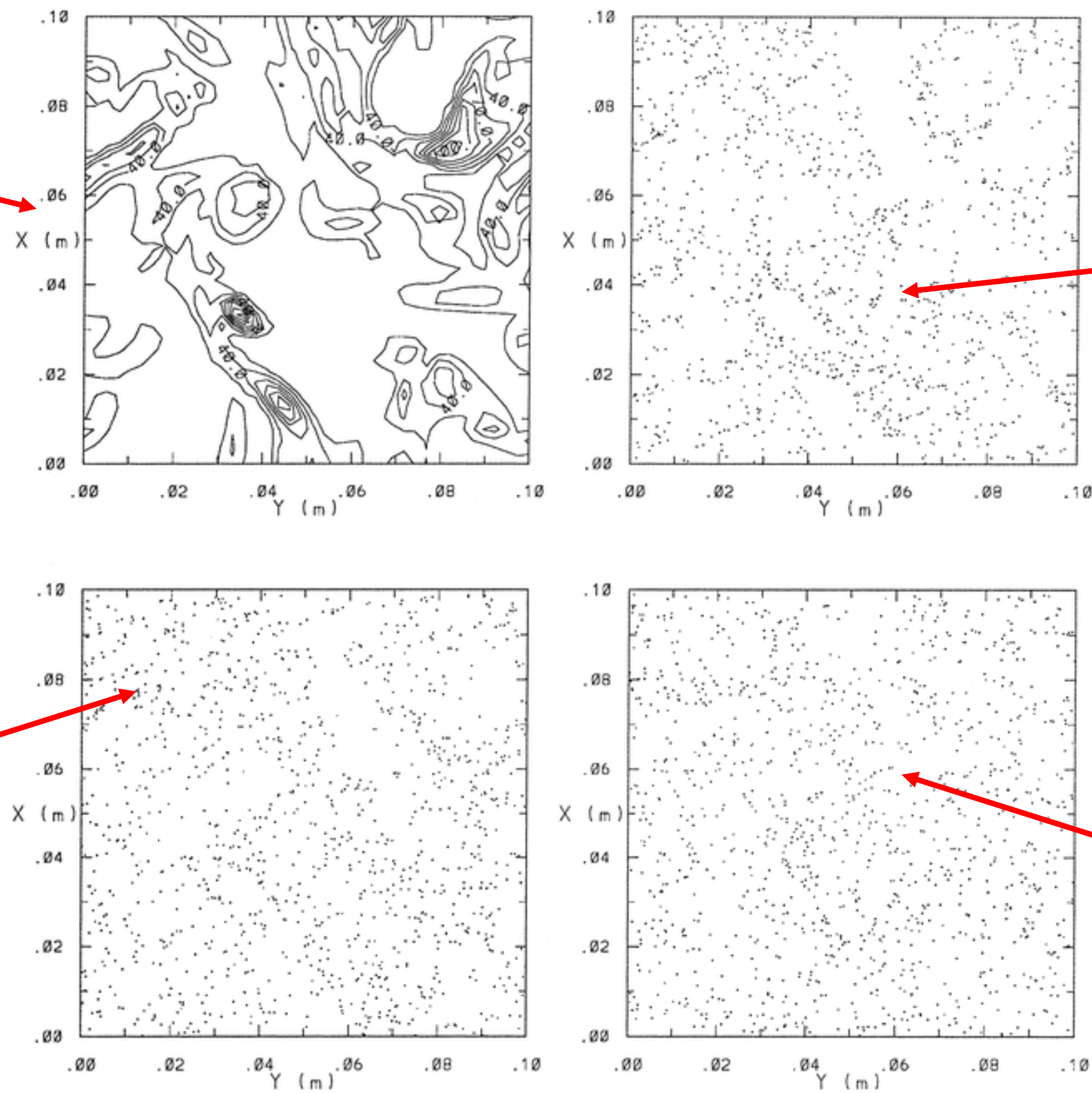
Can small-scale turbulence explain the width of the droplet spectra in (almost) undiluted cloudy volumes?

DNS simulations with sedimenting droplets *growing by the diffusion of water vapor* for conditions relevant to cloud physics (ε=160 $cm^2 s^{-3}$)

Vorticity

(contour 15 s^{-1})

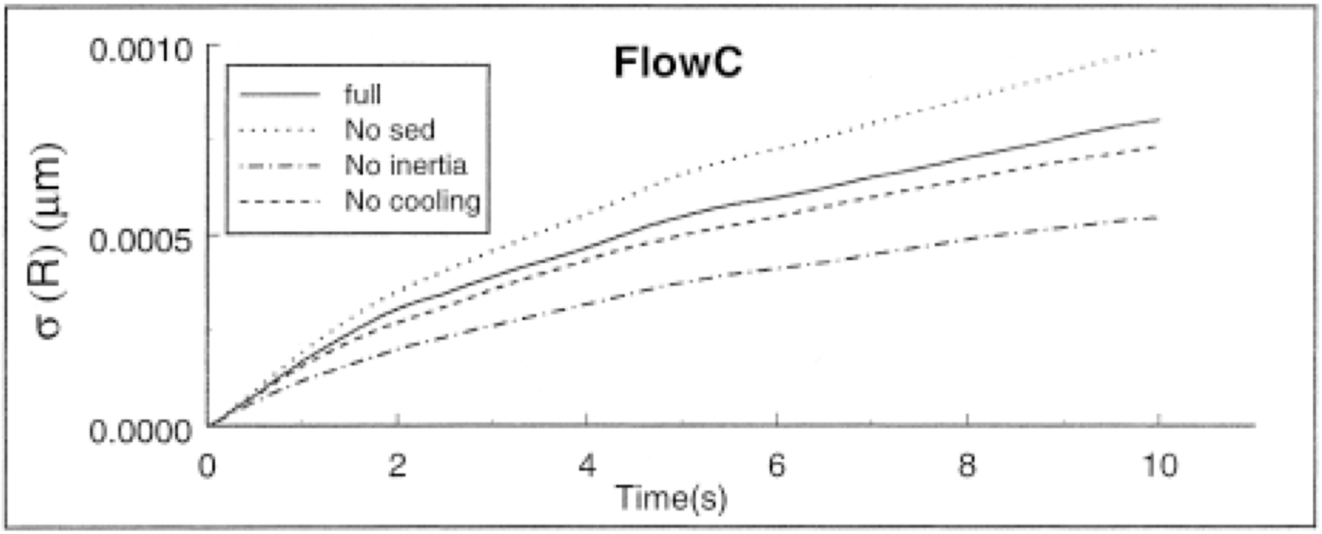
r=20 micron



r=15 micron

r=10 micron

Vaillancourt et al. *JAS* 2002



Main conclusion: small-scale turbulence has a very small effect…

Vaillancourt et al. *JAS* 2002

What about those DNS limitations?

Argument: if *Re* increases (i.e., the range of scales involved increases), can supersaturation fluctuation increase as well?

Label	N_{drops}/V (cm^{-3})	LWC (g m^{-3})	τ_s (s)	R (μm)	St
Series 1	130	1.2	2.5	13	3.5×10^{-2}
Series 2	130	0.07	7	5	5×10^{-3}

Label	N^3	L (cm)	T_L (s)	Re_λ	ε (m^2 s^{-3})	σ_s^0 (%)	v_{rms} (m s^{-1})	N_{drops} ($\times 10^5$)
A	64^3	9	2.0	40	10^{-3}	1.5×10^{-3}	4×10^{-2}	0.93
B	128^3	18	3.5	65	9.0×10^{-4}	3.4×10^{-3}	5.0×10^{-2}	8.2
C	256^3	38	5.5	105	10^{-3}	6.1×10^{-3}	7.0×10^{-2}	71.2
D	512^3	70	7.6	185	1.1×10^{-3}	1.2×10^{-2}	1.0×10^{-1}	320

Lanotte et al. *JAS* 2009

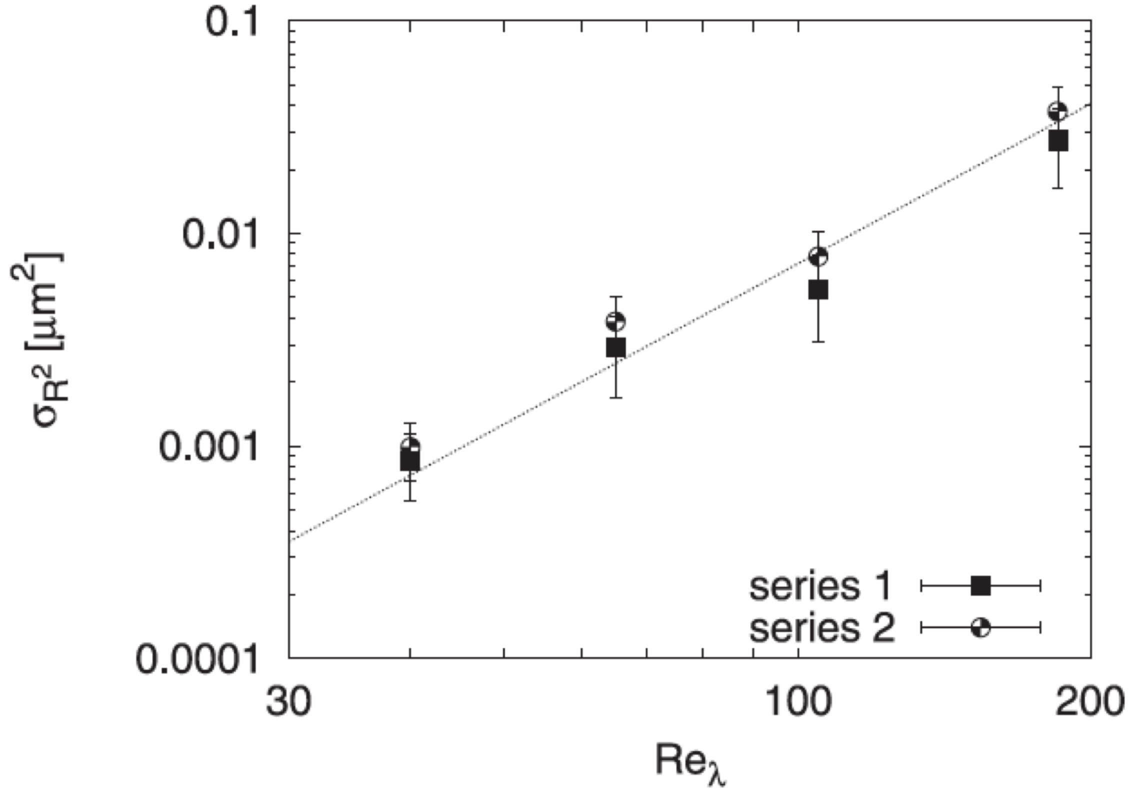


FIG. 3. Log–log plot of the spreading of droplet size distribution $\sigma_{R^2}(T_L)$ for the square radius R^2, measured after one large-scale eddy turnover time T_L as a function of the Reynolds number Re_λ.

Lanotte et al. *JAS* 2009

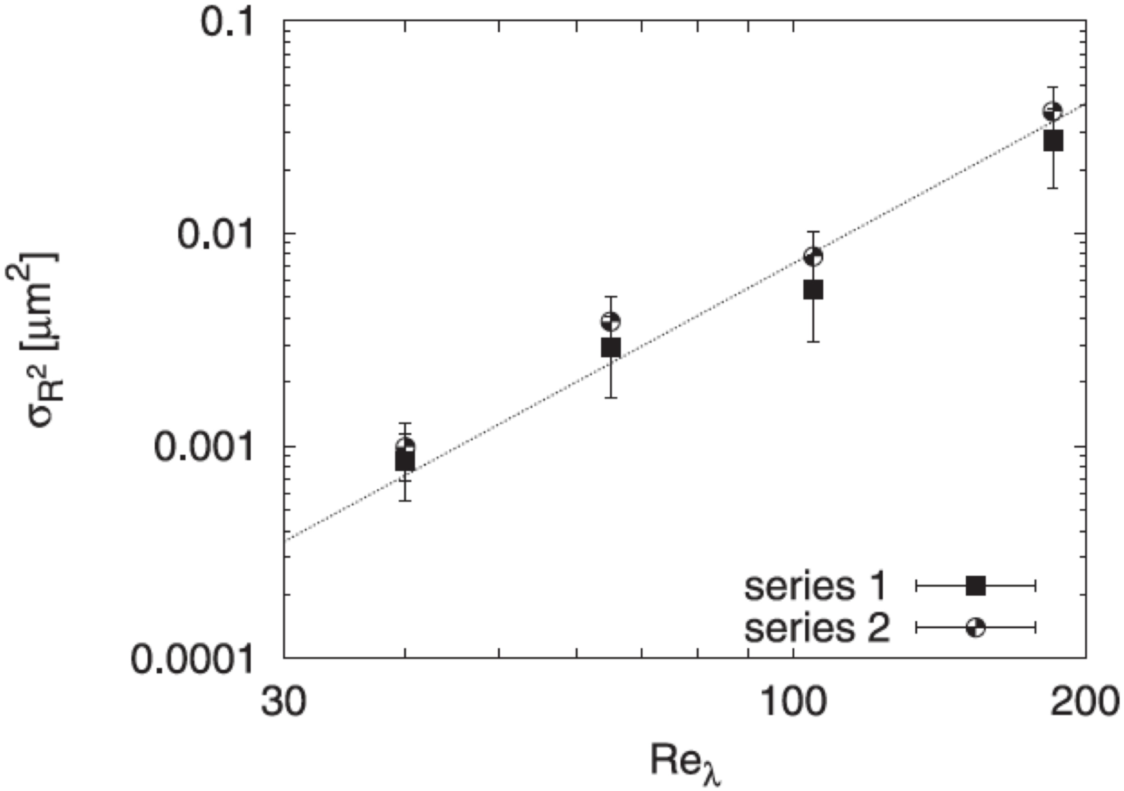


FIG. 3. Log–log plot of the spreading of droplet size distribution $\sigma_{R^2}(T_L)$ for the square radius R^2, measured after one large-scale eddy turnover time T_L as a function of the Reynolds number Re_λ.

Natural clouds feature $\mathrm{Re}_\lambda \sim 10^4$, then $\sigma_R{}^2 \sim 100\ \mu\mathrm{m}^2$!

Lanotte et al. *JAS* 2009

What about those DNS limitations?

Argument: if Re increases (i.e., the range of scales involved increases), can supersaturation fluctuation increase as well?

Yes, but only to some point…

The brake on supersaturation fluctuations:

$$\frac{dS}{dt} = \alpha w - \frac{S}{\tau_{qe}}$$

$$\tau_{qe} \sim 1\,\mathrm{sec}$$

$$\frac{dS}{dt} \equiv 0 \rightarrow S_{qe} = \alpha w \tau_{qe}$$

TABLE 1. Time constant characterizing supersaturation. (Values of $\tau = 1/(a_2 I)$ s for $p = 771$ mb, $T = 4.3°C$)

Radius (μm)	Droplet concentration (cm^{-3}) 100	300	500	1000
2	14.1	4.7	2.8	1.4
3	8.7	2.9	1.7	0.87
5	4.9	1.6	0.98	0.49
10	2.3	0.77	0.46	0.23

Politovich and Cooper, JAS 1988

For eddies with time-scale larger than τ_{qe}, S is limited to S_{qe} !!!

So within a uniform cloud (e.g., the adiabatic core), small-scale fluctuations of the supersaturation are likely have a small effect.

But what about the impact of larger eddies, meters and tens of meters?

VOL. 46, NO. 10 JOURNAL OF THE ATMOSPHERIC SCIENCES 15 MAY 1989

Effects of Variable Droplet Growth Histories on Droplet Size Distributions. Part I: Theory

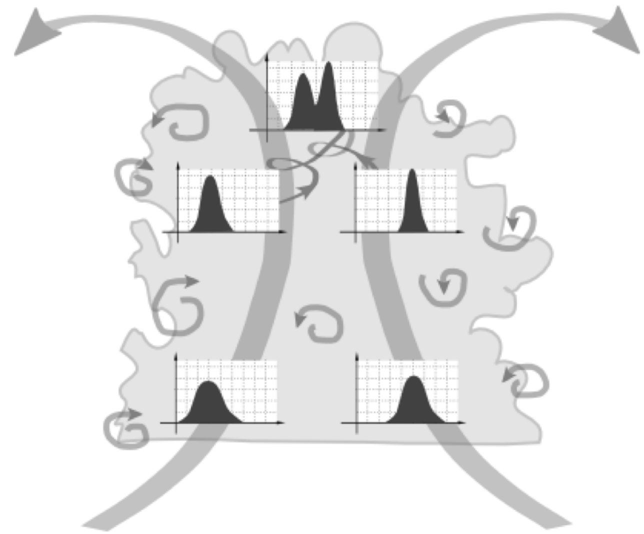
WILLIAM A. COOPER

National Center for Atmospheric Research, Boulder, Colorado*

(Manuscript received 4 April 1988, in final form 14 November 1988)

The key idea: droplets observed in a single location within a turbulent cloud arrive along variety of air trajectories…

Eddy-hopping mechanism (Grabowski and Wang ARFM 2013)



Droplets observed in a single location within a turbulent cloud arrive along variety of air trajectories:

- *large scales are needed to provide different droplet activation/growth histories;*

- *small scales needed to allow hopping from one large eddy to another.*

[see also Sidin et al. (*Phys. Fluids* 2009) for idealized 2D synthetic turbulence simulations]

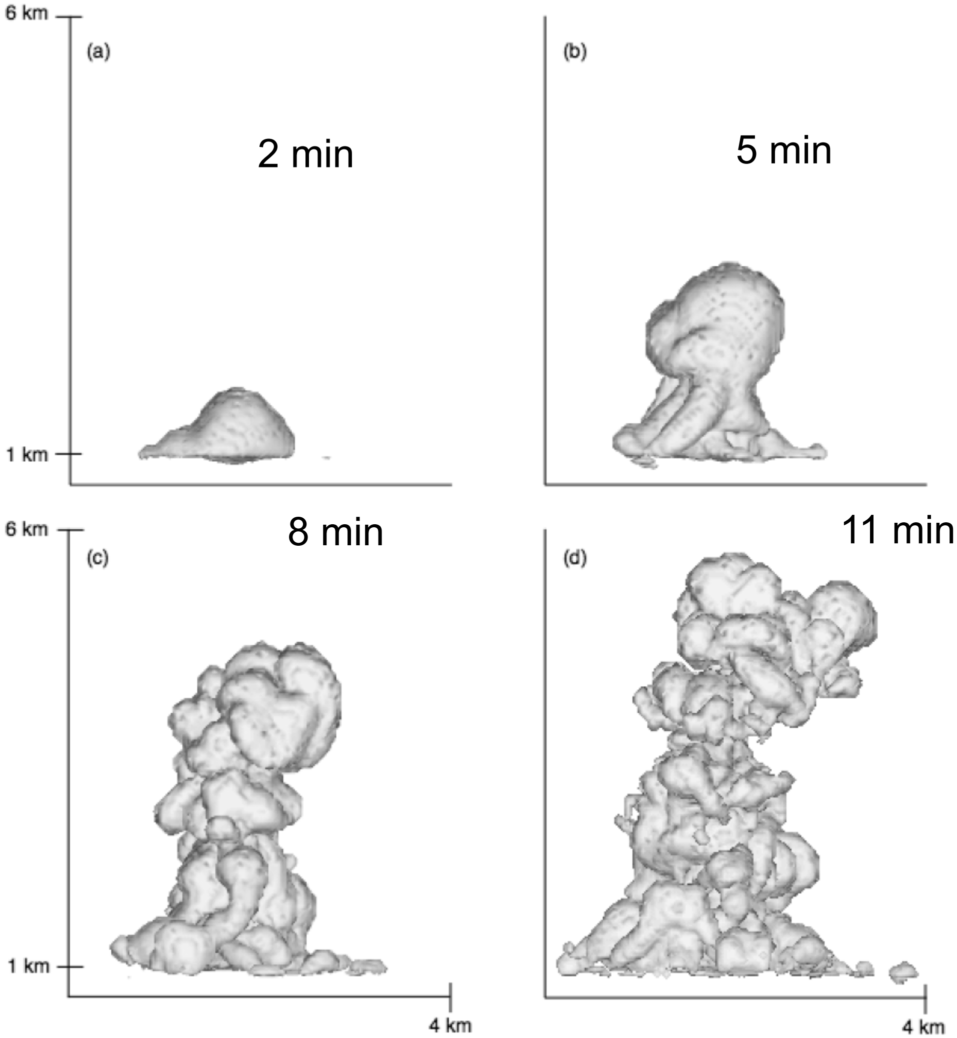
Q. J. R. Meteorol. Soc. (2005), **131**, pp. 195–220 doi: 10.1256/qj.03.199

Broadening of droplet size distributions from entrainment and mixing in a cumulus cloud

By SONIA G. LASHER-TRAPP†[1], WILLIAM A. COOPER[2] and ALAN M. BLYTH[3]

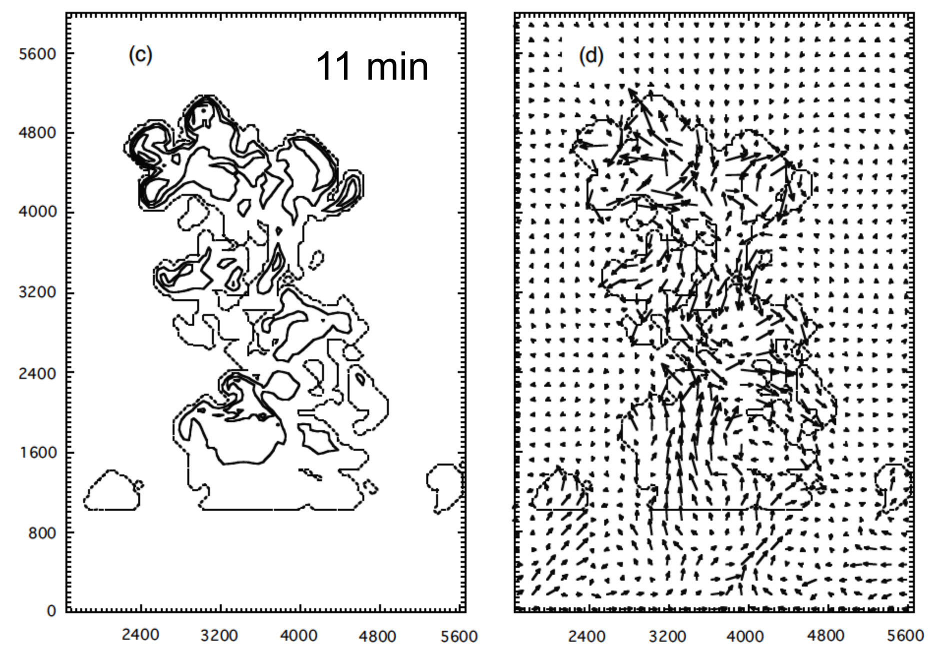
[1] *New Mexico Institute of Mining and Technology, Socorro, USA*
[2] *National Center for Atmospheric Research, Boulder, USA*
[3] *University of Leeds, Leeds, UK*

First, run a traditional Eulerian fluid dynamics cloud model…



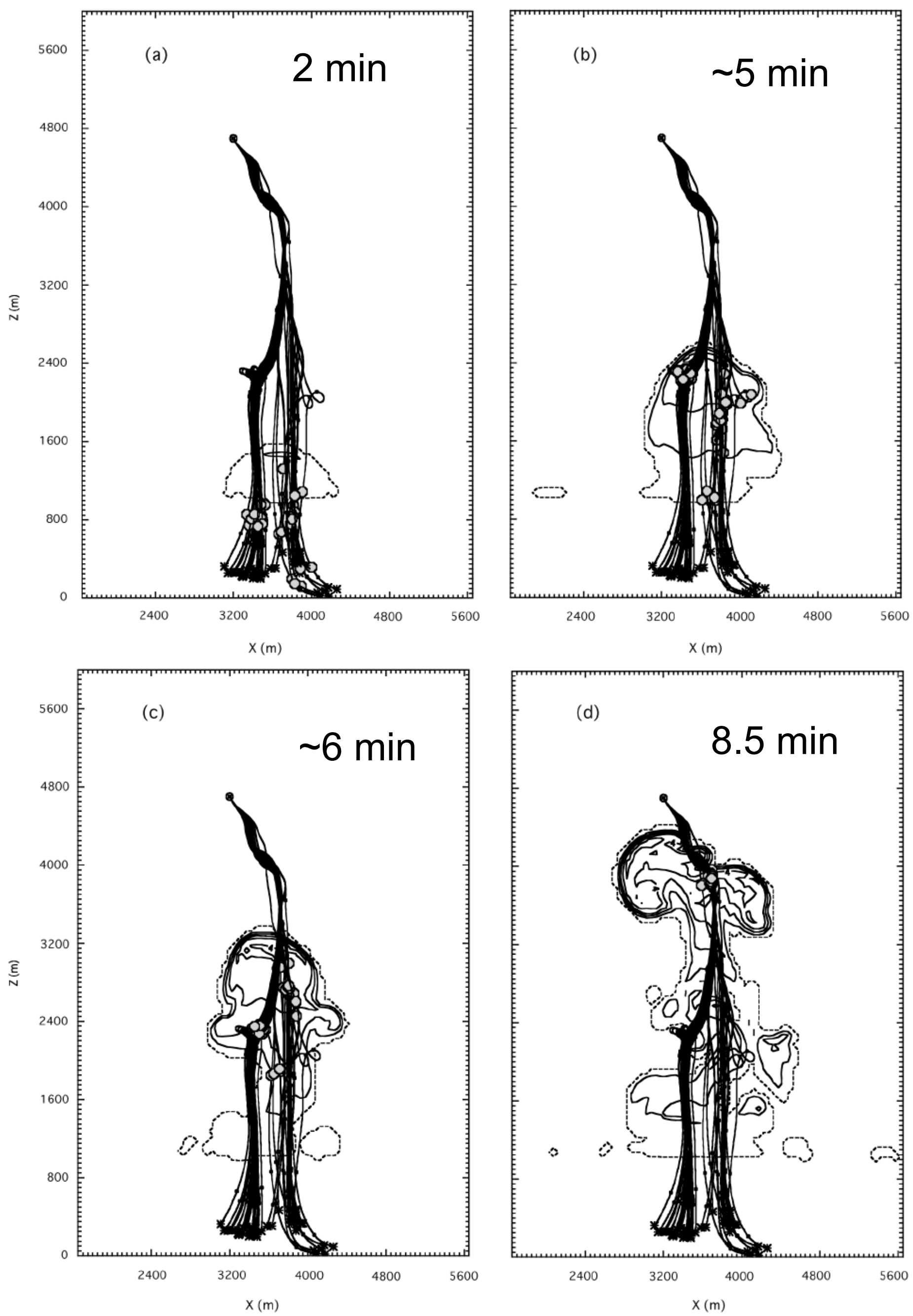
2. The simulated cloud used in this study. Shown is an isosurface of 0.005 g kg^{-1} of cloud-water mixing ratio at: (a) 2, (b) 5, (c) 8 and (d) 11 minutes after the first appearance of cloud.

$\Delta = 50$ m

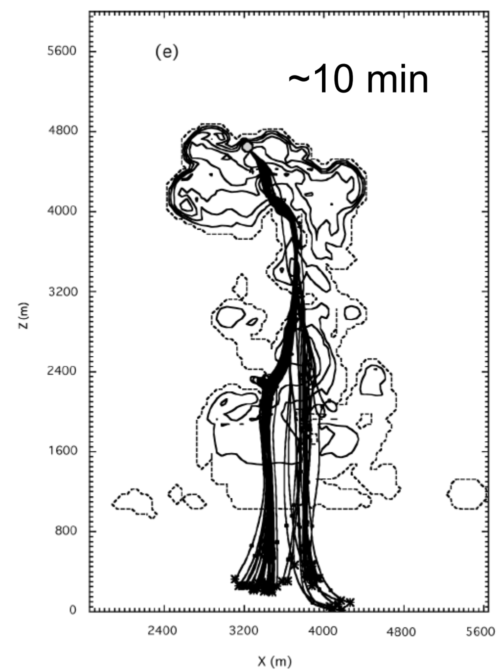


Lasher-Trapp et al. QJRMS 2005

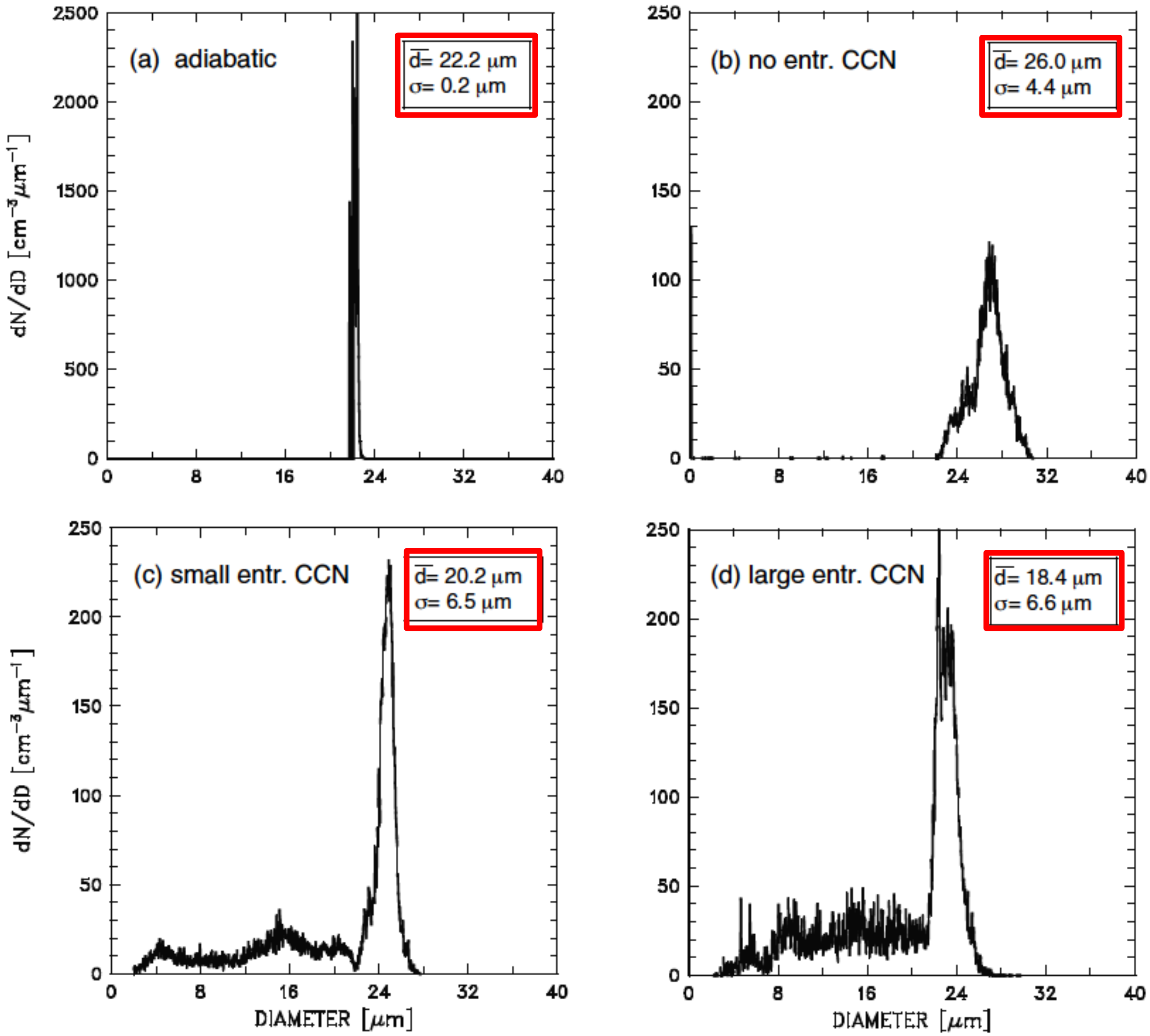
…second, run backward ensemble of trajectories from a selected point…



Lasher-Trapp et al. QJRMS 2005



...third, calculate activation and growth of cloud droplets along trajectories.



Lasher-Trapp et al. QJRMS 2005

This is really nice to illustrate the role of eddy hopping for the spectral broadening, but the method is cumbersome and thus not practical.

Is there any other methodology that would work better?

Lagrangian treatment of the condensed phase!
aka "Lagrangian Cloud Model", "Super-droplet method"…

The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model

S. Shima,[a]* K. Kusano,[c] A. Kawano,[a] T. Sugiyama[a] and S. Kawahara[b]

Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model

M. Andrejczuk,[1] W. W. Grabowski,[2] J. Reisner,[3] and A. Gadian[1]

Large-Eddy Simulations of Trade Wind Cumuli Using Particle-Based Microphysics with Monte Carlo Coalescence

SYLWESTER ARABAS

Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland

SHIN-ICHIRO SHIMA

Graduate School of Simulation Studies, University of Hyogo, Kobe, and Japan Agency for Marine-Earth Science and Technology, Kanagawa, Japan

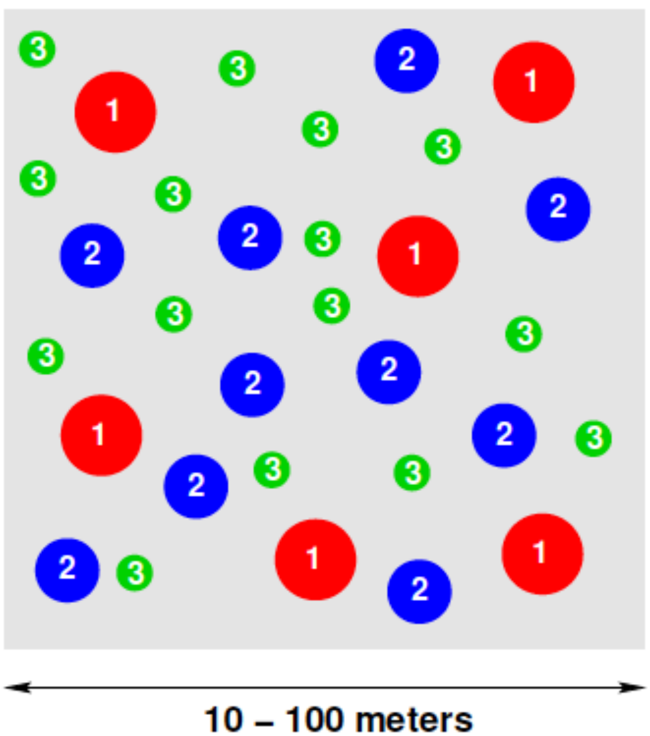
A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision

T Riechelmann[1,3], Y Noh[2] and S Raasch[1]

Super-droplets (SDs)

Shima *et al.* (2009), Arabas *et al.* (2015), Hoffmann *et al.* (2015)

$$N_{\text{droplets}} \sim 10^{11} - 10^{14}$$



- Multiplicities:

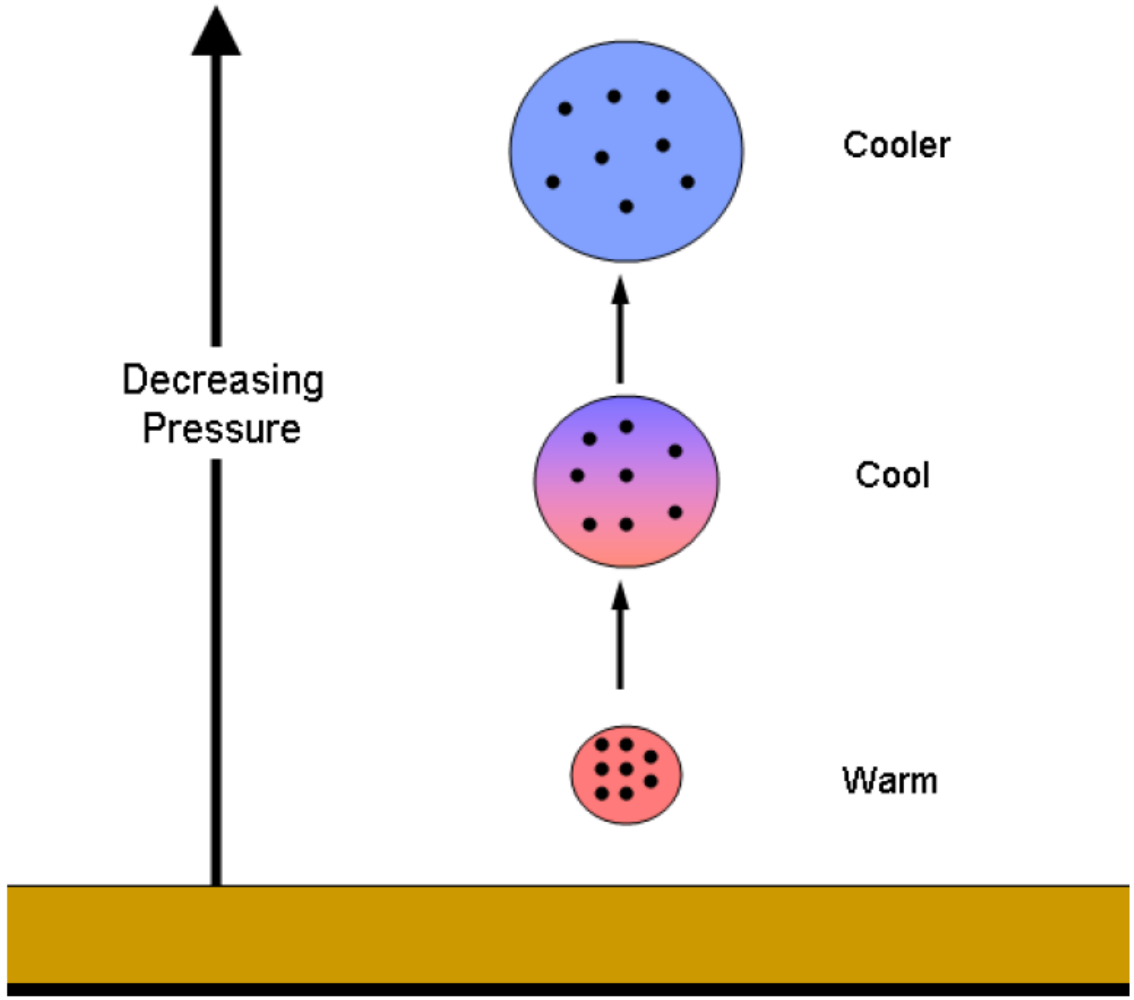
$$\xi_1 = 6,\ \xi_2 = 10, \ldots$$

- SDs have the same attributes

$$(r, S', W', \ldots)$$

- Well-mixed

The simplest model of cloud processes: the adiabatic parcel



Grabowski and Abade, 2017: Broadening of cloud droplet spectra through eddy hopping: Turbulent adiabatic parcel simulations. *J. Atmos. Sci.* **74**, 1485-1493.

$$c_p \frac{dT}{dt} = -gw + L_v C,$$

T – temperature
q_v – water vapor mixing ratio
w – updraft speed (1 m s^{-1})

C – condensation rate $C = \frac{d}{dt} \sum_i \frac{4}{3} \pi r_i^3 N_i \frac{\rho_w}{\rho_0}$

$$\frac{dq_v}{dt} = -C,$$

g = 9.81 ms^{-2} – gravitational acceleration
L_v = 2.5x10^6 J/kg - latent heat of condensation

$$\frac{dp}{dt} = -\rho_0 w g,$$

p – environmental pressure
ϱ_0 – environmental density (1 kg m^{-3})

Cloud droplets (super-droplets; a sample of real droplets)

$$\frac{dr}{dt} = \frac{1}{r + r_0} AS$$

r – droplet radius
S – supersaturation ($S = q_v/q_{vs} - 1$)
A = 0.9152x10^{-10} m^2 s^{-1}
r_0 = 1.86 μm

$T(t=0) = 288.16\,\mathrm{K}$, $p(t=0) = 900\,\mathrm{hPa}$, and relative humidity (RH) of 99%

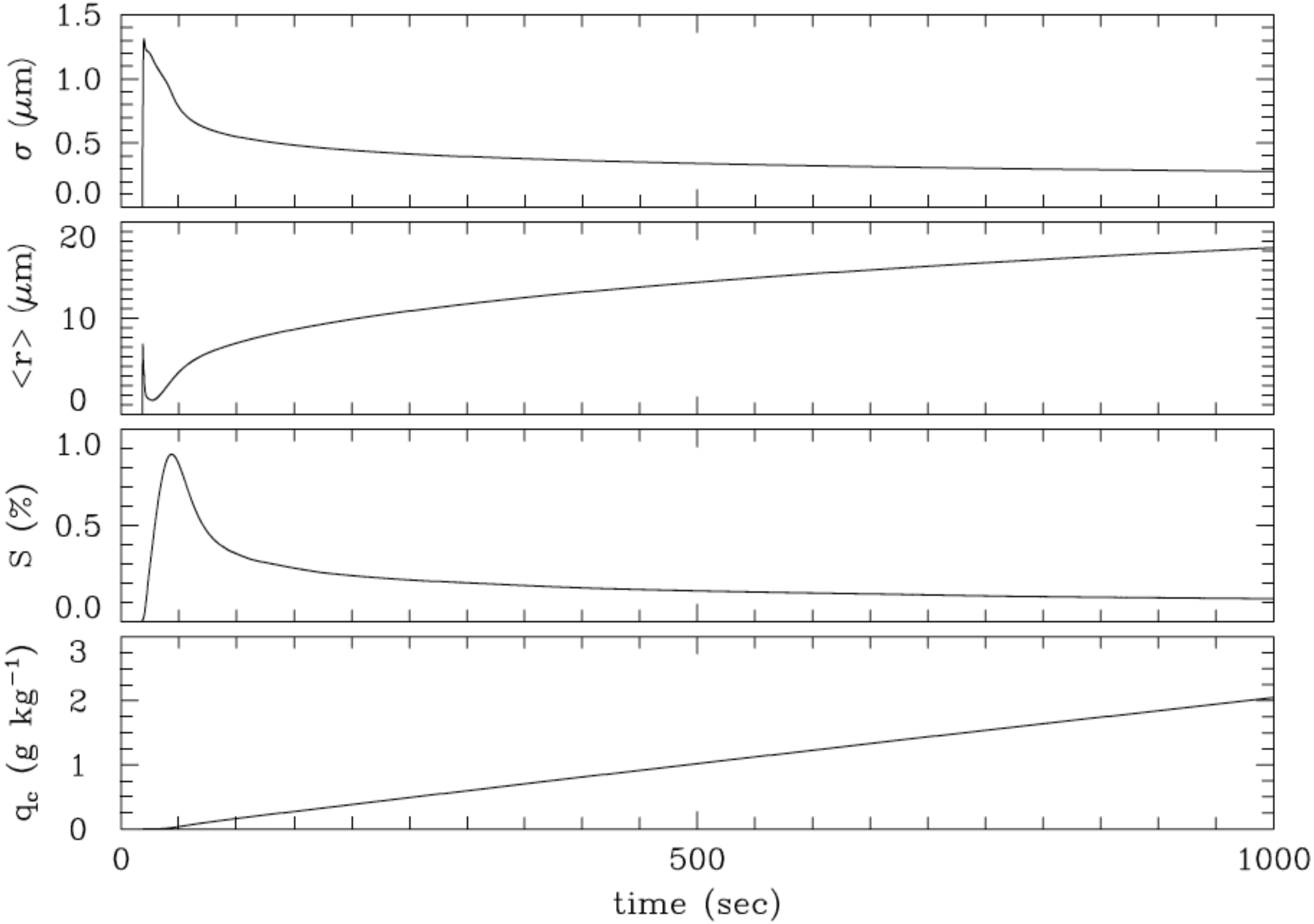
$$w = 1\,\mathrm{m\,s^{-1}}$$

spectral width

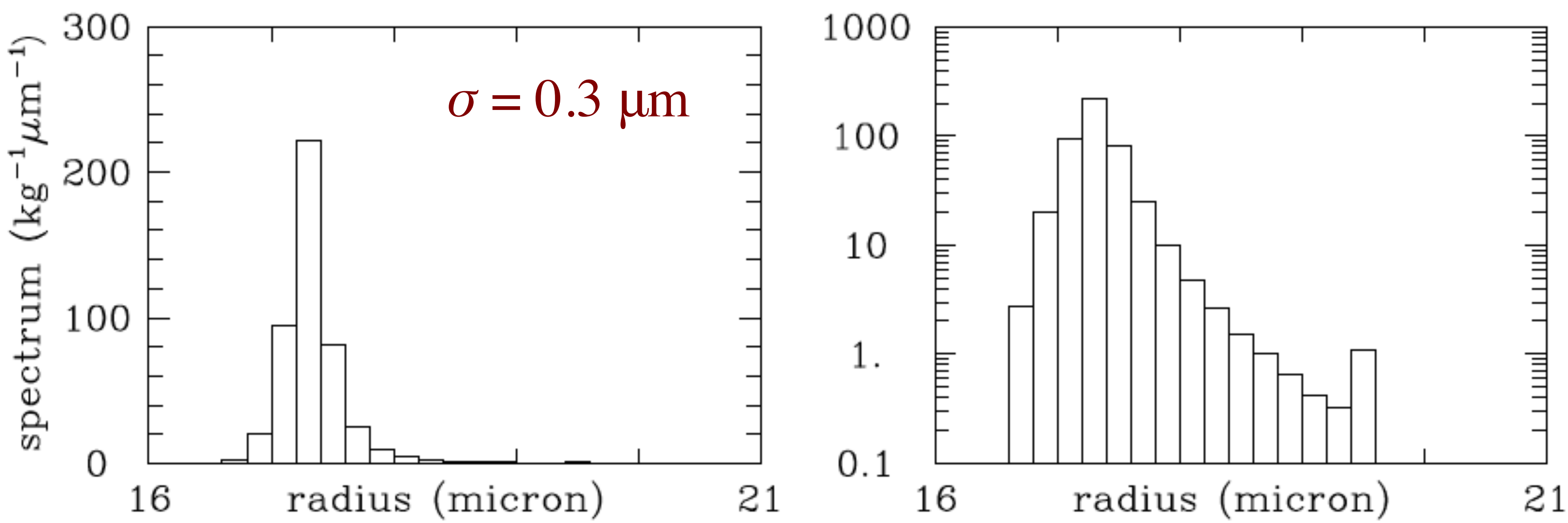
mean radius

supersaturation

$$\sum_i \frac{4}{3}\pi r_i^3 N_i \frac{\rho_w}{\rho_0}$$



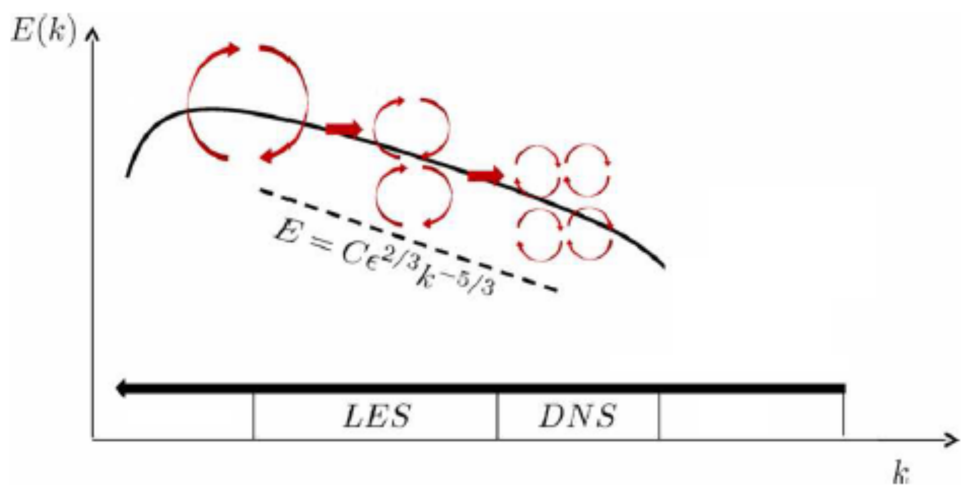
Spectrum of cloud droplets at t = 1000 s:



Turbulent adiabatic parcel model: adiabatic parcel as before, but now assumed to be filled with homogeneous isotropic turbulence.

Two parameters determining the turbulence:

1) dissipation rate of TKE, ε
2) scale (extent) of the parcel, L



turbulent kinetic energy, E

$$E = \left(\frac{L\varepsilon}{C_E}\right)^{2/3}$$

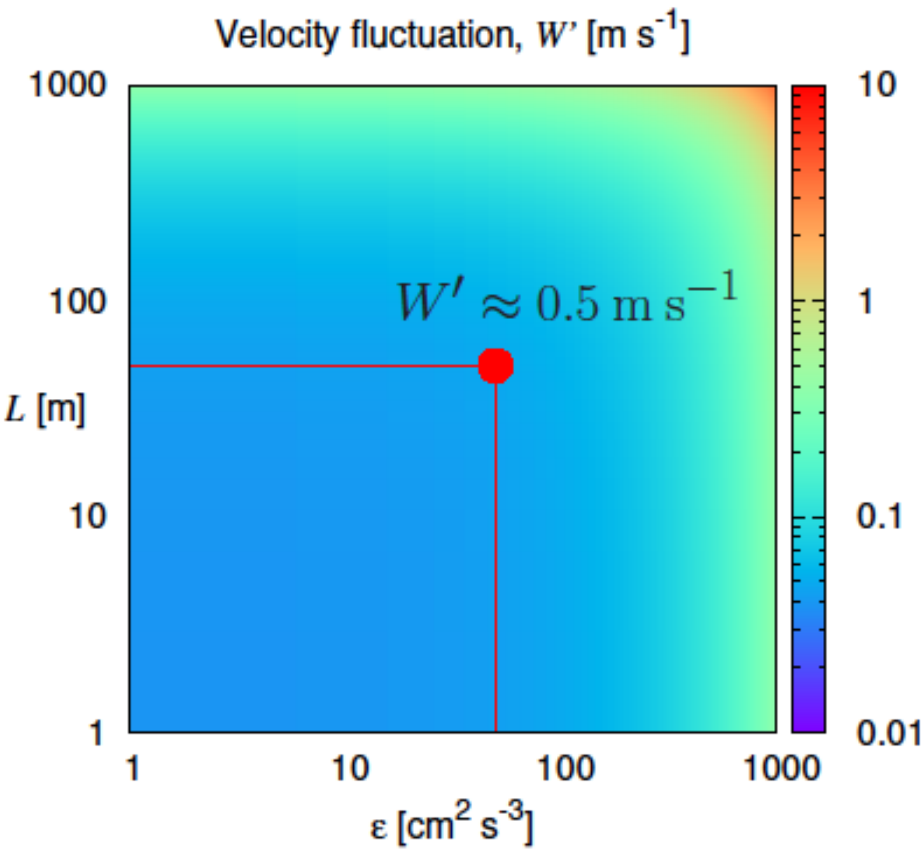
integral time scale, τ

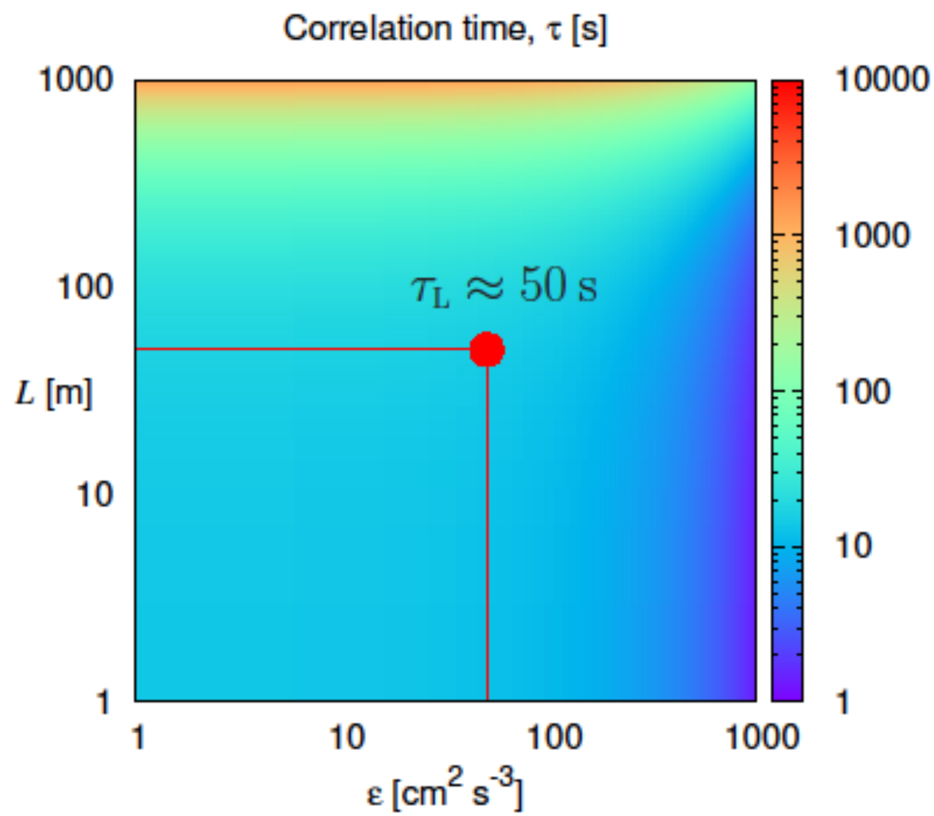
$$\tau = \frac{L}{(2\pi)^{1/3}} \left(\frac{C_\tau}{E}\right)^{1/2}$$

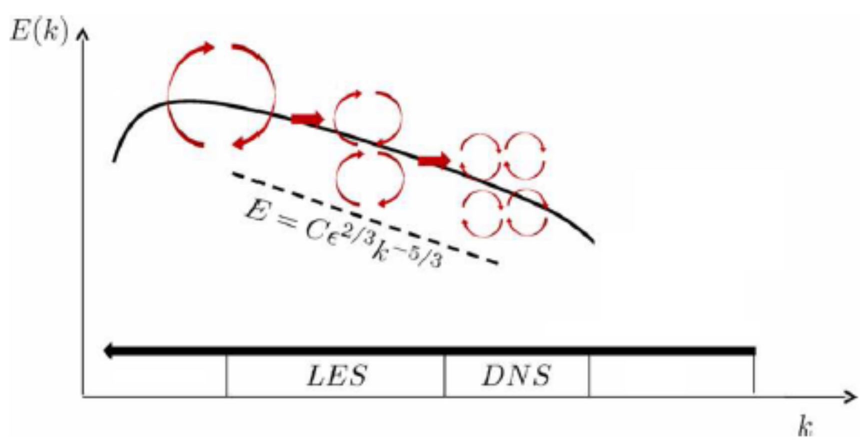
$C_E = 0.845$
$C_\tau = 1.5$

$$W' \sim (L\varepsilon)^{1/3}$$

$$\tau_L \sim (L^2/\varepsilon)^{1/3}$$







Supersaturation fluctuation S' (on top of the mean S) experienced by each superdroplet:

$$\frac{dS_i'}{dt} = a_1 w' - \frac{S_i'}{\tau_{\mathrm{relax}}}$$

i – superdroplet index

$$\tau_{\mathrm{relax}} = \left(a_2 \sum r_i N_i\right)^{-1}$$

$$a_2 = 2.8 \times 10^{-4}\ \mathrm{m}^2\,\mathrm{s}^{-1}$$

$$a_1 = 3 \times 10^{-4}\ \mathrm{m}^{-1}$$

Important note: phase relaxation time is the same for all droplets.

Hence, additional factors that may increase the impact (e.g., droplet concentration heterogeneities) are excluded…

Vertical velocity perturbation w' is assumed to be a random stationary processes and it is evolved in time as:

$$w'(t+\delta t) = w'(t)e^{-\delta t/\tau} + \sqrt{1-e^{-2\delta t/\tau}}\,\sigma_{w'}\psi$$

$$\sigma_{w'}^2 = \frac{2}{3}E$$

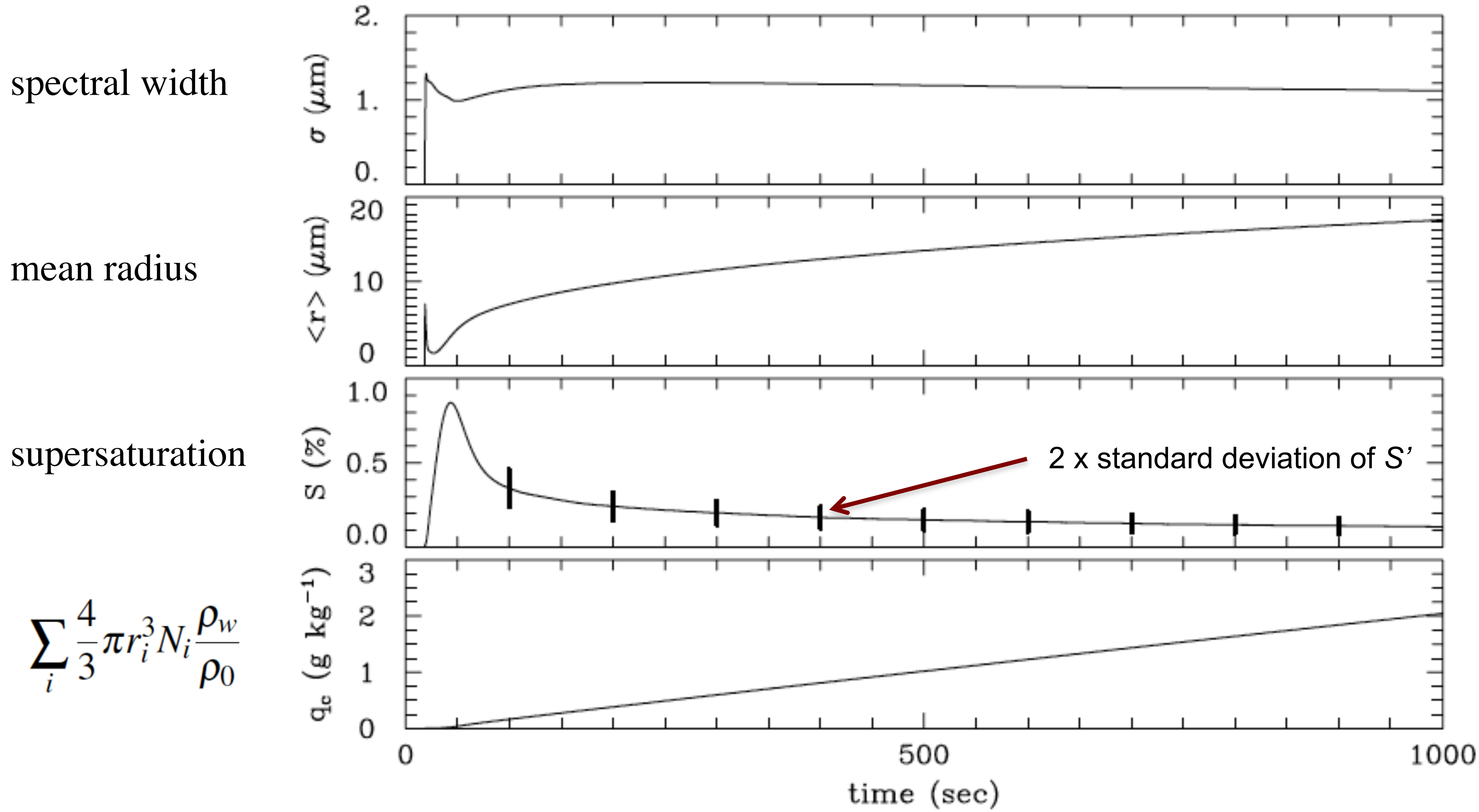
E - turbulent kinetic energy

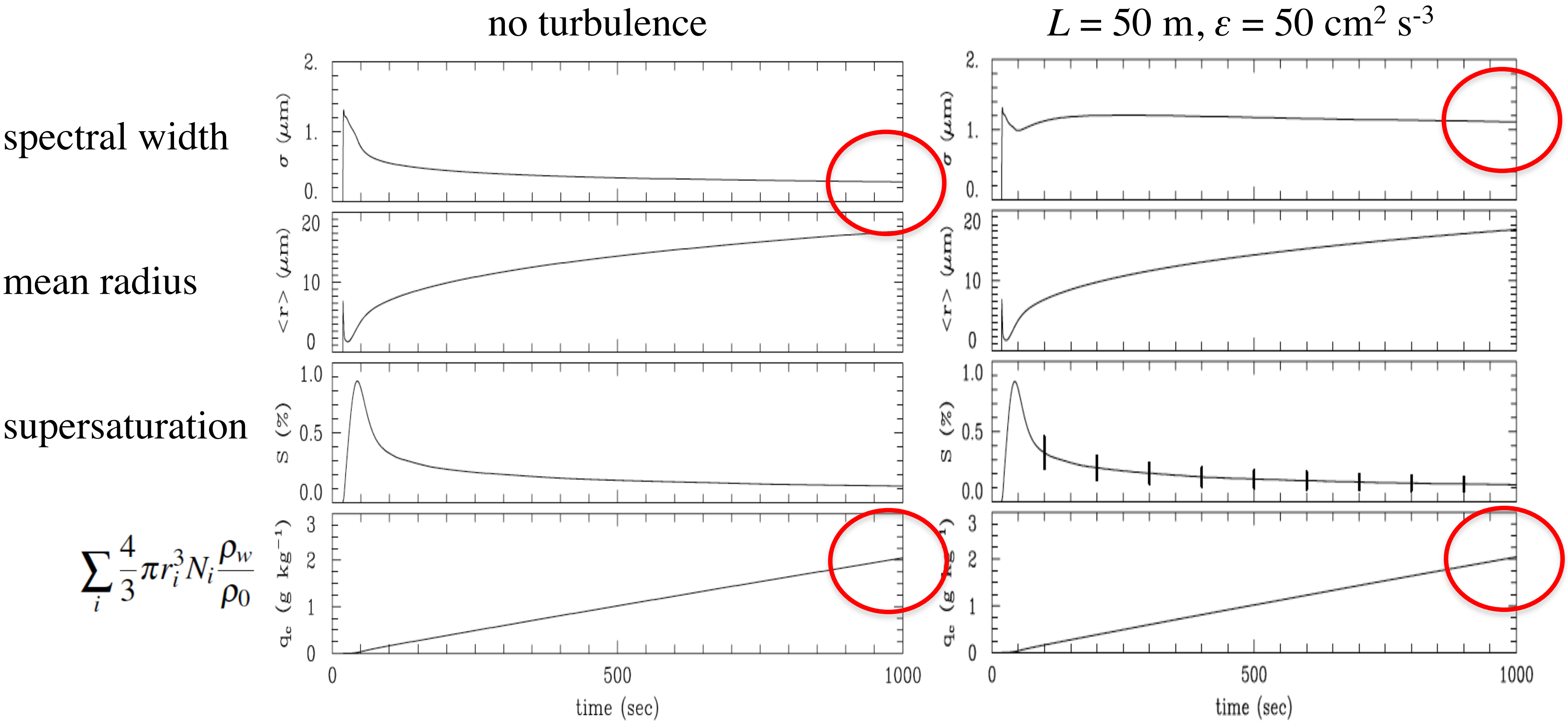
τ – turbulence integral time scale

ψ - Gaussian random number drawn every time step

δt - model time step

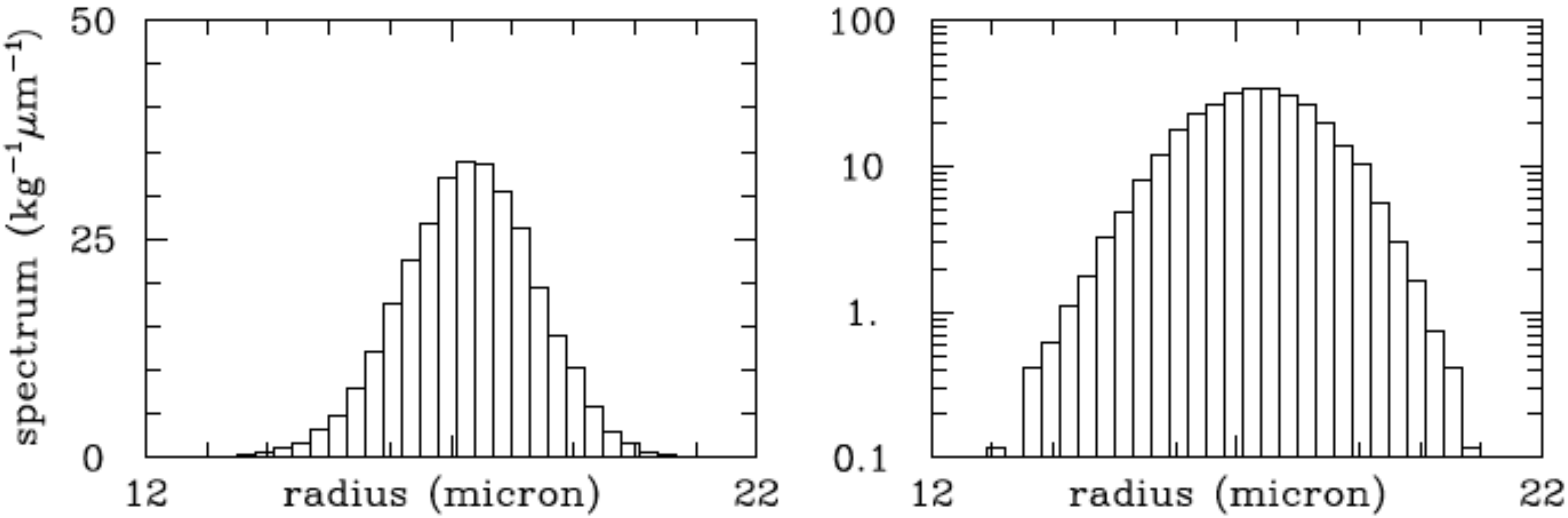
$L = 50$ m, $\varepsilon = 50$ cm^2 s^{-3}



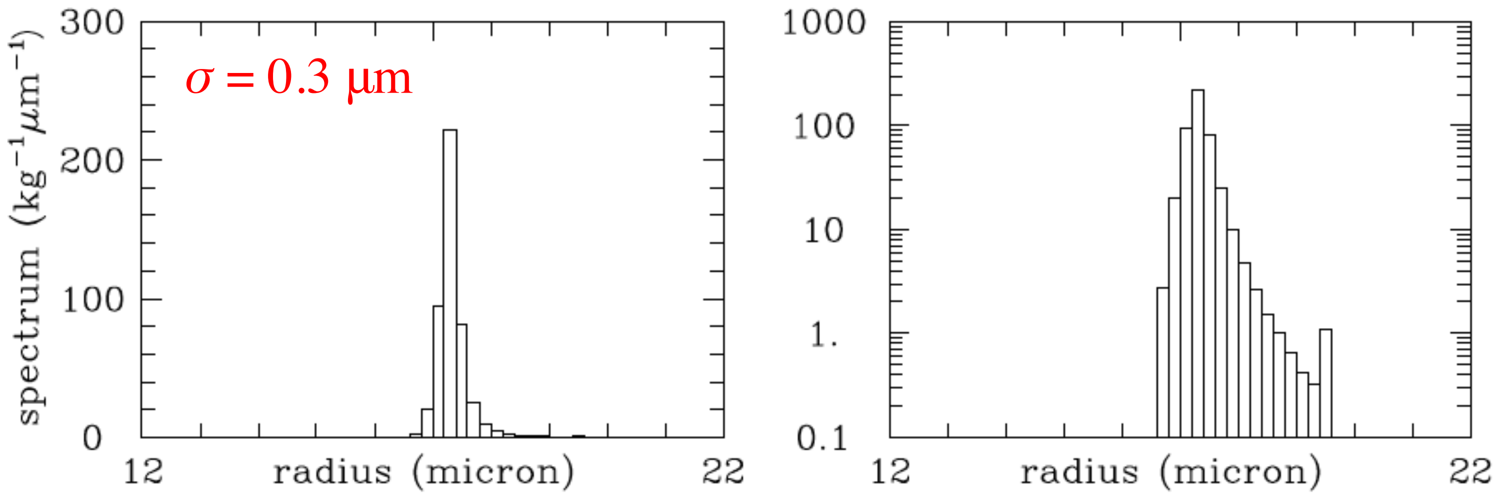

no turbulence
L = 50 m, ε = 50 cm² s⁻³
spectral width
mean radius
supersaturation
$\sum_i \frac{4}{3}\pi r_i^3 N_i \frac{\rho_w}{\rho_0}$
σ (μm)
<r> (μm)
S (%)
q_c (g kg⁻¹)
time (sec)

L = 50 m, ε = 50 cm^2 s^{-3}

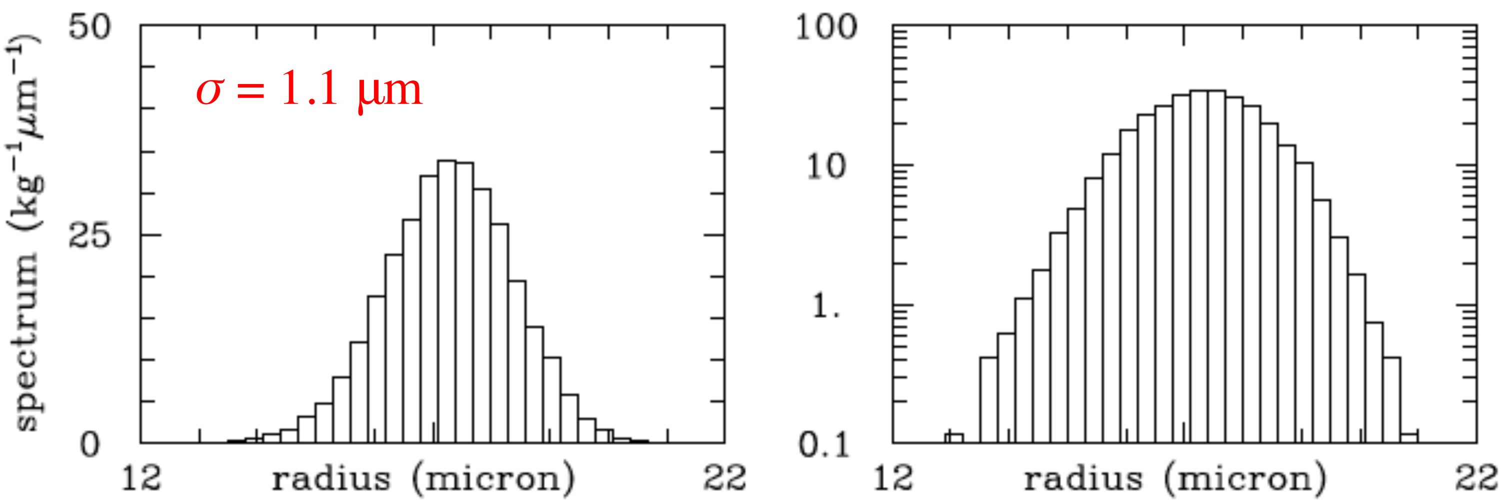
Spectrum of cloud droplets at t = 1000 s:



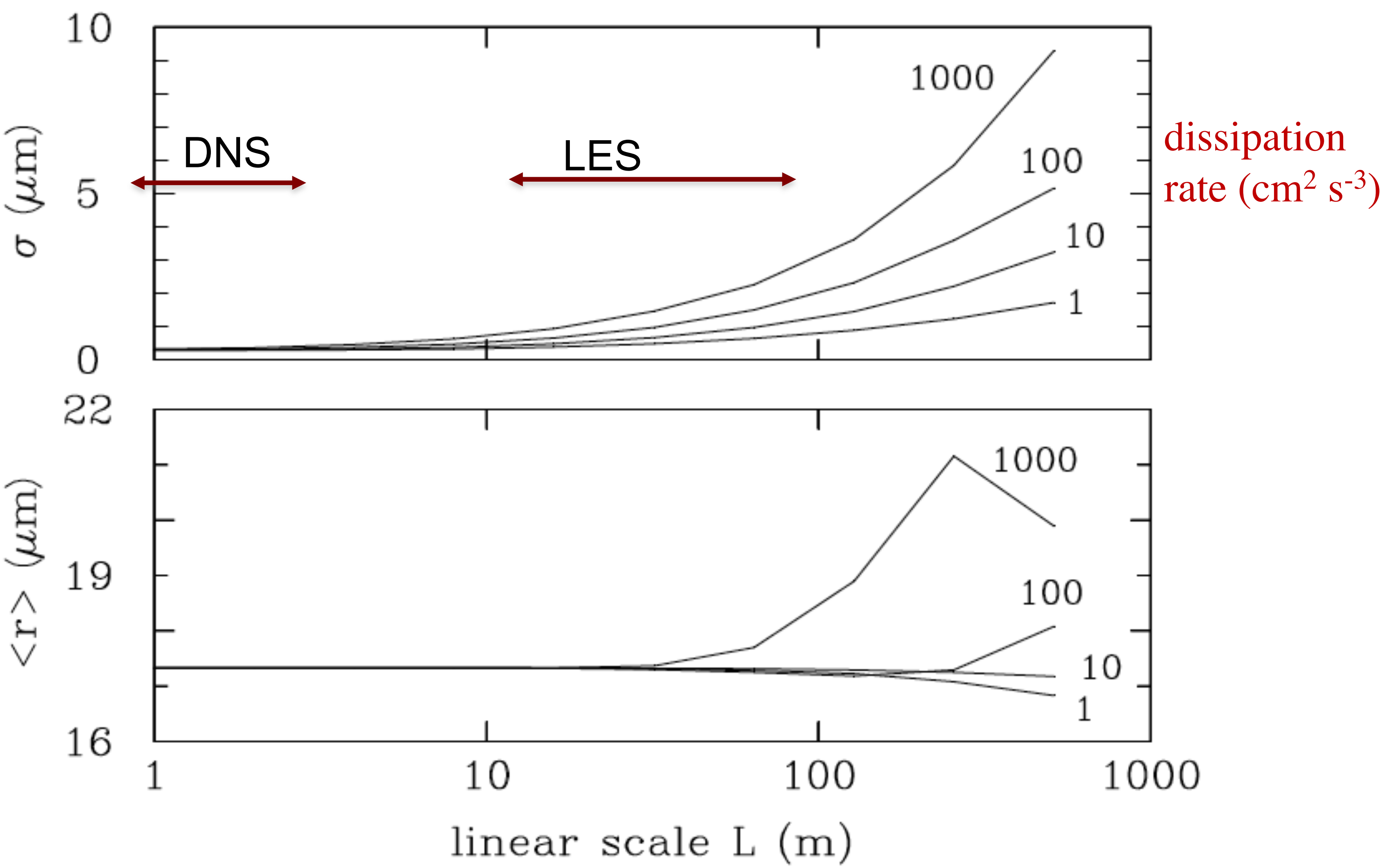
No turbulence



$L = 50$ m, $\varepsilon = 50$ cm^2 s^{-3}

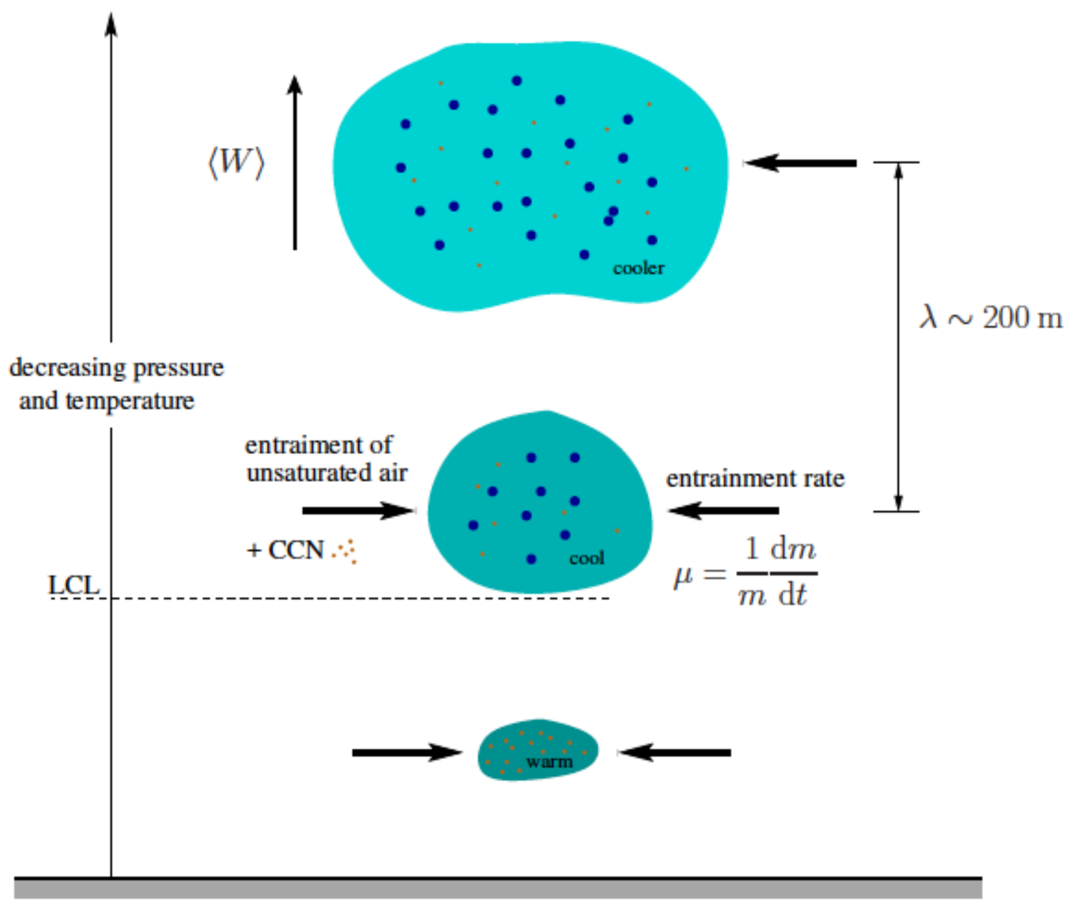


Spectral width and mean radius at t = 1000s as a function of L (m) for various dissipation rates (cm^2 s^{-3})



Entraining cloud parcel

stochastic entrainment events



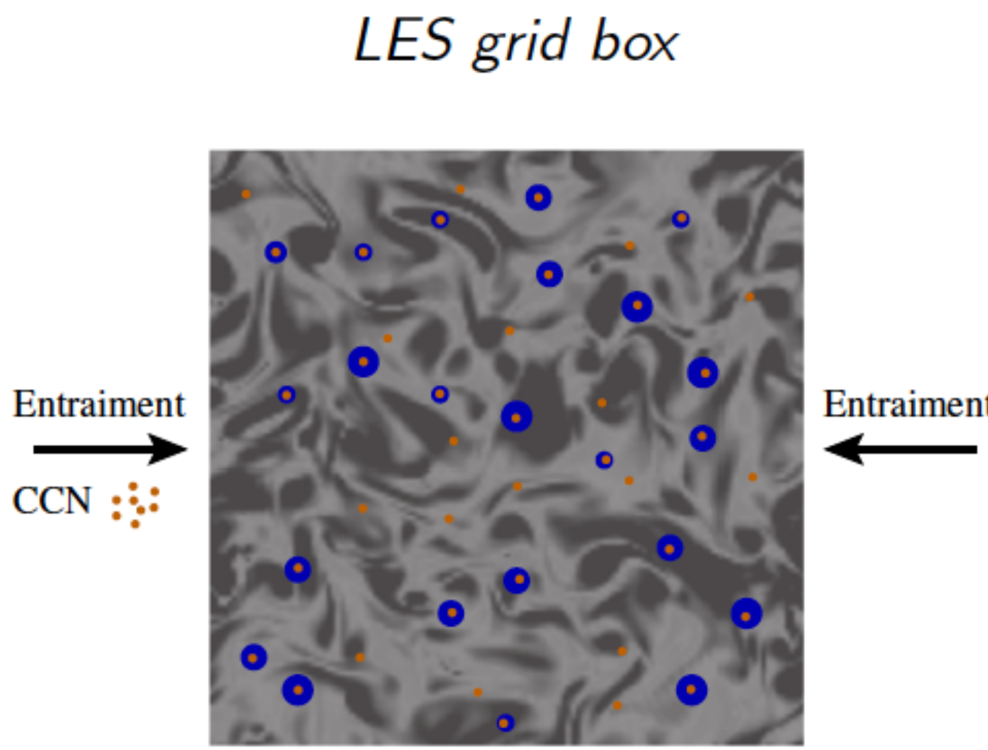
Krueger *et al.*, JAS, **54** (1997); Romps and Kuang, JAS, **67** (2010)

Abade, Grabowski, and Pawlowska *JAS* 2018 (in press)

Microphysical variability

at sub-grid scales (SGS)

- $S = \langle S \rangle + S'$
- Mixing
- Activation/deactivation
- Super-droplets



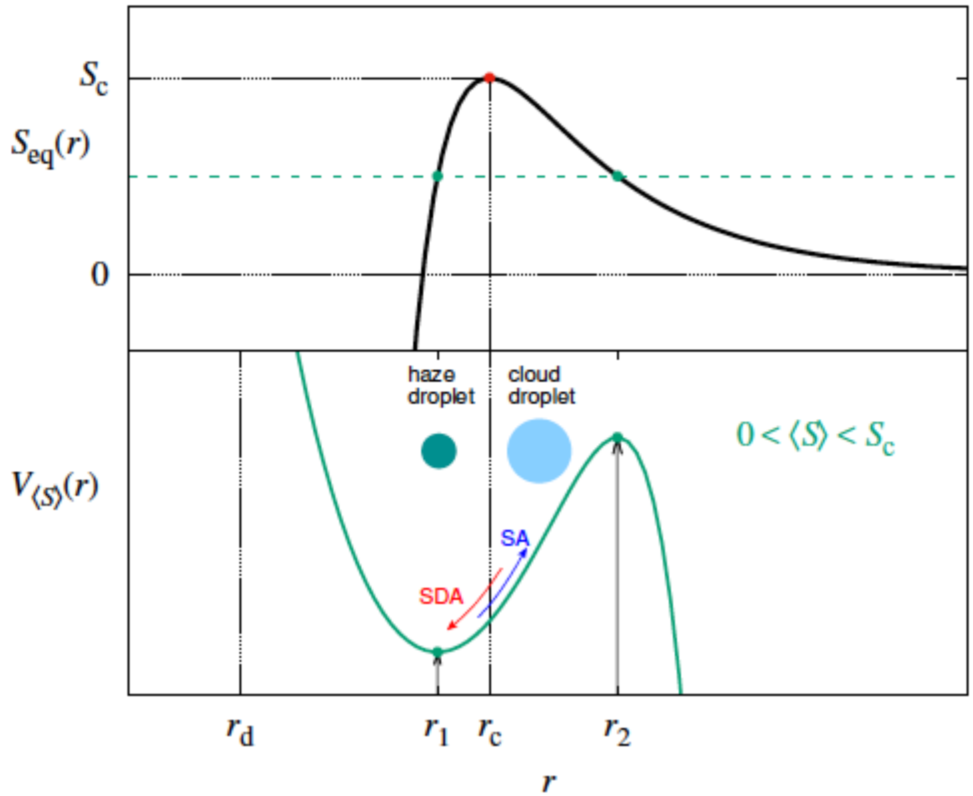
Stochastic activation and deactivation

Köhler curve and potential

$$r\frac{\mathrm{d}r}{\mathrm{d}t} = D\left(S - \frac{A}{r} + \frac{B}{r^3}\right)$$

$$x \equiv r^2 \qquad S = \langle S \rangle + S'$$

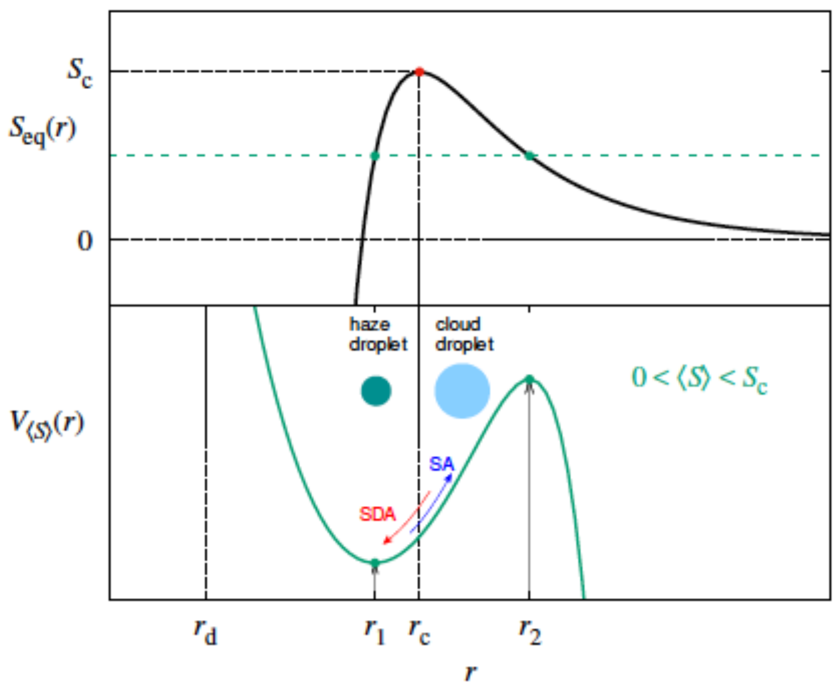
$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial V}{\partial x} + 2DS'$$



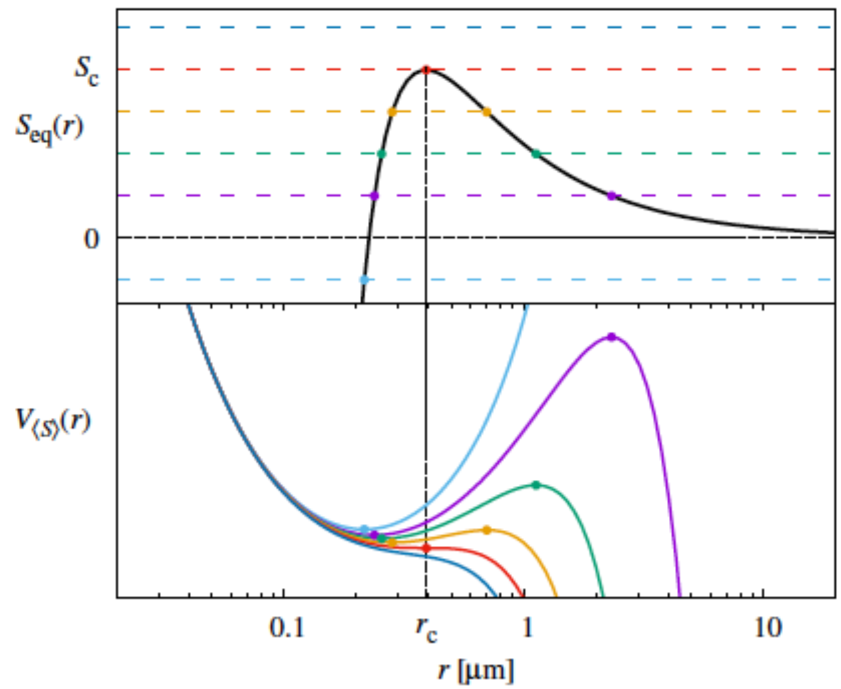
Arabas and Shima, *NPG* 2017
Abade, Grabowski, and Pawlowska *JAS* 2018 (in press)

Stochastic activation and deactivation

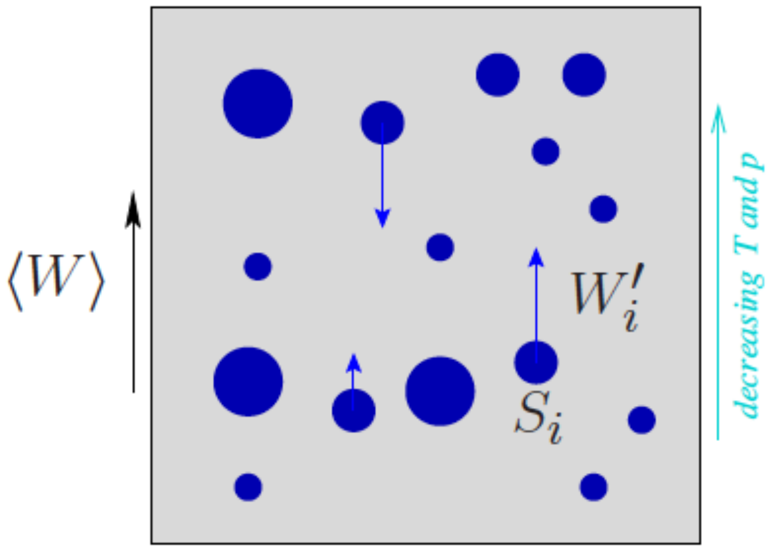
Köhler potential



Feedback on $\langle S \rangle$



Supersaturation and velocity fluctuations



$$\frac{\mathrm{d}S_i'}{\mathrm{d}t} = -\frac{S_i'}{\tau_{\mathsf{c}}} - \frac{S_i'}{\tau_{\mathsf{m}}} + aW_i'(t)$$

$$\tau_{\mathsf{c}} \sim \frac{1}{DN\langle r\rangle} \quad \text{(condensation)}$$

$\tau_{\mathsf{m}} \sim$ eddy turnover time (mixing)

- Statistical model for $W'(t)$

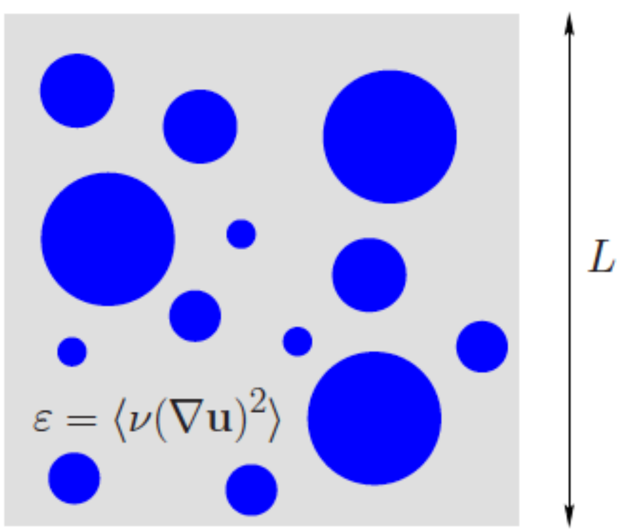
Celani *et al.*, EPL, **70** (2005); Grabowski and Abade, JAS, **74** (2017)

Vertical velocity fluctuations

Stationary homogeneous isotropic turbulence

$$\langle W'(t) \rangle = 0$$

$$\langle W'(0) W'(t) \rangle = \sigma_{W'}^2 \exp\left(-|t|/\tau_m\right)$$

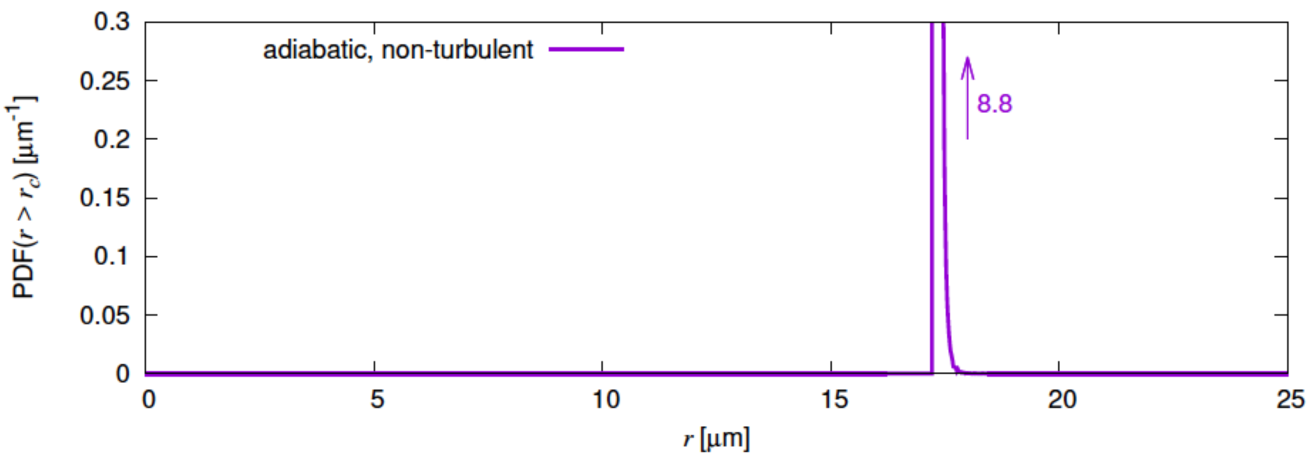


Kolmogorov scaling (inertial subrange)

$$\sigma_{W'}^2 \sim (L\varepsilon)^{2/3} \qquad \tau_m \sim \frac{L^{2/3}}{\varepsilon^{1/3}}$$

Droplet-size distribution

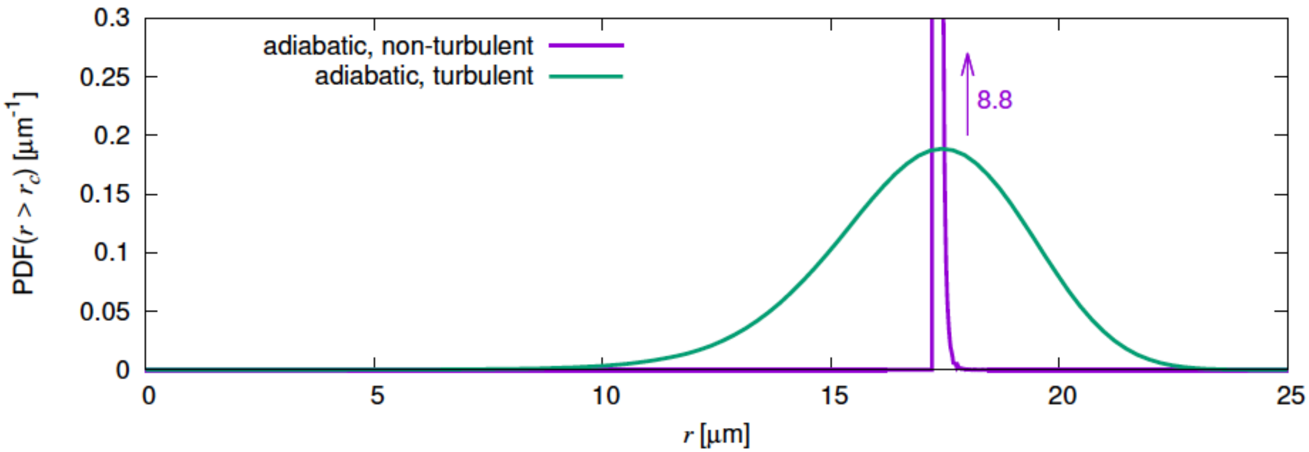
after a 1-km parcel rise



Droplet-size distribution

after a 1-km parcel rise

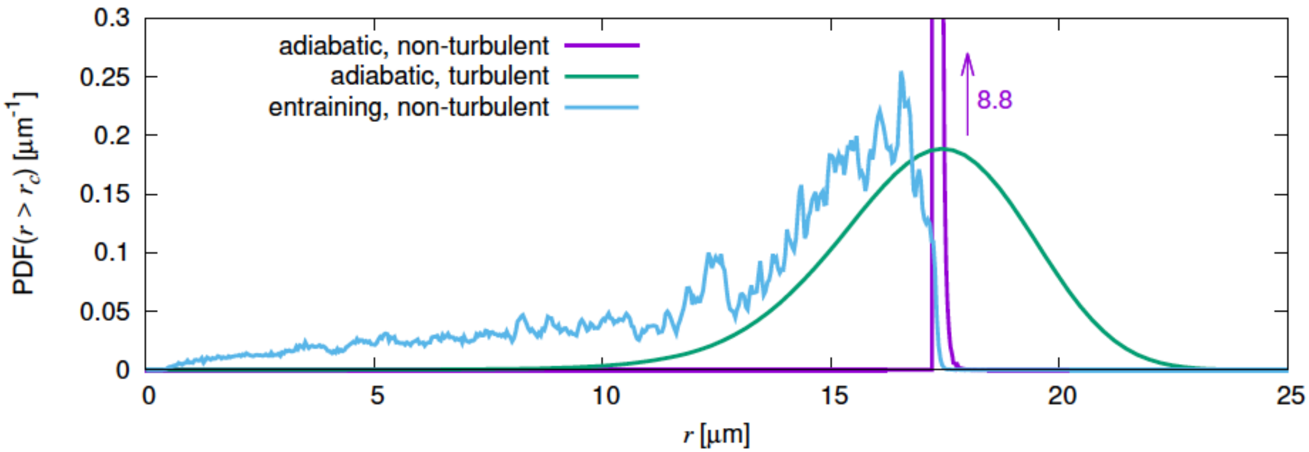
$L = 50$ m, $\varepsilon = 100$ cm^2 s^{-3}



Droplet-size distribution

after a 1-km parcel rise

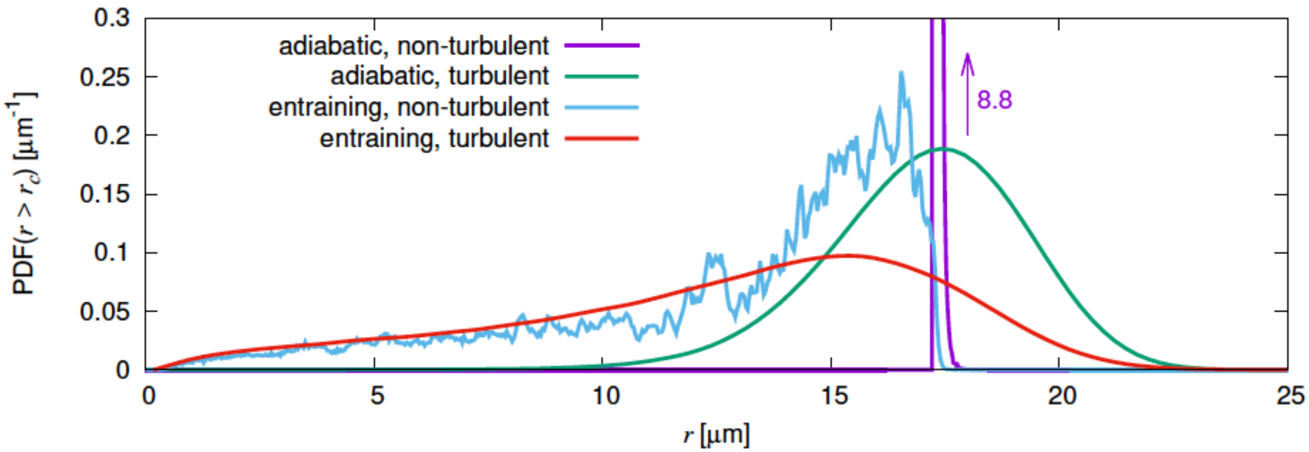
$L = 50$ m, $\varepsilon = 100$ cm^2 s^{-3}



Droplet-size distribution

after a 1-km parcel rise

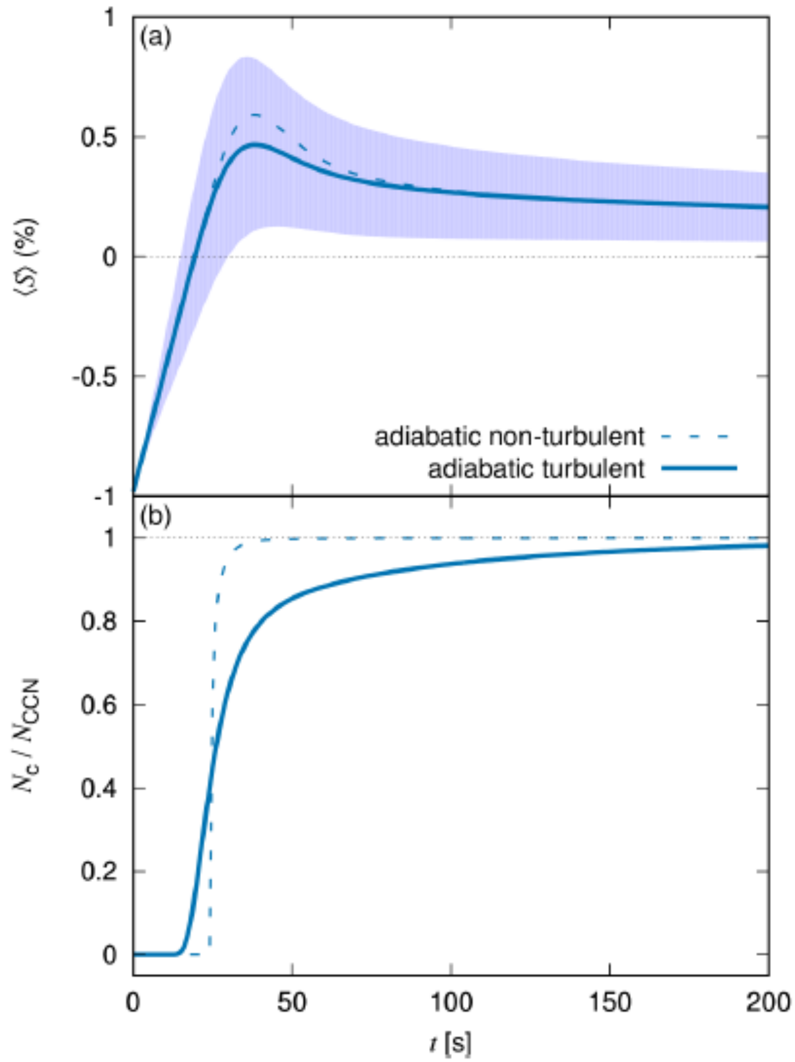
L = 50 m, ε = 100 cm^2 s^{-3}



Stochastic activation and feedback on $\langle S \rangle$

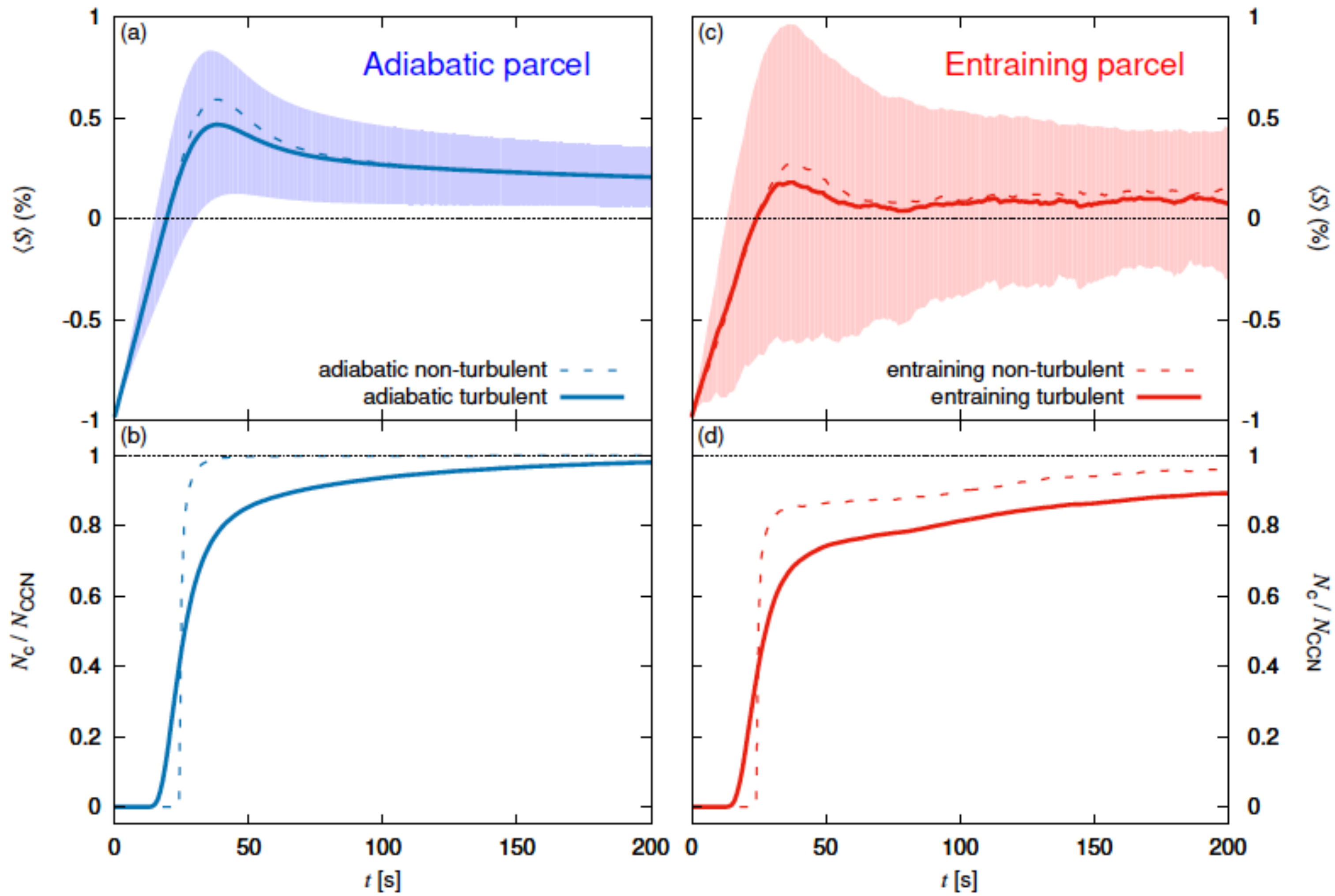
Adiabatic parcel

$L = 50$ m, $\varepsilon = 100$ cm^2 s^{-3}



Stochastic activation and feedback on $\langle S \rangle$

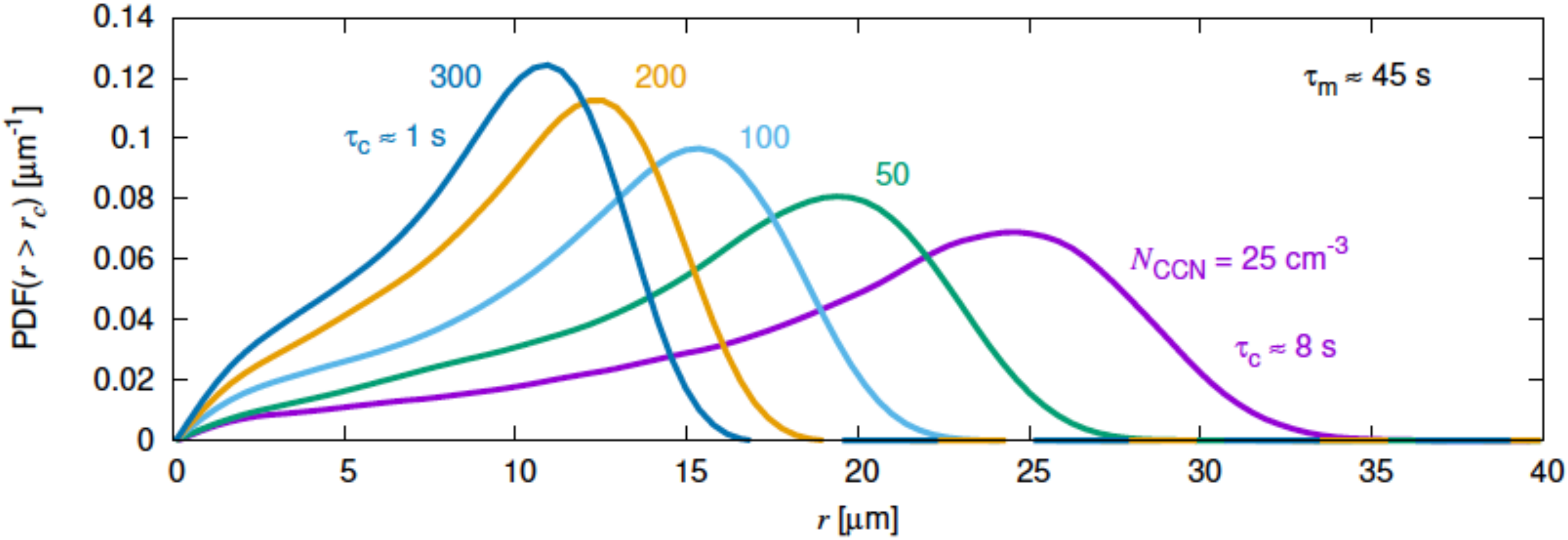
L = 50 m, ε = 100 cm^2 s^{-3}



Aerosol indirect effect

induced by turbulence

$L = 50$ m, $\varepsilon = 100$ cm^2 s^{-3}



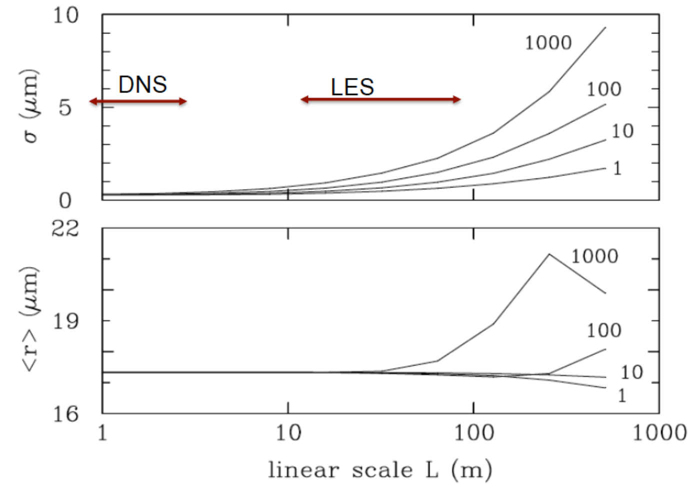
fast versus slow microphysics:

$$\frac{\mathrm{d}S'}{\mathrm{d}t} = -\frac{S'}{\tau_S} + aW'(t), \qquad \tau_S \sim \min\{\tau_{\text{condens}}, \tau_{\text{mixing}}\}$$

Chandrakar *et al.*, PNAS, **113** (2016); Siebert and Shaw, JAS, **74** (2017)

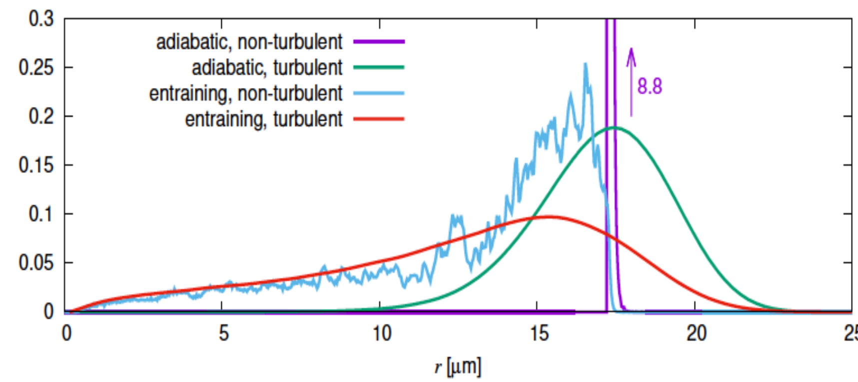
Summary and outlook:

Eddy-hopping mechanism plays a significant role in widening the droplet size distribution even for a homogeneous turbulent volume provided the the volume is large enough (i.e., *L* larger than several meters).



Even stronger effect is simulated when entrainment and mixing is considered in an entraining turbulent parcel model.

$L = 50$ m, $\varepsilon = 100$ cm^2 s^{-3}



Since typical grid lengths in LES cloud simulations are a few 10s of meters, the impact of eddy hoping on the droplet spectrum needs to be included. This is straightforward when the super-droplet method is used, but difficult (impossible?) for traditional Eulerian LES models.