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# A NOTE ON THE TURBULENT FLUXES OF HEAT AND MOISTURE IN THE BOUNDARY LAYER OVER THE ARABIAN SEA

by

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Abstract

This note attempts to compare some features of turbulent fluxes during the North-east and South-west Monsoons over the Arabian sea. It is shown that the coupling between mean and eddy motions, in the case of sensible heat and moisture flux, is much more variable during the N. E. Monsoon than during the S.W.Monsoon. In the case of moisture flux, however, the magnitude of the 'coupling' is in general greater at most levels, during the S.W.Monsoon. It is further shown, that the bulk of the energy is contained in larger scale vortices in the case of the N.E.Monsoon.

## 1. Introduction

The computations in this note were based on the data collected during the International Indian Ocean Expedition 1964, and subsequently presented as a technical report |1| by Bunker. In Table D of |1| is given the fluctuation and flux averages of heat and moisture over the whole Arabian sea, at various heights, for the N.E.Monsoon period. The sensible heat flux gives a quantity proportional to the covariance, $R_{wT}$, between the turbulent vertical velocity w and temperature T, where $C_p$ and $\rho$ are the specific heat and density respectively, of the air at a particular height and temperature. In the same way, utilising the data for the latent heat flux and root-mean-square fluctuations of vertical velocity and specific humidity at various heights, the corresponding quantity $LR_{wq}$, was obtained, where $R_{wq}$ is the covariance between the turbulent vertical velocity, $w$, and specific humidity, $q$, and $L$ is the latent heat of vaporisation. These values are plotted in figures 1(a) and 1(b) and indicated by curve (1). The heights were given as percentages of the the inversion height, but for the purpose of comparison, a mean inversion height of 253 metres was taken, on the basis of Bunker's data | 1 |, and the corresponding heights obtained.

In tables F and G of | 1 | are given the corresponding quantities for the S.W. Monsoon period. In this case the Arabian sea was divided into two parts by the $65^{\circ}$E meridian. Table F gives the averages over the eastern part of $65^{\circ}$E and table G, that of the western part. As mentioned above, the quantities $C_p \rho R_{wT}$ and $LR_{wq}$ were obtained and plotted in figures 1(a) and 1(b), shown by the curves (2) and (3).

The rates of changes of the quantities $C_p \rho R_{wT}$ and $LR_{wq}$ with height are plotted in figures 2(a) and 2(b) respectively for each of the three cases and the results compared.

## 2. Theoretical considerations :

We define the instantaneous wind velocity $\vec{V}$ as

$$\vec{V} = \vec{i}\, u + \vec{i}\, v + \vec{k}\, w \quad \ldots \quad (1)$$

where u, v and w are the components in the usual conventional directions. This instantaneous velocity may be considered as being composed of a mean and a fluctuating part. Thus

$$\vec{V} = \vec{\bar{V}} + \vec{V}' \quad \ldots \quad (2)$$

where the bar denotes the mean and the dash the fluctuating part. We now

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define two operators as follows

$$(a)\ \frac{d}{dt}\,(\ ) = \frac{\partial}{\partial t}\,(\ ) + \bar{\vec{V}}.\nabla(\ ) \qquad \ldots \qquad (3)$$

Here $\frac{d}{dt}$ represents differentiations following the mean velocity of the air particle.

$$(b)\ \frac{D}{Dt}\,(\ ) = \frac{\partial}{\partial t}\,(\ ) + \vec{V}\,.\,\nabla\,(\ ) \qquad \ldots \qquad (4)$$

Here $\frac{D}{Dt}$ represents differentiation following the instantaneous motion.

If 'S' represents the instantaneous value of a conservative scalar quantity, then it may be expressed as :

$$S = \bar{S} + S' \qquad \ldots \qquad (5)$$

where $\bar{S}$ represents the mean value of S and S', the fluctuating part. The equation for the mean state of 'S' may be written as :

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \bar{\vec{V}}\,.\,\nabla S \qquad \ldots \qquad \ldots \qquad (6)$$

If the fluid is regarded as incompressible then from the continuity condition we have

$$\frac{\partial S}{\partial t} = -\,\nabla\,.(\vec{V}\,S) \qquad \ldots \qquad (7)$$

$$\text{Also}\ |\ \nabla.(\vec{V}\,S)\ | = S\,(\nabla\,.\,\vec{V}) + \vec{V}\,.\,\nabla S$$

Substituting for V and 'S' from Equations (2) and (5) in Equation (6), we finally get :

$$\frac{dS}{dt} = -\,\bar{S}(\nabla.\,\bar{\vec{V}}) - \bar{S}\,(\nabla\,.\,\vec{V}') - S'\,(\nabla\,.\,\bar{\vec{V}}) - \vec{V}'.(\nabla\bar{S}) - \nabla\,.\,(\vec{V}'\,S')$$

Taking the mean we have

$$\frac{d\bar{S}}{dt} = \frac{d\bar{S}}{dt} = - \nabla \cdot (\overline{\vec{V}' S'}) - \bar{S} ( \nabla \cdot \bar{\vec{V}}) \quad \ldots \quad \ldots \quad (8)$$

Similarly, the equation for the departure flow can be written as

$$\frac{DS'}{Dt} = + \nabla \cdot (\overline{\vec{V}' S'}) - \nabla \cdot (\vec{V}' \bar{S}) - S' ( \nabla \cdot \bar{\vec{V}}) \quad \ldots \quad (9)$$

The first terms on the right hand side of Equations (8) and (9) are the same but with opposite signs. This term can therefore, be thought to represent a direct 'coupling' between mean and eddy flow.

We now let 'S' represent temperature T. Rewriting Equations (8) and (9) we have

$$\frac{d\bar{T}}{dt} = - \nabla \cdot (\overline{\vec{V}' T'}) - \bar{T} ( \nabla \cdot \bar{\vec{V}}) \quad \ldots \quad (10)$$

$$\frac{DT'}{Dt} = + \nabla \cdot (\overline{\vec{V}' T'}) - \nabla \cdot (\vec{V}' \bar{T}) - T' ( \nabla \cdot \bar{\vec{V}}) \quad \ldots \quad (11)$$

From | 3 | it is seen that :

$$\nabla_x \cdot ( \overline{u' T'} + \overline{v' T'} ) \approx 0$$

$$\nabla_x \cdot ( u' \bar{T} + v' \bar{T} ) \approx 0$$

where $\nabla_x$ is the operator for horizontal divergence. Further assuming that $( \nabla \cdot \bar{V} ) \approx 0$ we finally arrive at the equations :

$$\frac{d\bar{T}}{dt} = - \frac{\partial}{\partial Z} (\overline{w' T'}) \quad \ldots \quad \ldots \quad (12)$$

$$\frac{DT'}{Dt} = + \frac{\partial}{\partial Z} (\overline{w' T'}) - \frac{\partial}{\partial Z} (w' \bar{T}) \quad \ldots \quad (13)$$

From these equations it is evident that the strength of the coupling can be estimated from the magnitude of the correlation between the fluctuating vertical velocity and temperature. Likewise two similar equations can be

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written for the specific humidity :

$$\frac{d\bar{q}}{dt} = - \frac{\partial}{\partial Z} \left( \overline{w' q'} \right) \quad \ldots \quad \ldots \quad (14)$$

$$\frac{Dq'}{Dt} = + \frac{\partial}{\partial Z} \left( \overline{w' q'} \right) - \frac{\partial}{\partial Z} \left( w' \bar{q} \right) \quad \ldots \quad \ldots . \quad (15)$$

## 3. Discussion

From Fig.1(a) it is seen that above 150 metres, the magnitude of the correlation during the N.E.Monsoon is much greater than that during the S.W. Monsoon. The correlations are negative because of the downward heat flux. This high correlation between the temperature and vertical velocity fluctuations during the N.E.Monsoon period may be responsible for the rapidly changing characteristics of the boundary layer, especially the rapid changing of the height of the inversion layer, in this period.

Fig.2(a) shows the variation of the term $| C_p \rho R_{wT} |$ with height. It is seen that during the N.E.Monsoon this term varies widely with height while during the S.W.Monsoon period, $\frac{\partial}{\partial Z} | C_p \rho R_{wT} |$ shows a more uniform trend. From Equation (12), it is seen that $- \frac{\partial}{\partial Z} | C_p \rho R_{wT} |$ is equal to $C_p \rho \frac{d\bar{T}}{dt}$ which is the rate of change of the mean heat content of a parcel of the air of mass $\rho$ per unit volume. The sign of $\frac{d\bar{T}}{dt}$ is determined by the sign of $\frac{\partial}{\partial Z} | C_p \rho R_{wT} |$ .

In [3] Richardson found the rate of change of intrinsic energy in terms of the rate of eddying for dry air as :

$$\frac{C_p}{g} \int_0^G \frac{dT}{dt} \, dp = C_p \left| \frac{T}{\theta} \frac{\eta}{g} \frac{d\theta}{dp} \right|_0^G - \frac{1}{p_1^{0.29}} \frac{C_p}{g} \int_0^G \eta \frac{d\theta}{dp} \, d\left( p^{0.29} \right) \quad \ldots . (16)$$

where $\eta = g^2 \rho C$ is a measure of turbulence, $C$ being the eddy conductivity,

$\theta$ = Potential temperature

$p_s$ = Standard pressure adopted in defining $Q$.

$g$ = acceleration due to gravity

$G$ = Ground surface (in this case sea surface)

$T$ = Temperature

The first term on the R.H.S. of (16) is called by Richardson as the 'boundary activity' and gives the rate at which heat is being removed from the sea or ground surface by eddies per unit horizontal area. The second term is called 'body activity' and indicates whether heat is becoming eddying energy or vice versa depending upon whether $\frac{\partial\theta}{\partial p}$ is positive or negative. A positive $\frac{\partial\theta}{\partial p}$ implies that potential temperature increases downwards while a negative $\frac{\partial\theta}{\partial p}$ indicates that potential temperature increases upwards.

Fig.2(a) shown that during the N.E. Monsoon period the sign of $\frac{\partial}{\partial Z}\left| C_p \rho R_{wT} \right|$ is positive in the layers 175 m to 200 m and 350 m to 525 m. Equation (12) shows that for a positive value of $\frac{\partial}{\partial Z}\left| C_p \rho R_{wT} \right|$ the value of $\frac{d\bar{T}}{dt}$ should be negative. If the integration of Equation (16) be performed layer by layer then it is seen that for the L.H.S. of Equation (16) to be negative, indicative of negative value for $\frac{d\bar{T}}{dt}$ . The following two conditions must be fulfilled :

(i) $C_p \left| \frac{I}{\theta} \frac{\eta}{g} \frac{d\theta}{dp} \right|_{p_2}^{p_1}$ must be greater than the second term

because $\frac{d\theta}{dp}$ is negative

$$(ii)\quad C_p \left| \frac{T}{\theta}\ \frac{\eta}{g}\ \frac{d\theta}{dp} \right|_{p_1} > C_p \left| \frac{T}{\theta}\ \frac{\eta}{g}\ \frac{d\theta}{dp} \right|_{p_2}$$

$p_1$ is the pressure at lower level and $p_2$ that at the higher level.

Now $C_p \left| \frac{T}{\theta}\ \frac{\eta}{g}\ \frac{d\theta}{dp} \right|$ indicates the amount of heat given to the lower surface. Therefore the condition (ii) means that the heat efflux from the layer at the lower level must be greaterthan the heat influx into the layer at the upper level. Fig.1(a) shows that in the above mentioned layers such is indeed the case. In the case of the S.W.Monsoon also, except for very low levels, below 100 m,the above condition is satisfied.

Fig.1(b) shows that the covariance between the vertical turbulent velocity and the moisture is highest for the N.E.Monsoon through the major portion of the boundary layer, suggesting a rapid rate of evaporation. Figs. 3(a) and (b), reporduced from [1], show the kinetic energy production, transport and dissipation during both the monsoons. The dissipation of the energy, $\epsilon$. was computed from the production and flux, since the spectral range of the turbulent measurements does not extend into the inertial range. The formula used is given below [1]

$$\epsilon = u^{*2}\ \frac{\partial u}{\partial Z} + \frac{gH}{C_p\ \rho\ T} - \frac{\partial\ \overline{e'\ w'}}{\partial Z} \qquad \ldots \qquad (17)$$

where $u^*$ is the friction velocity, $\frac{\partial u}{\partial Z}$ is the vertical wind shear

$H$ is the sensible heat flux

$\frac{\partial\ \overline{e'\ w'}}{\partial Z}$ is the gradient of the kinetic energy flux.

It is seen from Figs. 3(a) and (b) that the dissipation is much greater during the S.W. monsoon, in the lowest 500 m, than during the

N.E.monsoon. The K.E. flux and buoyancy are about the same for the two Monscons but the mechanical energy production term, indicated by the first term on the right hand side of Equation (17), is much greater during the SW Monsoon and thus affecting the value of $\epsilon$. It is also pointed out in |1| that during the NE monsoon, the kinetic energy is transported upward away from the layer of generation by mechanical means. This suggests that the eddies are of a length scale belonging to the inertial sub-range. The energy is therefore transported, without dissipation, to a region where the scale of the eddies fall within the 'dissipation range'.

During the S.W. monsoon, on the other hand, it was shown in |1| that the energy is dissipated in the layer in which it is generated suggesting that the length scale of the eddies belong mostly to the lower part of the 'equilibrium region' where practically all the energy received is dissipated.

It may therefore be concluded that during the SW monsoon period, the bulk of the energy, in the lower 500 m is contained in much smaller scale eddies than that during the N.E.monsoon.

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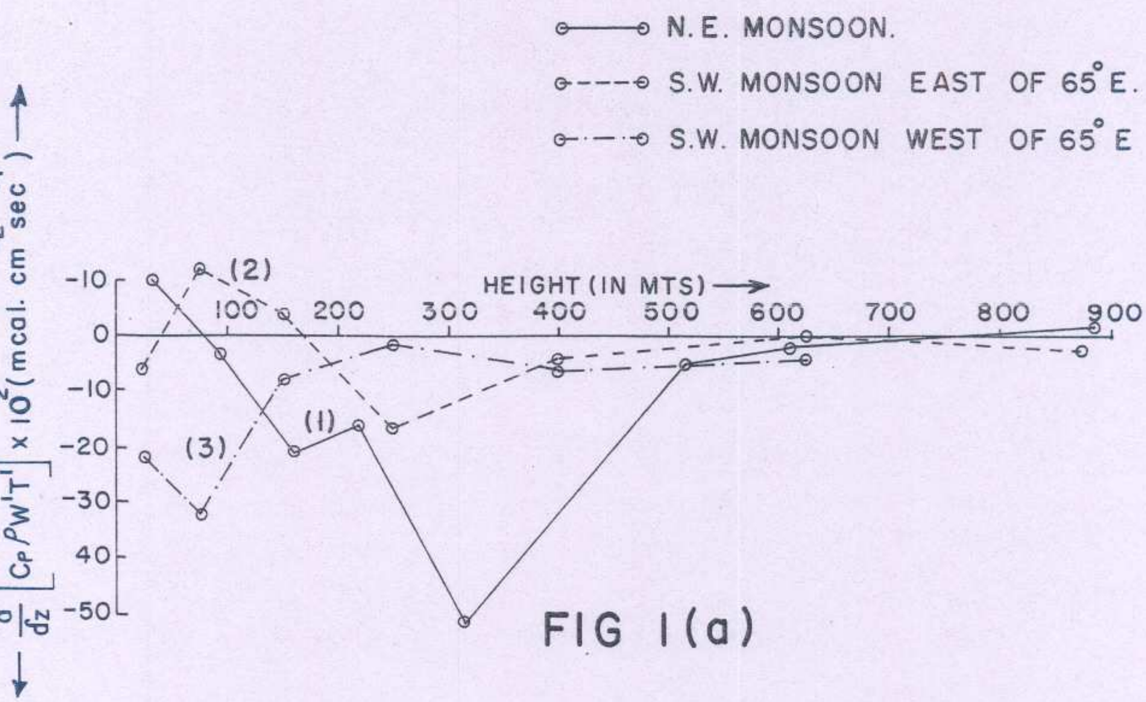
N. E. MONSOON.
S.W. MONSOON EAST OF 65° E.
S.W. MONSOON WEST OF 65° E
HEIGHT (IN MTS)
$\frac{d}{dz}\left[C_P \rho \overline{W'T'}\right] \times 10^2$ (mcal. cm$^{-2}$ sec$^{-1}$)
-10
0
-10
-20
-30
40
-50
100
200
300
400
500
600
700
800
900
(1)
(2)
(3)

FIG 1(a)

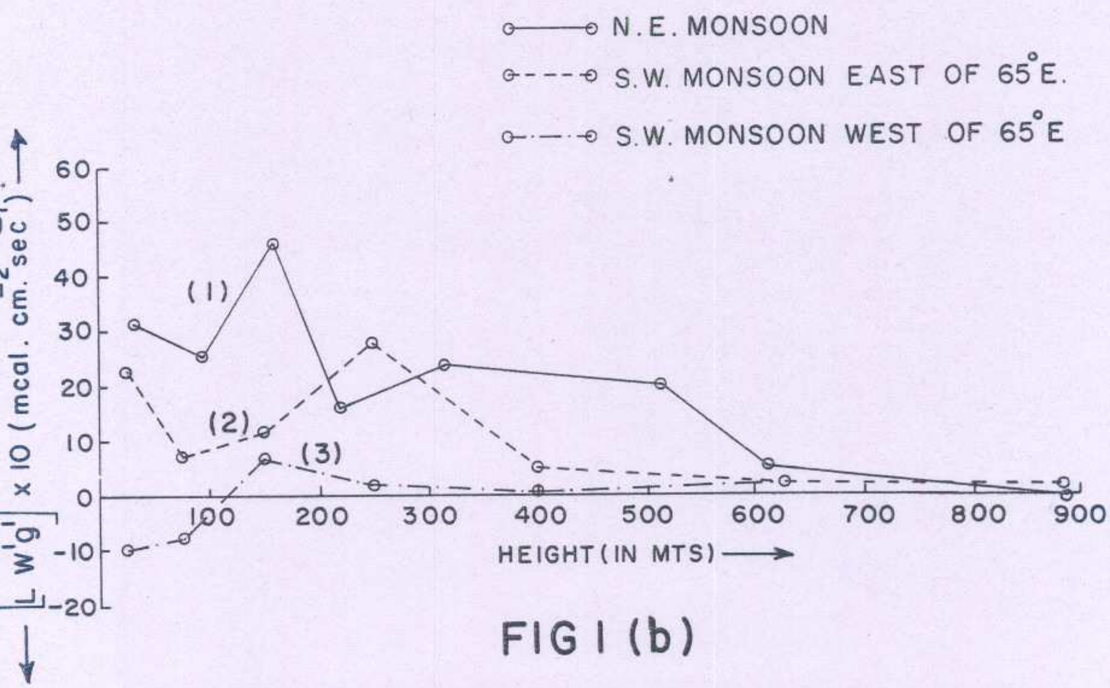
N.E. MONSOON
S.W. MONSOON EAST OF 65° E.
S.W. MONSOON WEST OF 65° E
$\left[L\,\overline{W'g'}\right] \times 10$ (mcal. cm.$^{-2}$ sec$^{-1}$)
60
50
40
30
20
10
0
-10
-20
100
200
300
400
500
600
700
800
900
HEIGHT (IN MTS)
(1)
(2)
(3)

FIG I (b)

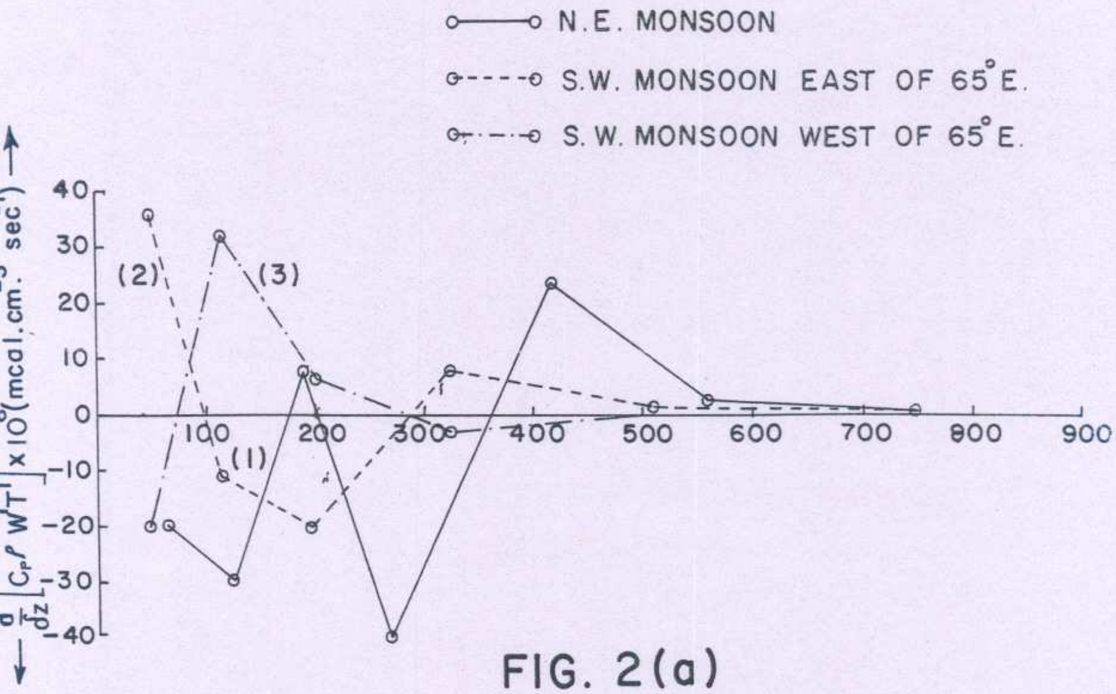
N.E. MONSOON
S.W. MONSOON EAST OF 65°E.
S.W. MONSOON WEST OF 65°E.
(1)
(2)
(3)
40
30
20
10
0
-10
-20
-30
-40
100
200
300
400
500
600
700
800
900

FIG. 2(a)

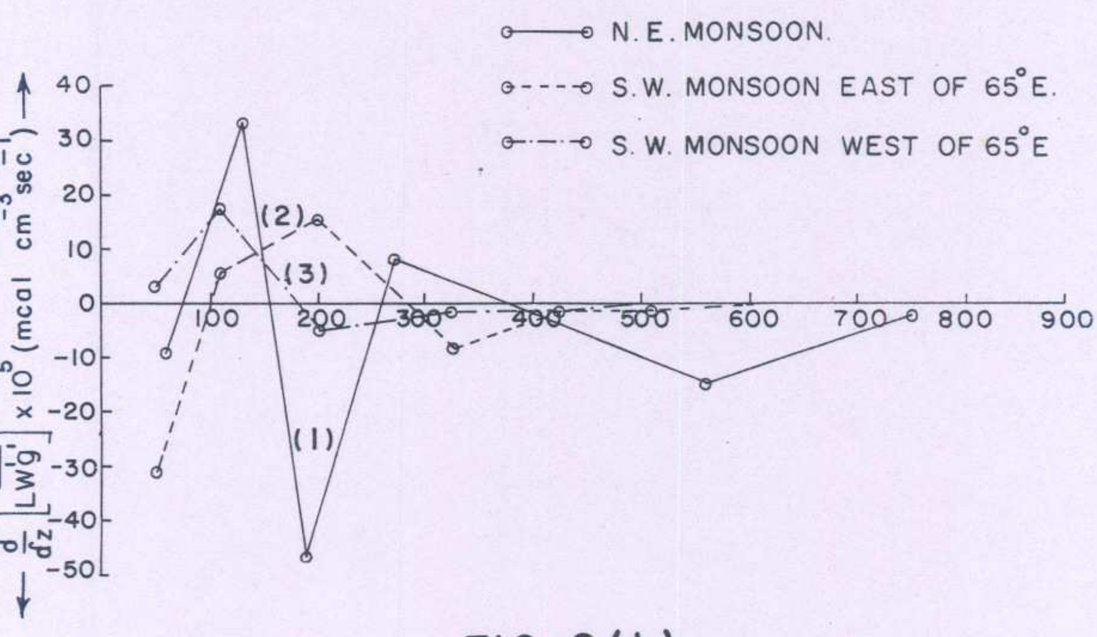
N.E. MONSOON.
S.W. MONSOON EAST OF 65°E.
S.W. MONSOON WEST OF 65°E
(1)
(2)
(3)
40
30
20
10
0
-10
-20
-30
-40
-50
100
200
300
400
500
600
700
800
900

FIG. 2(b)

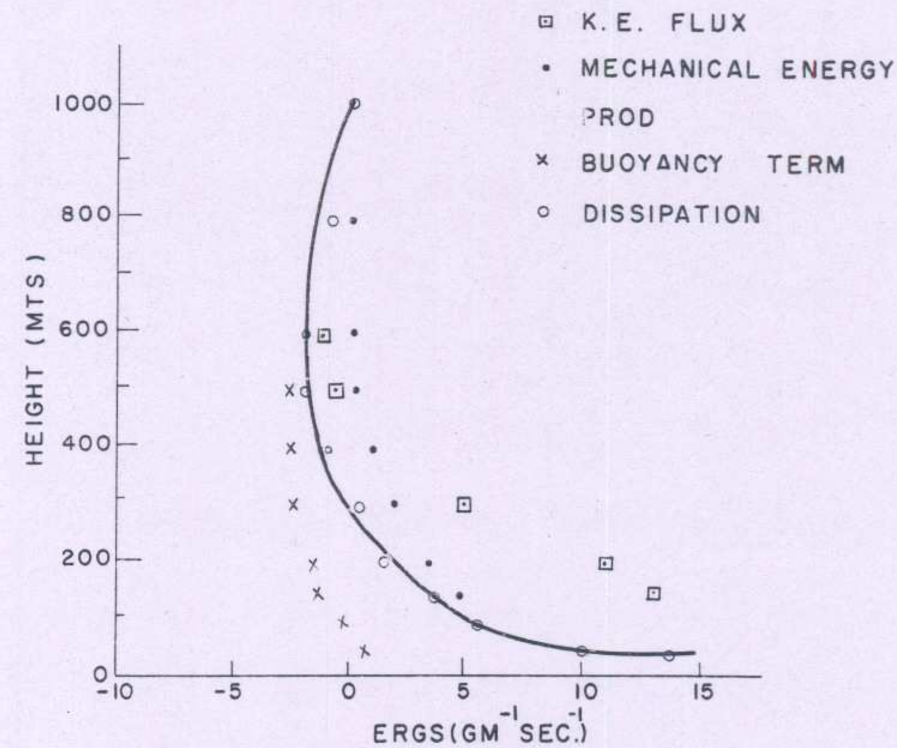
K. E. FLUX
MECHANICAL ENERGY PROD
BUOYANCY TERM
DISSIPATION
HEIGHT (MTS)
1000
800
600
400
200
0
-10
-5
0
5
10
15
ERGS(GM$^{-1}$ SEC.$^{-1}$)

FIG. 3(a)

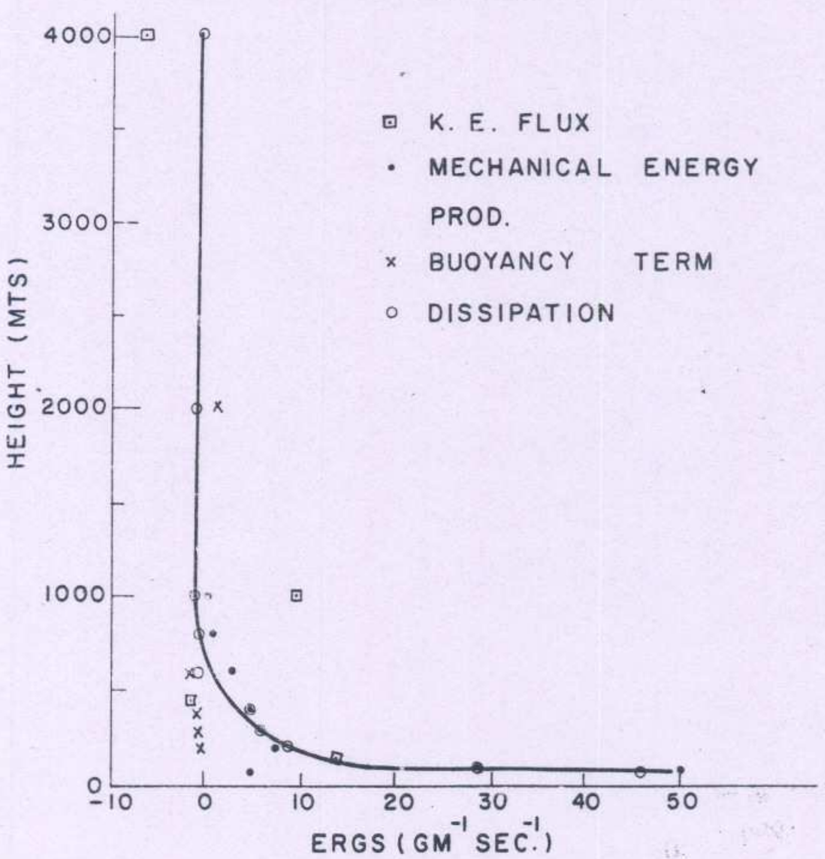
K. E. FLUX
MECHANICAL ENERGY PROD.
BUOYANCY TERM
DISSIPATION
HEIGHT (MTS)
4000
3000
2000
1000
0
-10
0
10
20
30
40
50
ERGS (GM$^{-1}$ SEC.$^{-1}$)

FIG. 3(b)