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MECHANISM FOR GROWTH OF TROPICAL DISTURBANCES

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Abstract

Parameterisation of cumulus convection is suggested through "effective static stability". It is shown that if A be the relative area of clouds with saturated static stability S_2 and 1-A be the cloud-free area with dry static stability S_1, then the effective static stability is given by $\left\{\frac{A\omega_2}{\overline{\omega}} S_2 + \left(1-\frac{A\omega_2}{\overline{\omega}}\right) S_1\right\}$ where W_2 is the vertical p -velocity inside the cloud area and $\overline{\omega}$ is the horizontally average p -velocity. From the definition of $\overline{\omega}$, it is shown that a reasonable value for $\frac{A\omega_2}{\overline{\omega}}$ would be around 0.8, i.e. S_2 receives considerably high weightage.

With this reduced value of static stability, even weak vertical shears in tropical zonal currents manifest baroclinic instability. It is shown that there is much in common between this reduction in the value of static stability and the recent methods of parameterising cumulus convection and explaining the growth of tropical disturbances.

1. Introduction

The purpose of this note is three-fold :-

i. to offer a formula for parameterising static stability when there is a cumulus convection (section 2);

ii. to show that with this parameterisation of cumulus convection and with other rain and cloud conditions which generally prevail in regions found climatologically favourable for growth of tropical disturbances, the value of effective static stability gets sufficiently reduced to account for growth of tropical disturbances through classical baroclinic instability (section 3);

and

iii. to show that there is much in common between the reduction of static stability through condensation as presented in this paper and the recent methods of parameterising cumulus convection and explaining the growth of tropical disturbances.

2. Parameterisation of cumulus effect on static stability

2.1. Under adiabatic conditions, an individual parcel conserves its equivalent potential temperature θ_e

$$\text{i.e.}, \quad \frac{d\theta_e}{dt} = 0 \qquad (1)$$

If the parcel does not experience condensation and evaporation, then the process represented by equation (1) can, for all practical purposes, be represented by

$$\frac{d\theta}{dt} = 0 \qquad (2a)$$

or by its equivalent forms

$$\frac{\partial \alpha}{\partial t} + \mathbb{V}\cdot\nabla\alpha - S_1\,\omega = 0 \qquad (2b)$$

$$\frac{\partial T}{\partial t} + \mathbb{V}\cdot\nabla T - \frac{\omega\alpha}{g}\left(\gamma_d - \gamma\right) = 0 \qquad (2c)$$

when $S_1 \equiv -\frac{\alpha}{\theta}\frac{\partial \theta}{\partial p}$ denotes static stability for dry atmosphere,

$\gamma_d \equiv \frac{g}{c_p}$ denotes dry-adiabatic lapse rate, and

$\gamma \equiv -\frac{\partial T}{\partial z}$ denotes actual lapse rate in the atmosphere.

Other symbols have their usual meaning.

If the parcel experiences condensation and evaporation, then the process represented by equation (1) can be expressed as equation (3a) or its equivalent forms (3b) and (3c) given below.

$$\frac{d\theta}{dt} = \frac{\theta}{c_pT}\left(-L\frac{dq_s}{dt}\right) \doteq \frac{\theta}{c_pT}\left(-L\omega\frac{\partial q_s}{\partial p}\right) \text{---------} \quad (3a)$$

$$\frac{\partial \alpha}{\partial t} + \mathbb{V}.\nabla\alpha - S_2\omega = 0 \text{-----------------} \quad (3b)$$

$$\frac{\partial T}{\partial t} + \mathbb{V}.\nabla T - \frac{\omega\alpha}{g}\left(\gamma_s - \gamma\right) = 0 \text{-----------} \quad (3c)$$

q_s is saturation specific humidity,

$S_2 = -\frac{\alpha}{\theta_e}\frac{\partial\theta_e}{\partial p}$

γ_s = Saturation adiabatic lapse rate.

2.2 In a region having cumulus clouds, there are sub-regions, some with cumulus clouds and some without cumulus clouds. For the mass between two isobaric surfaces separated by infinitasimal pressure differential δp we postulate that relative proportion A of the total mass has cumulus cloud and relative proportion (1-A) is free from cumulus cloud. Set of equations 2(a, b, c) will be applicable to the area (1-A) and the set of equations 3(a, b, c) will be applicable to the area A. Our parameterisation problem consists of finding a constant value s for static stability such that equation sets (2) and (3) can be replaced by one common set with sufficient degree of accuracy. In particular,

$$\frac{\partial \alpha}{\partial t} + \mathbb{V}.\nabla\alpha - S\omega = 0 \text{----------------------} \quad (4)$$

2.3 We are not in a position to take account of each cumulus cloud in the region. As a zero-th order approximation, we consider one set of uniform conditions to prevail throughout cloud region A and another set of uniform conditions to prevail throughout cloud-free region (1-A). Averaging equation (4) over the whole region, we get

$$\frac{\partial \bar{\alpha}}{\partial t} + \overline{V \cdot \nabla \alpha} - \bar{\omega}\left(\bar{s} + \frac{\overline{s'\omega'}}{\bar{\omega}}\right) = 0 \qquad (5)$$

where bar denotes area average and prime denotes departure from area average. The value of static stability s given by

$$s = \bar{s} + \frac{\overline{s'\omega'}}{\bar{\omega}} \qquad (6)$$

may be considered to be the parameterised value which we are searching for. Now,

$$\bar{\omega} = (1-A)\,\omega_1 + A\,\omega_2 \qquad (7)$$

$$\bar{s} = (1-A)\,s_1 + A\,s_2 \qquad (8)$$

where ω_2 and s_2 refer to the region A and ω_1 and s_1 refer to region 1-A

$$\overline{s'\omega'} = (1-A)(s_1-\bar{s})(\omega_1-\bar{\omega}) + A(s_2-\bar{s})(\omega_2-\bar{\omega})$$

$$= A(1-A)(s_1-s_2)(\omega_1-\omega_2) \qquad (9)$$

$$\begin{aligned} s_1 - s_2 &= -\alpha\left(\frac{1}{\theta}\frac{\partial \theta}{\partial p} - \frac{1}{\theta_e}\frac{\partial \theta_e}{\partial p}\right) \\ &= \frac{R}{p}\frac{L}{c_p}\left(\frac{\partial q_s}{\partial p} - \frac{1}{T} q_s \frac{\partial T}{\partial p}\right) \\ &\doteq \frac{R}{p}\frac{L}{c_p}\frac{\partial q_s}{\partial p} \end{aligned} \qquad (10)$$

Substituting in (6), we get

$$\left.\begin{aligned} s &= \left(1 - \frac{A\omega_2}{\bar{\omega}}\right) s_1 + \frac{A\omega_2}{\bar{\omega}}\, s_2 \\ \text{or} \quad s &= s_1 - (s_1 - s_2)\frac{A\omega_2}{\bar{\omega}} \end{aligned}\right\} \qquad (11)$$

In general terms, $\frac{A\omega_2}{\overline{\omega}}$ is the ratio

$$\frac{\text{Vertical flux of mass in cloud area}}{\text{Vertical flux of mass in whole area}}$$

It is interesting to find that whereas for calculating area average of static stability, we use the weighting factors (1-A) for S_1 and A for S_2 as in equation (8) above; but to get effective static stability S, we have to replace A by $\frac{A\omega_2}{\overline{\omega}}$ as in equation (11). In general vertical velocity inside cumulus cloud will be numerically large compared to average vertical velocity $|\overline{\omega}|$. <u>Hence relatively small area of convective cloud is responsible for considerable modification in the value of effective static stability, forgreater than simple area-wise wèightage for convective clouds would suggest.</u>

In general, we have

$$\left.\begin{array}{ll} S_1 > 0 & \text{in lower troposphere in tropics} \\ S_2 < 0 & \\ S_2 > 0 & \text{in upper troposphere in tropics in entire troposphere in extra-tropics.} \\ S_1 - S_2 > 0 & \\ \omega_2 < 0 & \\ \omega_2 - \overline{\omega} < 0 & \\ \omega_1 - \overline{\omega} > 0 & \\ 0 < A < 1 & \end{array}\right\} \qquad (12)$$

These inequalities combined with equation (11) lead to a few very interesting conclusions. We shall examine these conclusion for two cases separately, firstly for $\overline{\omega} > 0$ and secondly for $\overline{\omega} \leq 0$.

2.4 (i) Case $\overline{w} > 0$

From equation (11), it follows that in the presence of clouds,

$$s > s_1 \quad \text{------------} \quad (13)$$

We know that higher static stability inhibits growth of clouds. Hence the very existence of clouds in an area of general descending motion inhibits further growth of clouds.

(ii) Case $\overline{w} < 0$, Now

$$s < s_1 \quad \text{------------} \quad (14)$$

i.e. existence of clouds accelerates the growth of clouds, further reduction in the value of static stability and further growth of clouds and so on. Hence combination of clouds and proper direction of average vertical motion defines a sort of stability criteria for the value of effective static stability s.

2.5 We shall now consider the case $\overline{w} < 0$ in some greater detail,

(i) We ask ourselves : can $\frac{Aw_2}{\overline{w}}$ be greater than 1 ? If so, what would that imply ? From equation (7), it is easy to see that for the case $\frac{Aw_2}{\overline{w}} < 1$ the condition

$$\frac{Aw_2}{\overline{w}} < 1$$

is both a necessary and sufficient condition that w_1 be negative (proof given in appendix). Similarly,

$$\frac{Aw_2}{\overline{w}} > 1$$

is both a necessary and sufficient condition for w_1 to be positive. In other words, $\frac{Aw_2}{\overline{w}} = 1$ defines a transition state between upward and downward motion in the cloud-free region. In vigorous convection, there would be downward motion in cloud-free region, then $\frac{Aw_2}{\overline{w}} > 1$. In equation (11),

S_2 then gets weightage greater than 1 and S_1 gets negative weightage. Since S_2 is negative in the lower atmosphere of the tropics, s itself may become negative. When S is negative, cumulus-scale convection grows exponentially with respect to time and there is no scope for development of large-scale tropical disturbances (kuo, 1961; Lilly, 1960 and others).

ii) In this paper, we are Chiefly interested in the parameterisation of cloud effect on large scale disturbances. For this purpose, the quantity $\frac{A\omega_2}{\bar{\omega}}$ has emerged as an important parameter. At present, we can only speculate about the range of values of $\frac{A\omega_2}{\bar{\omega}}$. To provide room for growth of large-scale disturbances, we should exclude the case when s becomes negative; i.e. $\frac{A\omega_2}{\bar{\omega}}$ should be sufficiently less than 1. At the same time, it will not be desirable to assign to $\frac{A\omega_2}{\bar{\omega}}$ a value much less than 1 because we should be close to the case where vertical velocity in cloud-free region is in downward direction. In other words, the value assigned to $\frac{A\omega_2}{\bar{\omega}}$ should be less than 1 and at the same time close to 1.

iii) These calculations and consequent results which follow lead one to suspect that in a region of general upward motion ($\bar{\omega} < 0$), cloud growth will get accelerated and the net effect of these clouds will be give to s a value very nearly equal to S_2. If s becomes negative, vigorous convection leads to precipitation which cools the lower layers and restores the static stability to positive values. But the prevailing general upward motion and clouds again initiate more cloudiness and the lowering of the value of static stability and so on.

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iv) It is known that lowering of the value of static stability is favourable for bacoclinic instability and for the growth of synoptic scale disturbances. Hence the existence of gsneral upward motion and the presence of sufficient moisture in the region so as to lead to condensation in the region of upward motion will trigger the growth of synoptic scale disturbances.

3. Baroclinic instability of zonal currents in the tropics

3.1 It is known that the vertical shears of zonal winds in the tropics are so weak that the currents are baroclinically stable with reference to wave perturbations in which thermodynamic processes are dry and therefore are represented by equation (2b). It is also known that if by some process, the governing thermodynamic equation (2b) could be replaced by equation (4) such that $s \leqslant 0.1\ S_1$, then the same zonal currents with the same weak vertical shears would become baroclinically unstable. The question is; can we justify such reduction in the value of effective static stability ?

From what we have seen in section 2 above, such a reduction seems not only possible but even inevitable provided that we have a general upward motion ($\overline{w} < 0$) and also supply of sufficient moisture to the air to yield condensation in the generally upward moving air.

We cannot invoke condition $\overline{\omega} < 0$ from the wave perturbation itself because the upward flux of mass in one half of the wave will cancel the downward flux of mass in the other half of the wave. We therefore need to invoke vertical upward motion on a much larger scale than the perturbation wave itself. If possible, we should show that the zonal current with weak vertical shear and the wave perturbation superimposed on it lies in the region of this vertical upward motion.

Here, observations come to our help. It is observed that tropical disturbances generally grow over oceanic regions or other extensive water surfaces close to seasonal troughs. The seasonal trough may be ITCZ or the other near-equatorial trough (Asnani, 1968). These trough regions are the regions of general upward motion forming the limb of rising motion of the Hadley cell or the Equatorial cell in the vertical meridional plane. Thus the two conditions of general upward motion and of availability of sufficient moisture for condensation are both satisfied.

3.2 Figures 1 a, b, and c are the instability diagrams for tropical region (latitude 15 deg.) for values of effective static stability $s = 0.006$, 0.004 and 0.002 $m^2\ sec^{-2}\ mb^{-2}$ respectively. These diagrams are based on 2-layer linearised quasi-geostrophic model (Thomson, P.D., 1961; Sreeramamurthy, D. 1967). The vertical shears of wind which are frequently encountered in the Indian monsoon tropics are of the order of 20 m sec^{-1}/500 , mb. Zonal currents with this and even weaker shears become baroclinically unstable if $s < 0.004$ in the two-layer model. S_1 for the layer 800-300 mb is 0.037 $m^{-2}\ sec^{-2}\ mb^{-2}$ (Asnani and Rao, 1963); $S_2 \approx -0.1\ S_1$. For some plausible values of $\frac{A\omega_2}{\omega}$, we give the values of s in Table 1.

4. Relationship between present work and other studies on parameterisation of cumulus convection

4.1 Due to inability of the dry process to create conditions for baroclinic instability in tropics, search was made for other processes

Table 1 : Values of s for different values of $A w_2/\overline{w}$

($s_1 = 0.037$, $s_2 = -0.005$).

$\frac{A w_2}{\overline{w}}$	0.6	0.7	0.8	0.9
s	0.0120	0.0079	0.0037	-0.0005

to account for the observed growth of tropical disturbances. Wholesale replacement of dry static stability s_1 by s_2 led to the growth of cumulus scale convection in the tropics (Kuo, 1961; Lily, 1960). A plausible combination of s_1 and s_2 was essential. Attempts to replace s_1 by $\bar{S} = AS_2 + (1-A)S_1$ led only to a very small and inadequate reduction in the value of static stability becuase observations showed that even in the wall region of tropical cyclone, less than 10% of the mass could be considered to be participating in active convection at a time (Malkus et al, 1961). Theory of conditional instability of second kind (CISK) first advanced by Charney and Eliassen (1964) to explain the growth of symmetric circular vortices and later extended (Yamasaki, 1969) to explain the growth of wave perturbations superimposed on tropical zonal currents with weak shears along the vertical has proved to be a very fruitful approach to explain the growth of tropical disturbances. From what has been stated in sections 2 and 3 above, it is natural to expect a close link to exist between the method of CISK and the method of reducing the value of effective static stability.

4.2 CISK discussions generally take thermodynamic equation explicitly in terms of θ . We shall outline derivation of corresponding effective stability from equations (2a) and (3a). We have

$$\frac{\partial \theta}{\partial t} + \mathbb{V}\cdot\nabla\theta - \sigma_1 w = 0 \qquad \text{(2a)}$$

$$\frac{\partial \theta}{\partial t} + \mathbb{V}\cdot\nabla\theta - \sigma_2 w = 0 \qquad \text{(3a)}$$

where $\sigma_1 \equiv -\frac{\partial \theta}{\partial p}$

$$\sigma_2 \equiv \sigma_1 - \frac{\theta}{c_p T} L \frac{\partial q_s}{\partial p}$$

σ_1 is always +ve; σ_2 is −ve in the lower troposphere of the tropics but is +ve elsewhere.

$\sigma_1 - \sigma_2$ is always positive. As in section 2, we define effective static stability σ by

$$\sigma = \bar{\sigma} + \frac{\overline{\sigma' w'}}{\overline{w}}$$

Following the steps taken in section 2.3, we easily obtain

$$\left.\begin{aligned} \sigma &= \left(1 - \frac{Aw_2}{\overline{w}}\right)\sigma_1 + \frac{Aw_2}{\overline{w}}\sigma_2 \\ \text{or}\quad \sigma &= \sigma_1 - (\sigma_1 - \sigma_2)\frac{Aw_2}{\overline{w}} \end{aligned}\right\} \qquad (15)$$

and the thermodynamic equation becomes

$$\left.\begin{aligned} \frac{\partial \bar{\theta}}{\partial t} + \overline{\mathbb{V}\cdot\nabla\theta} - \overline{w}\,\sigma_1 &= -\frac{Aw_2}{\overline{w}}\frac{\theta}{c_pT}L\overline{w}\frac{\partial q_s}{\partial p} \\ \frac{d\bar{\theta}}{dt} &= -\frac{Aw_2}{\overline{w}}\frac{\theta}{c_pT}L\overline{w}\frac{\partial q_s}{\partial p} \end{aligned}\right\} \qquad (16)$$

It would be seen that equation (16) is different from the corresponding expression obtained by Yanai (1964). For $\frac{Aw_2}{\overline{w}} = 0$, equation (16) represents completely dry process; for $\frac{Aw_2}{\overline{w}} = 1$, for $\frac{A\omega_2}{\overline{\omega}}=1$, it represents completely saturated process. CISK parameterisation of condensation has consisted in replacing R.H.S. of equation (16) by alternative expressions corresponding to $0 \leq \frac{A\omega_2}{\overline{\omega}} < 1$. In Charney and Eliassen's (1964) work, the total heating is taken proportional to

$$-\int_0^{\alpha} \bar{\rho}\, \overline{\omega}\, \frac{\partial}{\partial z}\left(\mu q_s\right) dz$$

Charney and Eliassen found that μ = 0.7 or 0.8 gave realistic growth rates for tropical cyclone-scale disturbances. This is somewhat similar to our taking $\frac{A\omega_2}{\overline{\omega}} = 0.7$.

Following Ooyama's work (1964), Yamasaki (1969) replaced the R.H.S. of equation (16) by $h(p)\frac{\partial\bar{\theta}}{\partial p}\omega^*$ where $h(p)$ is a non-dimensional function of pressure and ω^* is the vertical p -velocity at the top of frictional layer. He assigned a few plausible values to h and by numerical modelling studied the effect of these distribution on growth rates of different wave-lengths.

Following Kuo (1965), Krishnamurti (1969) denoted the rate of heating due to condensation by $\frac{\theta}{c_pT}\, a\left(T_c - T\right)$ where $T_c - T$ is the temperature difference between the cloud and the environment, 'a' being a measure of the fraction of the area covered by convective cloud ($a \leq 0.01$). Vertical velocity is induced by low level friction, orography and dynamical forcings at all levels.

More and more numerical models incorporating latent heat of condensation, friction and also other forcings, all simulating growth of tropical disturbances are coming in the field (Manabe et al, 1970; Murakami, 1972) and the common feature of all the models is introduction of latent heat due to condensation in regions of rising motions of air, the rising motion being induced essentially by friction, other forcings for vertical motion being sometimes present and sometimes abxeht.

It would be seen that in all the recent treatments of parameterising cumulus convection in the theory of tropical disturbances, there is much in common between equation (16) and the corresponding thermodynamic energy equations used by other authors.

With the multiplicity of numerical models, it becomes often diffi̇cult to decide which are the minimum requirements for the growth of tropical disturbances. It appears to us that one of the essential requirements is considerable reduction in the value of effective static stability through condensation.

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Case : $\overline{\omega}$ upwards

Prove that in a region of negative $\overline{\omega}$, necessary and sufficient condition for ω_1 to be negative is that

$$\frac{A\omega_2}{\overline{\omega}} < 1$$

Ans. (1) Prove necessity of the condition :

Given $$\frac{A\omega_2}{\overline{\omega}} < 1$$

(1.1) Multiply throughout by $\overline{\omega}$ knowing that $\overline{\omega}$ is negative

$$A\omega_2 > \overline{\omega}$$

(1.2) Add $(1-A)\,\omega_1$ throughout

$$\underbrace{A\omega_2 + (1-A)\,\omega_1}_{\text{This is } \overline{\omega}} > \overline{\omega} + (1-A)\,\omega_1$$

$$\therefore \overline{\omega} > \overline{\omega} + (1-A)\,\omega_1$$

(1.3) Add $-\overline{\omega}$ throughout

$$0 > (1-A)\,\omega_1$$

(1.4) This shows that $(1-A)\,\omega_1$ is $-ve$

Now $(1-A)$ is $+ve$: $\therefore \omega_1$ is negative

Necessary condition is proved.

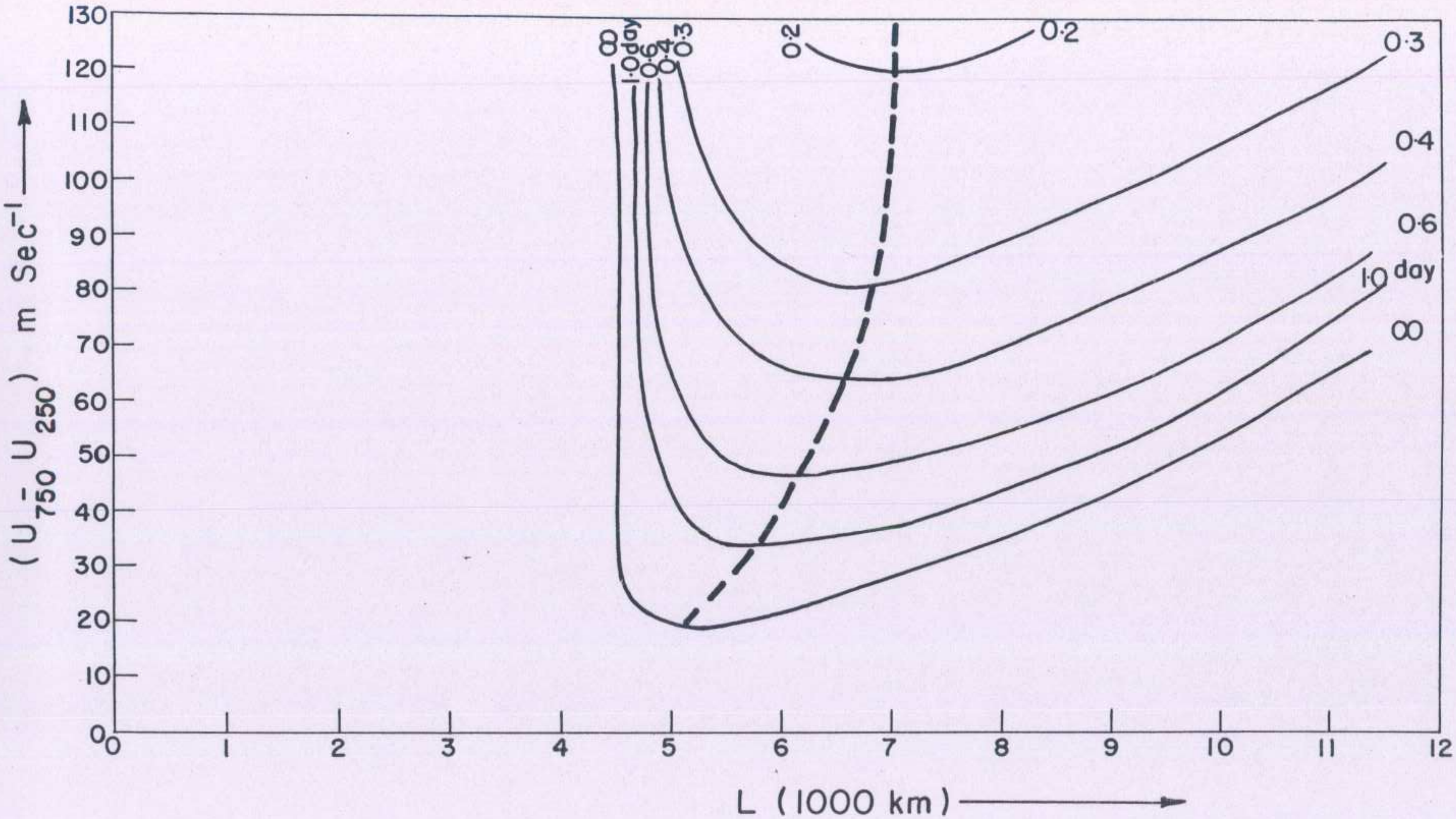
For this, we have effectively used only. $\dfrac{A\omega_2}{\overline{\omega}} < 1$

(2) Prove sufficiency of this condition :

Given $\omega_1 < 0$

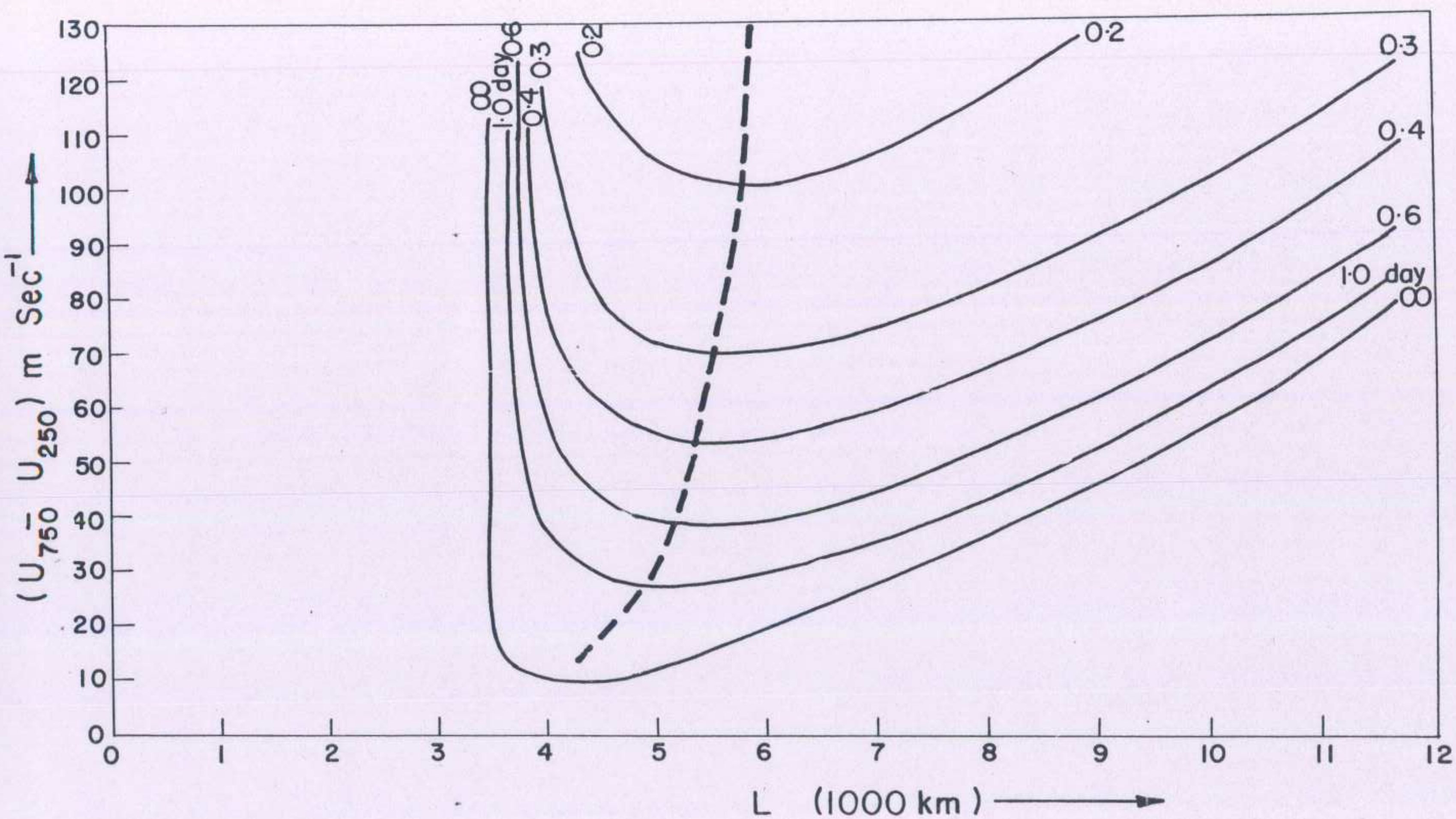
(2.1) $$0 > \omega_1$$

multiply by $(1-A)$ which is $+ve$



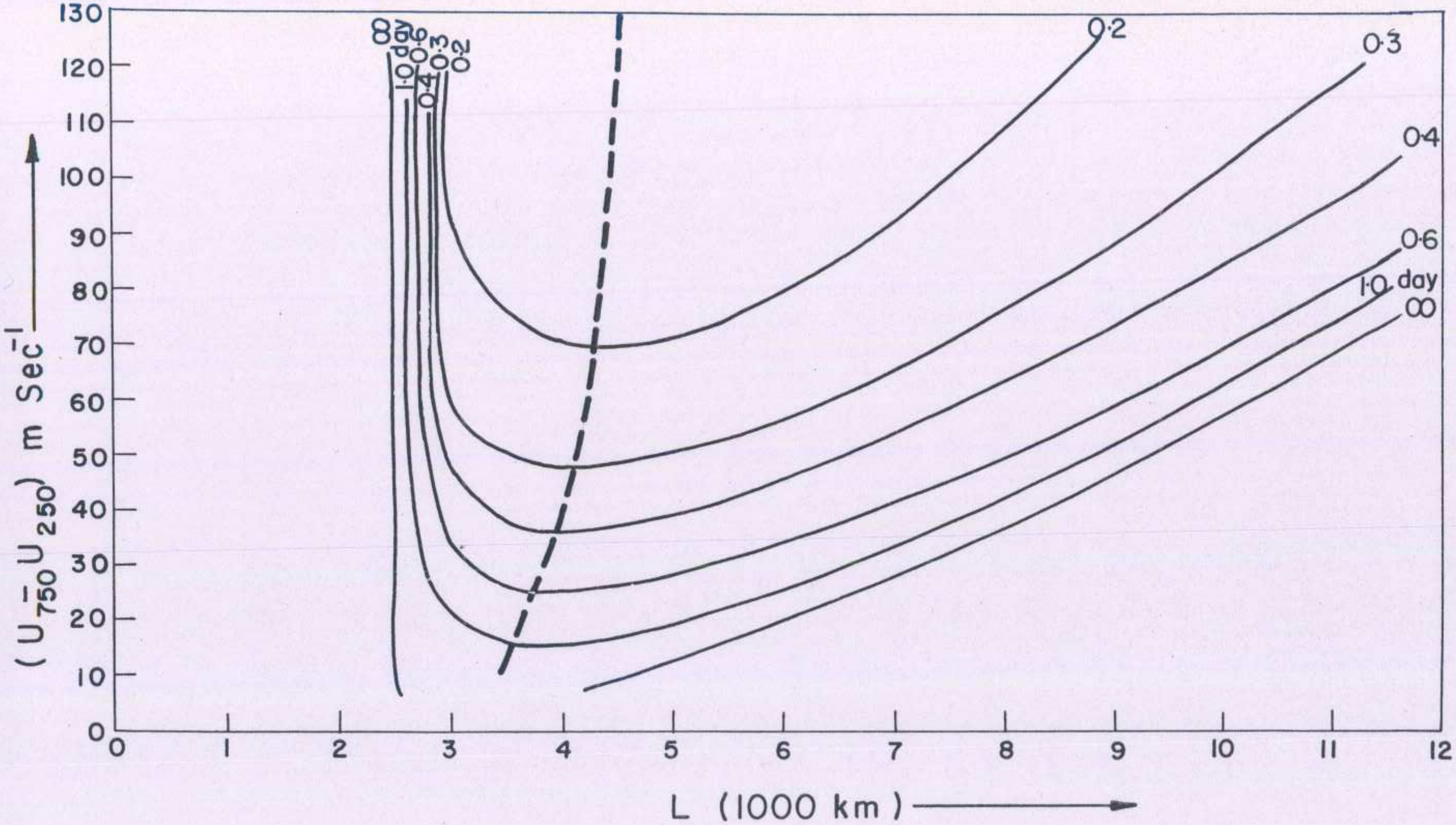
Doubling Time in days ; $S = 0{\cdot}006\ m^2\ Sec^{-2}\ mb^{-2}$

Fig. I (a)



Doubling Time in days ; $S = 0{\cdot}004\ m^2\ Sec^{-2}\ mb^{-2}$

Fig 1 (b)



Doubling Time in days; $S = 0{\cdot}002 \ m^2 \ Sec^{-2} \ mb^{-2}$

Fig 1 (c)