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A NOTE ON "APPLICABILITY OF QUASI-GEOSTROPHIC BAROTROPIC MODEL IN THE TROPICS"

by

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1. Introduction

Due to low value coriolis parameter in the tropics, it is generally believed that quasi-geostrophic models are not suitable for the tropics in general and for near equatorial regions in particular. We show in this note that this is not necessarily so; for barotropic case in particular, quasi-geostrophic wave velocity is very close to the one given by prinitive equation model and this closeness increases as we approach the equator.

Treating linearised P.E.Model, Matsuno (1966) analysed the configuration of isobars and streamlines in the equatorial region and from the appearance of the configurations came to the conclusion : "It is noteworthy that we can get 'quasi-geostrophic' motions in the domain including the equator". He did not compare the performance of quasi-geostrophic model relative to the P.E.Model. The present linearised model is more restricted in y direction than Matsuno's model but it compares quantitatively the quasi-geostrophic model with P.E.Model, both having identical y -variations.

2. Model

We take a simple barotropic model with constant zonal velocity U and an infinitesimal sinusoidal perturbation having the following restrictions :

i) Divergent componant of perturbation wind consists of only u' referred to as u'_3 after the terminology of Lorenz (1960). Non-divergent component of perturbation wind consists of only v' referred to as v'_2.

ii) $u'_3, v'_2, \phi'_1 \propto e^{ik(x-ct)}$

and are independent of y.

The linearised forms of the usual complete vorticity, divergence and continuity equations become

$$ikfu'_3 + \left\{\beta - k^2(U-c)\right\} v'_2 + 0\cdot\phi'_1 = 0 \qquad (1)$$

$$\left\{\beta - k^2(U-c)\right\} u'_3 - ikf v'_2 - k^2\phi'_1 = 0 \qquad (2)$$

$$ikg\bar{h}\, u'_3 - fUv'_2 + ik(U-c)\,\phi'_1 = 0 \qquad (3)$$

When we start with complete vorticity, divergence and continuity equations as above, we consider ourselves to be dealing with primitive equation (P.E.) model.

For non-trivial solution, we get the frequency equation as

$$Y^3 - \frac{2\beta}{k^2} Y^2 + \left\{\left(\frac{\beta}{k^2}\right)^2 - g\bar{h} - \frac{f^2}{k^2}\right\} Y + \left(\frac{\beta}{k^2} g\bar{h} + \frac{f^2}{k^2} U\right) = 0 \qquad \ldots (4)$$

If instead of taking P.E.Model as in set of equations (1), (2) and (3) we were to take the energetically consistent quasi-geostrophic model (Lorenz, 1960), we would get the linearised vorticity, divergence and continuity equations in the form

$$ikf\, u'_3 + \left\{\beta - k^2(U-c)\right\} v'_2 + 0\cdot\phi'_1 = 0 \qquad \ldots (1')$$

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$$0 \cdot u_3' - ikf v_2' - k^2 \phi_1' = 0 \qquad \cdots (2')$$

$$ikg\bar{h}\, u_3' - fU v_2' + ik(U-c)\phi_1' = 0 \qquad \cdots (3')$$

For non-trivial solution, this set yields

$$\alpha \equiv (U-c)_{\text{geostrophic}} = \frac{\frac{\beta}{k^2} g\bar{h} + \frac{f^2}{k^2} U}{g\bar{h} + \frac{f^2}{k^2}} \qquad \ldots (5)$$

This equation can also be arrived at easily from the work of Doos (1965) by combining his equation (4.4) with his equation (3.24) and taking one layer model ($\epsilon = 0$) and $f(\xi + f) \approx f^2$.

3. Root of equation (5) compared with Rossby root of equation (4)

We got the three roots y_1, y_2, y_3 of equation (4) on digital computer numerically by Cardan's method, for a wide range of variation of U (which was varied from -40 to +40) metres sec^{-1} and of latitude (which was varied from equator to pole). Wavelength was changed over a range of values from 100 km to 40,000 km. We varied $\bar{h}$ from 10 km to 5 km. For the same values of U, L, f, β and $\bar{h}$, we also got the numerical values of analytical expression α. The typical results of computation are shown in Table 1, for U = 20 metres sec^{-1}, $\bar{h}$ = 10.0 km and for wave lengths between 500 km and 20,000 km, from equator to 80°N. Of course, wave lenghts as large as 20,000 km are of no physical significance in very high latitudes but figures corresponding to high latitudes have also been shown here just to compare the Rossby root y_1 of the primitive equation model (4) with the Rossby root α of the quasi-geostrophic model (5). The values have been given to 5 significant digits as obtained from the computer after rounding off. The computer performed the operations with 28 digits.

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Table 1 : Quasi-geostrophic root ... (Eq.5) and the three roots of cubic frequency equation ... (Eq.4) of primitive equation model.

$U = 20\ \mathrm{m\ sec^{-1}}$ $\bar{h} = 10.0$ km

Wave length km	Latitude Deg.N	α (Q.G.) $\mathrm{m\ sec^{-1}}$	γ_1 (P.E.) $\mathrm{m\ sec^{-1}}$	γ_2 (P.E.) $\mathrm{m\ sec^{-1}}$	γ_3 (P.E.) $\mathrm{m\ sec^{-1}}$
500	Equator	.14496	.14496	313.12	- 312.98
"	10	.14358	.14358	313.13	- 312.99
"	20	.13941	.13941	313.14	- 313.01
"	40	.12233	.12233	313.19	- 313.09
"	60	.09300	.09300	313.24	-.313.18
"	80	.05176	.05176	313.26	- 313.26
1000	Equator	.57984	.57984	313.34	- 312.76
"	10	.57425	.57425	313.36	- 312.79
"	20	.55738	.55738	313.42	- 312.88
"	40	.48850	.48850	313.60	- 313.20
"	60	.37085	.37085	313.80	- 313.59
"	80	.20622	.20622	313.88	- 313.89
10000	Equator	57.984	57.984	343.38	- 285.40
"	10	56.498	56.498	345.80	- 288.10
"	20	52.403	52.406	352.47	- 295.90
"	40	39.899	39.905	372.08	- 323.15
"	60	26.367	26.368	388.17	- 356.55
"	80	13.522	13.524	390.93	- 384.31
20000	Equator	231.94	231.94	449.81	- 217.87
"	10	215.45	215.78	465.29	- 224.25
"	20	177.45	179.59	499.85	- 243.55
"	40	102.61	105.55	569.24	- 319.44
"	60	56.225	56.988	601.09	- 426.14
"	80	26.472	26.489	581.60	- 527.54

The gravity wave roots (Y_2 and Y_3) of (4) have also been given in Table 1.

Comparing the P.E. root Y_1 with the quasi-geostrophic root α at various latitudes, we easily see that there is very good agreement between the two roots at all latitudes for all wave-lengths from 500 km to 20,000 km. Difference between the roots, if any, tends to be <u>minimum</u> at the equator, increasing towards middle latitudes. This feature is highlighted in Table 2 which presents the ratio $\frac{Y_1 - \alpha}{Y_1}$ for two wave-lengths, 1,000 km and 10,000 km at latitudes from equator to 80°N. As before, the computer performed floating point arithmetic using 28 digits.

Table 2 : $\frac{Y_1 - \alpha}{Y_1}$ for wave lengths 1000 km and 10000 km.
$U = 20\ m\ sec^{-1}$; $\bar{h} = 10.0$ km.

Latitude deg.	$(Y_1 - \alpha)/Y_1$ Wave length 1000 km	Wave length 10000 km
Equator	-3.2×10^{-25}	-7.2×10^{-27}
10	1.1×10^{-10}	3.7×10^{-6}
20	1.6×10^{-9}	4.2×10^{-5}
30	7.3×10^{-9}	1.2×10^{-4}
40	2.0×10^{-8}	1.7×10^{-4}
50	4.1×10^{-8}	1.4×10^{-4}
60	6.7×10^{-8}	5.0×10^{-5}
70	9.3×10^{-8}	2.1×10^{-8}
80	1.1×10^{-7}	7.9×10^{-5}

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4. Conclusion

This analysis has its limitations in so far as it is based on linearised model of a restricted type. To that extent, the interpretation of results has to be taken with caution. Nevertheless, it is quite interesting to find that in respect of wave-velocity, the quasi-geostrophic model of this restricted type is at its best in the tropics, if the merit is to be judged by closeness of the geostrophic root to Rossby-type root of the P.E. model. In N.W.P. work, barotropic model is essentially used for forecasting the movement of large-scale meteorological waves in the middle troposphere rather than for forecasting the development of these systems. This analysis suggests that smallness of coriolis parameters f in the tropical latitudes need not deter us from using quasi-geostrophic barotropic model in the tropics. Obviously, β is a very important parameter for synoptic scale waves in this model and this term is largest at the equator, partly overcoming the influence of smallness of f. For wave-propagation in barotropic model, the essence of Rossby wave dynamics appears to be pretty well caught by simple quasi-geostrophic model at all latitudes including the tropics.

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