RESEARCH REPORT RR-022

RR-22

DYNAMICAL PARAMETERS DERIVED FROM ANALYTICAL FUNCTION REPRESENTING NORMAL JULY ZONAL FLOW ALONG 87·5°E

by

S. T. AWADE & G. C. ASNANI

INDIAN INSTITUTE OF TROPICAL METEOROLOGY

Ramdurg House
Ganeshkhind Road
Poona-5, India

May 1978

DYNAMICAL PARAMETERS DERIVED FROM ANALYTICAL FUNCTION REPRESENTING NORMAL JULY ZONAL FLOW ALONG 87.5°E

RR-22

By

S.T. Awade and G.C. Asnani
Indian Institute of Tropical Meteorology, Poona-5.

Abstract :

In an earlier paper, Awade and Asnani (1973) fitted analytical function to normal July zonal wind along longitude 77.5°E between the latitudes 5°N and 25°N from 850 mb to 150 mb levels. From this analytical function, they derived dynamical parameters like vorticity, thermal wind shear, static stability, etc. Similar analysis of normal July zonal wind component is now presented for longitude 87.5°E between 5°N and 25°N.

It is found that there is good agreement between the observed and the fitted zonal winds and temperatures, confirming the soundness of the analytical function chosen for fitting purposes and also of the assumption of geostrophic balance in this case. From the analysis of absolute vorticity of the zonal flow, it is concluded that the zonal flow is inertially stable and further would also be barotropically stable if it is subjected to infinitesimal perturbations of the type

... 2

considered by Kuo (1949). The maximum wind at 150 mb level is found to be easterly 60 kts at latitude 9.2°N. Extrapolation to 130 mb give maximum easterly wind of 72 knots at 10°N.

Compared to conditions along 77.5°E, the horizontal meridional temperature gradient and consequently the vertical shear of the zonal wind is slightly weaker along 87.5°E; static stability $-\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p}$ is slightly more along 87.5°E.

Utilising fitted zonal wind components along 77.5°E and 87.5°E, it is found that $\frac{\partial u}{\partial x}$ contributes to velocity convergence in the lower levels and to velocity divergence aloft. Magnitude of $\frac{\partial u}{\partial x}$ is of the order of 10^{-6} sec^{-1}.

1. Introduction :

There is considerable merit in fitting smooth analytical functions to climatological data and then studying some of their large scale characteristics. With this motivation, the authors (Awade and Asnani 1973) fitted, by least square method, the perabola $u = a + b \log_{10} p + c(\log_{10} p)^2$ ..... (1) to the zonal wind u along longitude 77.5°E between latitude 5°N and 25°N. The co-efficients a,b,c are again functions of the latitude φ found by fitting the curve of the type

$$a = a_0 + a_1\varphi + a_2\varphi^2 \qquad ..... (2)$$

... 3

p denotes the pressure level and φ denotes the latitude. The fitted values agreed very well with the observed values. The authors could then easily construct vertical meridional profiles of important parameters like temperature, vertical wind shear, relative vorticity, absolute vorticity, static stability etc.

Longitude 77.5°E is representative of the meridian passing through the southern tip of the Indian peninsula and thus gives the largest length of Indian landmass in north-south direction. It was considered equally important to have such profiles also for the Bay of Bengal and for the Arabian sea regions. It is true that we do not have observing stations along these meridians in the sea region. Still with the help of normal winds available for east coast of India, Bay Islands, west coast of Burma and Bangaladesh, it is possible to make analysis of u -field for the Bay of Bengal. We did this analysis and it looked reasonable. The zonal wind values obtained from this analysis were further smoothened by drawing smooth graphical profiles along the vertical. These were taken as the observed winds at different levels and latitudes along the meridian 87.5°E. To these winds, we fitted the analytical function :

$$u = (a_o + a_1\varphi + a_2 \varphi^2) + (b_o + b_1\varphi + b_2 \varphi^2)(\log_{10}p) + (c_0 + c_1\varphi + c_2 \varphi^2)(\log_{10}p)^2 \quad \ldots [3]$$

For further details of fitting, one may refer to Awade and Asnani (1973).

2. Fitted values and the goodness of fit

An analysis of variance for the parabolic curve against $\log_{10}p$ (Eq.1) showed that the linear and second degree terms are significant for all latitudes at 5 percent level.

Similar analysis of variance for parabolic curve fitted to the coefficients a,b and c against latitude (Eq.2) also showed that the first degree and second degree terms are significant for all the coefficients at 5 percent level.

The values of the coefficients are given below :

a_o = -106.44 , a_1 = -140.21 , a_2 = 5.58
b_o = - 18.24 , b_1 = 106.89 , b_2 = -4.17
c_o = 20.42 , c_1 = - 19.99 , c_2 = 0.76

Table 1 gives observed and fitted zonal winds. The fitted values agree fairly well with the observed values. Except at four points in the upper troposphere, the difference between observed and fitted winds does not exceed 3 knots. All subsequent calculations referred to in this paper for

... 5

latitude belt 5°N to 25°N were done on the basis of the Eq.3. The vertical meridional cross-section of the fitted u is shown in Fig.1. Positive u's indicate westerlies and negative u's indicate easterlies.

3. Calculation of derived parameters

3.1 Calculation of the vertical wind shear :

From Eq.3, it follows that

$$\frac{\partial u}{\partial p} = [(b_o + b_1\varphi + b_2\varphi^2) + 2 (c_o + c_1\varphi + c_2 \varphi^2) \log_{10}p] \times \text{long}_{10}e \quad ..... (5)$$

The vertical wind shear calculates from Eq.(5) and expressed in knots per 100 mb is shown in Table 2. It is seen that the vertical wind shear increases with height and has considerable magnitude (of the order of 50 knots per 100 mb) in the upper troposphere.

3.2 Calculation of the temperature :

As in the previous paper (Awade and Asnani, 1973), we introduced geostrophic approximation through the thermal wind relationship

$$\frac{\partial u}{\partial p} = \frac{R}{fP}\frac{\partial T}{\partial y} \quad ... (6)$$

Substituting from Eq.(5) for $\frac{\partial u}{\partial p}$ in (6), we get

$$\frac{\partial T}{\partial y} = \frac{f}{R} \left[(b_o + b_1 \varphi + b_2 \varphi^2) + 2 (c_o + c_1 \varphi + c_2 \varphi^2) \log_{10} p \right] \log_{10} e \qquad \ldots (7)$$

Integrating Eq.(7) with respect to φ , we get

$$\bar{T}_{\psi,p} - \bar{T}_{\psi_o,p} = \log_{10} e \ \bar{r} \ 2 \Omega \left[-b_o \operatorname{Cos} \psi + b_1 (\operatorname{Sin} \psi - \psi \operatorname{Cos} \psi) + b_2 (2 \psi \operatorname{Sin} \psi - (\psi^2 - 2) \operatorname{Cos} \psi) + 2 \log_{10} p \left\{ -c_o \operatorname{Cos} \psi + \psi + c_1 (\operatorname{Sin} \psi - \psi \operatorname{Cos} \psi) + c_2 (2 \psi \operatorname{Sin} \psi + (\psi^2 - 2) \operatorname{Cos} \psi) \right\} \right]_{\psi_o} \qquad \ldots (8)$$

where ψ is the latitude in radians, φ is the latitude in degrees, Ω is the angular velocity of the earth and r is the radius of the earth. We took ψ_o corresponding to the latitude 5^{O}N. The R.H.S. of the Eq.(8) then gives the latitudinal variation of the temperature from 5^{O}N to any latitude for which Eq.(8) is valid. These profiles for various isobaric levels are shown in Fig.2. In this diagram, 5^{O}N has been taken as the reference latitude.

The values of the temperature at 5^{O}N were obtained from climatological charts. From these we obtained the temperature values at various latitudes at different pressure levels. These were compared with the climatological temperarature values and were found to agree well. This gives an indirect comfirmation that the thermal wind relationship which is based on geostrophic relationship, holds very well over the region 5^{O}N to 25^{O}N along 87.5^{O}E for the climatological zoral winds in the monsoon season.

3.3 Calculation of static stability :

The static stability was calculated using the formula

$$S = -\frac{\bar{\alpha}}{\theta}\frac{\partial \theta}{\partial p} = \frac{R}{p^2}\left(KT - \frac{\partial T}{\partial \log_e p}\right) \qquad \ldots (9)$$

since against log at all latitudes showed a linear relationship between 850 mb and 150 mb levels, we can use finite different form

$$S \approx \frac{R}{p^2}\left(KT - \frac{T_{850} - T_{150}}{\log_e 850 - \log_e 150}\right) \qquad \ldots (10)$$

The values of S are given in Table 3.

3.4 Calculation of vorticity on the isobaric surfaces :

From Eq.(3), we obtain $\left(-\frac{\partial u}{\partial y}\right)$ i.e. relative vorticity of the zonal motion. Fig.3(a) gives the vertical meridional cross section of this quantity. The following points are noteworthy.

i] Below 300 mb level, there is a zero line of transition north of which there is cyclonic vorticity and south of which there is anticyclonic vorticity. This line of transition has steep alopes between 8°N and 13°N but weak alope from 15°N to 25°N.

ii] On any isobaric surface below 300 mb level, relative vorticity increases from south to north, the largest cyclonic vorticity occuring at 25°N at 600 mb level. At 200 and 150 mb levels relative vorticity continuously decreases from south to north, the largest anticyclonic vorticity occurring at 25°N at 150 mb level.

iii] At all latitudes, we see the reversal of the sign of relative vorticity along the vertical, with cyclonic vorticity below and anticyclonic vorticity aloft or vice versa.

Fig.3 (b) gives the variation of ($f - \frac{\partial u}{\partial y}$) with the latitude and height. It is seen that this quantity increases from south to north at all latitudes and at all levels including 200 and 150 mb, levels. This is because β is larger than $-\frac{\partial^2 u}{\partial y^2}$. The values of $-\frac{\partial^2 u}{\partial y^2}$ as a function of pressure calculated from Eq.(3) and given below. Since u is a second degree polynomial in y , $\frac{\partial^2 u}{\partial y^2}$ is independent of latitude.

Level (mb)	850	800	700	600	500	400	300	200	150
$-\frac{\partial^2 u}{\partial y^2}$ (Unit = $10^{-11} m^{-1} sec^{-1}$)	0.68	0.74	0.85	0.92	0.93	0.83	0.52	−0.26	−1.06

The value of β varies from 2.07 x 10^{-11} m^{-2}, sec^{-1}. It is, therefore, clear that $\beta - \frac{\partial^2 u}{\partial y^2}$ is always positive in the region of our investigation. In other words, absolute vorticity corresponding to the zonal motion does not reach any maximum value in this region. In other words, by Kuo's (1949) criterion based on linearised theory, the zonal current is barotropically stable with respect to the infinitesimal perturbations.

In these calculations, we have not considered meridional component of wind v at all. It is known that over the head Bay of Bengal in the lower troposphere, the persistent climatological wind pattern has considerable meridional component during July. Kuo's (1949) linearised theory of barotropic instability may not be applicable in these circumstances. It will be seen that static stability increases with latitude and also with height throughout the region of our analysis.

3.5 Calculation of $-\frac{\partial u}{\partial y}$ on Isentropic surfaces :

We have calculated $f-\left(\frac{\partial u}{\partial y}\right)_\theta$, where subscript θ denotes differentiation along an isentropic surfaces. These values are plotted in Fig.4. A fluid layer is inertially stable $f+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)_\theta \geq 0$ (Kuo, 1956).

Assuming that the contribution due to the term   is small north of 5°N, one can infer from Fig.4 that the absolute vorticity on an isentropic surface north of 5° is positive and hence the normal zonal flow in the monsoon season is inertially stable between 5°N and 25°N along 87.5°E.

3.6 Calculation of Jet maximum

We use Eq.(3) for the analysis of the wind maximum. From the condition $\frac{\partial u}{\partial \varphi} = 0$ for maxima or minima, one gets the latitude of the maximum wind at constant pressure surface as

$$\varphi = \frac{a_1 + b_1 \log 10p + c_1 (\log 10p)^2}{2\{a_2 + b_2 \log 10p + c_2 (\log 10p)^2\}} \quad \ldots (11)$$

At 150 mb level, the maximum wind then comes out to be 59.9 knots at latitude 8.17°N. Inspection of vortical profile of zonal wind show that the easterly wind maximum in the vertical occurs above 150 mb level but below 100 mb, perhaps around 130 mb level. Eq.(1) is strictly valid at and below 150 mb level. If we extend the application of this equation in the vertical upto 130 mb, then maximum easterly wind would be 71.7 kts at the latitude 9.98°N.

4. Comparison of parameters at 77.5°E and 87.5°E

Earlier, Awade and Asnani (1973) computed similar along 77.5°E. We can compare those values with the present values along 87.5°E.

(a) Compared to conditions along 77.5°E, the temperatures along 87.5°E are generally higher (by about 0.5°K) at southern latitudes and generally lower (by about 0.5°K) at northern latitudes. Thus, the meridional horizontal temperature gradient is weaker along 87.5°E than along 77.5°E. Hence the vertical wind shear values are generally on lower side for all latitudes and levels at 87.5°E as compared to the values at 77.5°E.

(b) The static stability parameters are on slightly higher side at 87.5°E as compared to the values obtained at 77.5°E.

(c) Along both meridians 77.5°E and 87.5°E, the normal zonal wind flow between 5°N and 25°N is inertially and barotropically stable - with respect to infinitesimal perturbations.

(d) Comparison of zonal winds at 77.5°E and 87.5°E showed that there is systematic gradient between the values of u along the two meridians. Fig.3(c) shows the distribution of $\dfrac{u_{87.5} - u_{77.5}}{\text{Distance between meridian}}$ at various latitudes and levels. These values can be interpreted as approximately the value of $\frac{\partial u}{\partial x}$ along longitude 82.5°E. It is interesting to find that the zonal components of the wind is contributing systematically to velocity convergence in lower levels and divergence aloft.

5. Conclusions :

i] There is good agreement between the observed and the fitted zonal winds. It shows that the method of smoothing adopted in this paper is reasonably sound.

ii] From the zonal winds, temperature profiles have been computed on the assumption of geostrophic balance. The computed temperatures are in good agreement with the observed temperatures. Hence, geostrophic assumption for the climatological zonal wind in this region appears justified.

iii] Absolute vorticity relating to zonal wind component is positive throughout the region. Hence climatological zonal wind flow is inertially stable in this region.

iv] Absolute vorticity relating to zonal wind component does not reach a maximum in the region investigated. Such zonal flow subjected to infinitesimal perturbations would be barotropically stable (Kuo, 1949).

v] The fitting of analytical function has been done from 850 to 150 mb level. At 150 mb level, the maximum wind is 60 kts at latitude 9.2°N. If the application of fitted function could be extended upto 130 mb level, then the maximum wind at 130 mb level would be 72 kts at 10°N.

vi] Computed to conditions along longitude 77.5°E, the horizontal meridional temperature gradient and consequently the vertical shear of the zonal wind is slightly weaker along longitude 87.5°E.

vii] Static stability parameter $-\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p}$ is slightly higher along 87.5°E than along 77.5°E.

viii] If we use the fitted values of u along 77.5°E and 87.5°E, level for level, to calculate $\frac{\partial u}{\partial x}$ at various points along 82.5°E, then it is seen that this contributes to velocity convergence in the lower levels and to velocity divergence aloft.

Acknowledgements

The authors have greatly benefited from discussion with their colleagues Dr. R.N. Keshavamurty and Mr. S.K. Mishra. Mrs. Leela George gave valuable assistance in computation. We acknoledge our sincere thanks to all of them.

REFERENCE

Awade, S.T. and Asnani, G.C. 1973 Indian J. Met. Geophys. Vol. 24, No. 4. pp 345 - 352.

...

BHEGADE / 26.9.

Table 1 : Observed and fitted zonal wind (kts) along 87.5°E

	5°N		7.5°N		10°N		12.5°N		15°N		17.5°N		20°N		22.5°N		
	Obs.	fitted	Obs.	fitted	Obs.	fitted	Obs.	fitted	Obs.	fitted	Obs.	fitted	Obs.	fitted	Obs.	fitted	Oted
850	17.5	20.0	19.0	20.8	19.3	20.6	18.7	19.4	17.5	17.2	12.0	14.0	77.0	9.8	1.8	4.6	- 1.5
800	16.0	18.4	18.2	19.6	18.5	19.7	18.5	18.8	18.1	16.7	15.7	13.6	11.0	9.3	2.0	4.0	2.4
700	16.0	14.7	17.0	16.7	17.2	17.4	16.9	16.9	15.5	15.1	10.0	12.1	7.5	7.8	0.5	2.3	-4.4
600	11.2	9.8	13.0	12.5	13.4	13.8	13.4	13.8	11.6	12.4	8.0	9.6	4.5	5.5	-2.0	0.0	-6.7
500	4.6	3.5	6.9	6.6	7.2	8.4	7.5	8.8	6.3	7.9	5.0	5.6	0.5	1.9	-6.0	-3.1	-9.5
400	- 4.0	- 5.3	- 2.5	- 2.0	- 1.4	0.0	- 0.2	0.9	- 1.6	0.5	- 3.0	- 0.9	- 6.6	- 3.7	-11.0	-7.7	12.9
300	-24.5	-18.4	-17.8	-15.6	-14.7	-13.6	-13.0	-12.3	-12.9	-11.8	-13.0	-12.1	-14.5	-13.1	-16.5	-14.9	17.4
200	-39.5	-40.1	-40.0	-39.4	-38.5	-38.4	-37.5	-36.9	-35.0	-35.1	-33.0	-32.9	-31.0	-30.3	-28.0	-27.3	23.9
150	-55.5	-57.7	-58.0	-59.5	-58.0	-59.8	-56.8	-58.5	-54.0	-55.7	-51.0	-51.2	-45.0	-45.3	-38.0	-37.7	28.6

Table 2 : Vertical Wind Shear in kts per 100 mb

p mb / Lat. °N	850	800	700	600	500	400	300	200	150
5	2.9	3.3	4.2	5.5	7.4	10.6	16.3	29.0	43.1
7.5	2.1	2.5	3.5	4.9	7.1	10.7	17.4	32.7	49.8
10	1.5	1.9	2.9	4.4	6.7	10.5	17.7	34.5	53.4
12.5	1.5	1.4	2.4	3.9	6.2	10.1	17.4	34.5	54.0
15	0.8	1.2	2.1	3.5	5.7	9.4	16.4	32.8	51.6
17.5	0.7	1.1	1.9	3.1	5.1	8.4	14.7	29.3	46.0
20	0.9	1.2	1.9	2.9	4.5	7.2	12.2	24.0	37.4
22.5	1.2	1.4	1.9	2.7	3.8	5.7	9.1	16.9	25.7
25	1.7	1.9	2.1	2.5	3.1	3.9	5.3	8.1	11.0

Table 3 : Static Stability $\left(S = -\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p}\right)$
(Unit : $10^{-4} m^2 sec^{-2} mb^{-2}$)

Lat. in degree / p in mb	5°N	10°N	15°N	20°N	25°N
850	140	143	148	154	158
700	193	197	205	215	221
500	328	338	354	376	388
300	674	703	756	821	859
200	1065	1135	1265	1422	1517
150	1459	1590	1833	2130	2306

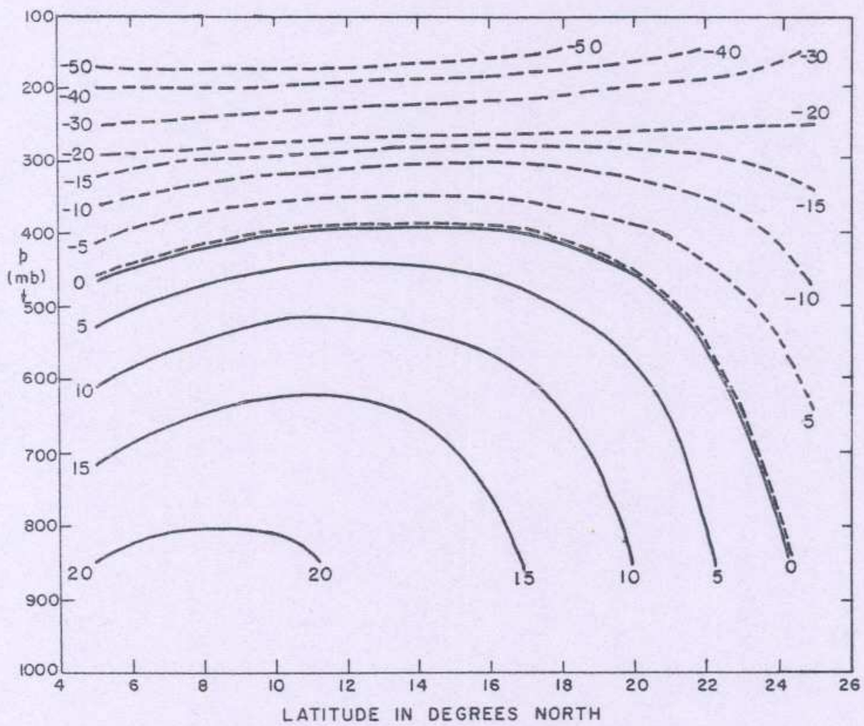

FIG. 1 : VERTICAL MERIDIONAL CROSS-SECTION OF ZONAL WIND (FITTED VALUES) ALONG 87·5°E, +VE VALUES INDICATE WESTERLIES, IN KNOTS.

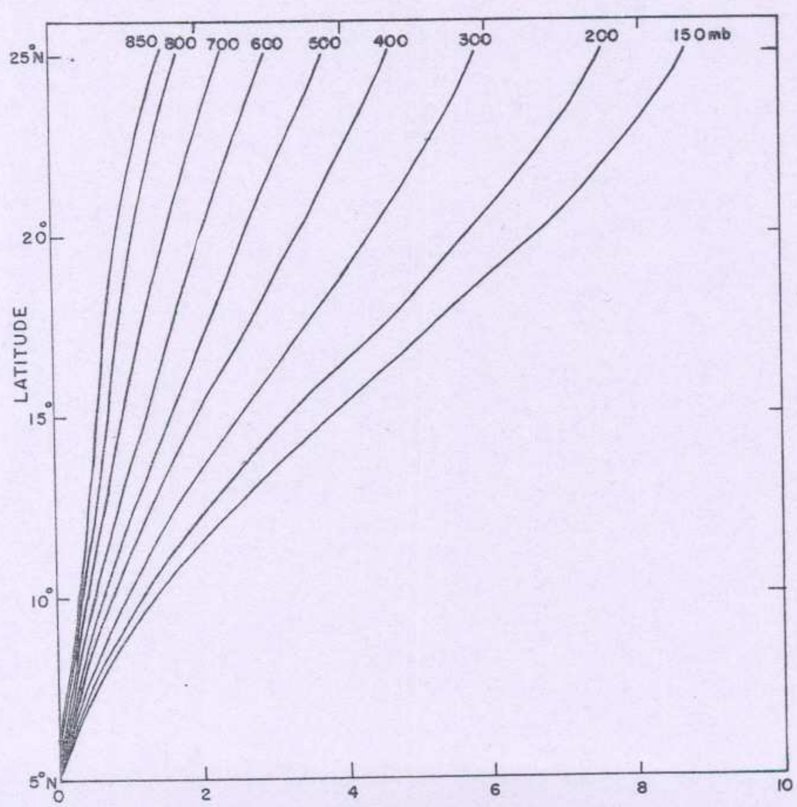

FIG. 2 : LATITUDINAL VARIATION OF TEMPERATURE ALONG 87·5°E AT DIFFERENT ISOBARIC LEVELS

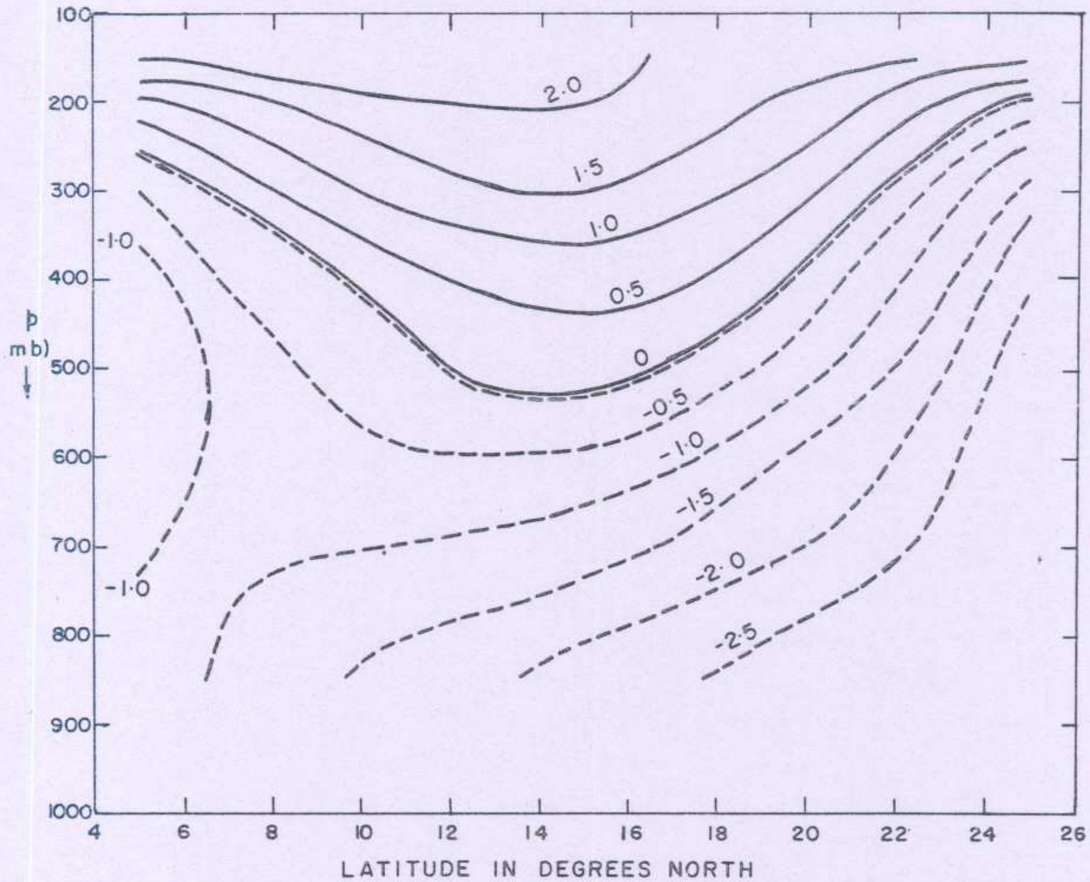

FIG. 3(c) : VERTICAL MERIDIONAL CROSS-SECTION OF $\frac{\partial u}{\partial x}$ ON ISOBARIC SURFACES ALONG 82·5° E (UNITS : 10^{-6} SEC^{-1})

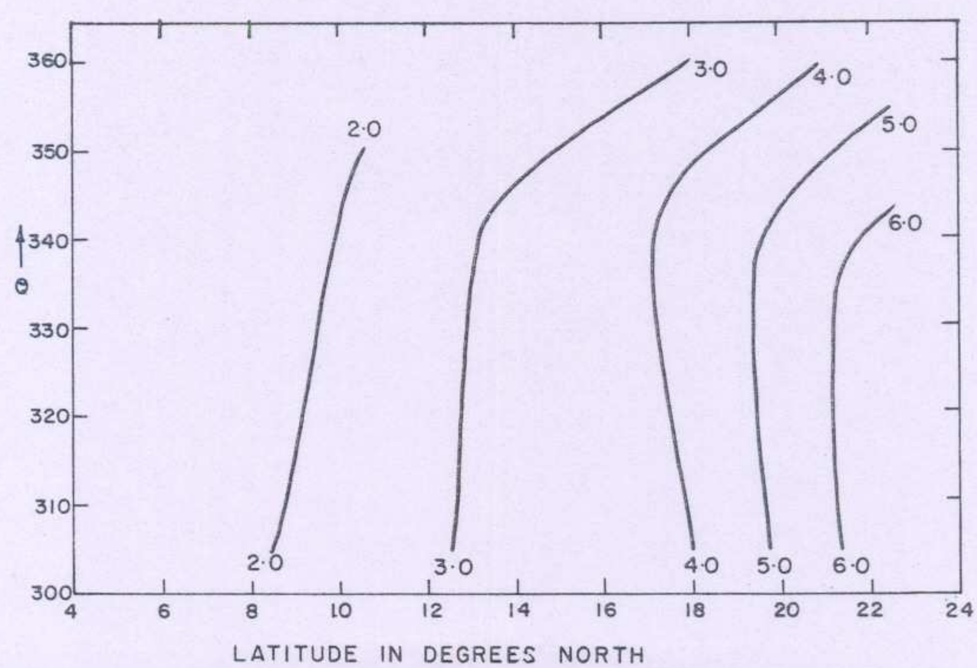

FIG. 4 : VERTICAL CROSS-SECTION OF $(f - \frac{\partial u}{\partial y})$ ON ISENTROPIC SURFACES ALONG 87·5°
(UNITS: 10^{-5} SEC^{-1})